USN

18EE81

Eighth Semester B.E. Degree Examination, June/July 2024
Power System Operation and Control

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Discuss the various operating states of power system with neat block diagram. **(06 Marks)**

b. What is energy control center? Explain the functions of energy control center. **(07 Marks)**

c. List out the objectives of power system control. Explain the various controls involved. **(07 Marks)**

OR

2 a. With a neat diagram, explain the components of RTU (Remote Terminal Unit). **(08 Marks)**

b. What are Intelligent Electronic Devices [IED's]? Explain its functional block diagram. **(07 Marks)**

c. Discuss the classification of SCADA system with neat sketches wherever necessary. **(05 Marks)**

Module-2

3 a. Explain the AVR and ALFC control loops with schematic block diagram. **(07 Marks)**

b. Explain the different modes of Governer operation. **(05 Marks)**

c. Draw the schematic diagram of a steam turbine governing system and explain the functions of various components. **(08 Marks)**

OR

4 a. Obtain the transfer function for the complete ALFC system. **(10 Marks)**

b. Obtain the overall expression of an AGC with PI controller from its relevant block diagram representation of ALFC. **(10 Marks)**

Module-3

5 a. Obtain the state space model of an isolated system with necessary equations. **(10 Marks)**

b. Explain the two area load frequency control with neat block diagram and necessary equations. **(10 Marks)**

OR

6 a. With a schematic block diagram, explain Automatic Voltage Control (AVR). With necessary equations and mathematical models. **(10 Marks)**

b. Explain the decentralized control of AGC. **(04 Marks)**

c. Two generators rated 200 MW and 400 MW are operating in parallel. Their droop characteristics are 4% and 5% respectively from no load to full load. The speed changers are so set that the generators operate at 50 Hz sharing a full load of 600 MW in the ratio of their ratings. If the load reduces to 400 MW, how will it be shared among the generators and what will be the system frequency? **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-4

7 a. Explain briefly the various elements of power system that can generate or absorb reactive epower. **(10 Marks)**

b. Show that the real power flow between two nodes is determined by the transmission angle, and the reactive power flow is determined by the scalar voltage difference between the nodes. **(10 Marks)**

OR

8 a. Explain the different methods of voltage control by reactive power injection. **(10 Marks)**

b. With neat diagram, explain Booster transformers and phase shift transformers used for voltage control. **(06 Marks)**

c. Discuss the process of voltage collapse with a neat sketch. **(04 Marks)**

Module-5

9 a. Explain the security constrained optimal power flow with the help of an example showing various states involved. **(07 Marks)**

b. List out the factors affecting the Power System Security. **(05 Marks)**

c. With a neat flow chart, discuss the process involved in AC power flow security analysis with contingency case selection. **(08 Marks)**

OR

10 a. With neat diagrams and necessary equations, explain:

(i) Generation shift factors

(ii) Line outage distribution factors **(10 Marks)**

b. Explain the linear least square estimation technique used for state estimation in power system with flow chart. **(10 Marks)**

* * * * *

Solutions

1A

12.2 Components of SCADA System

The general configuration is shown in Fig. 12.1[2]. Basically, SCADA systems collect information from the site (field) of the equipment, transfer it to a central computer facility and display the information to the operator to facilitate the control of the entire system from the central control center. In a SCADA system, the geographically dispersed sites contain either a remote terminal unit (RTU), which is a computer, or a programmable logic controller (PLC), which controls local actuators and monitor the sensors. The communication equipment allows transfer of information or data from the RTU/PLC to the central control center which houses a master terminal unit (MTU).The communication could be via telephone, radio, cable or satellite. The software of the SCADA system is programmed to tell the system what to monitor, what are the operating ranges, when to initiate alarms, controls, etc. Further, the system may consist of intelligent electronic devices (IEDs) that are smart sensors, at times combining a sensor, low level intelligent control, a communication system and program memory in one device. The IEDs can communicate directly with the MTU. Other components are the human–machine interface(HMI), also called the man–machine interface (MMI)that allows the operator to monitor the state of a process under control, modify

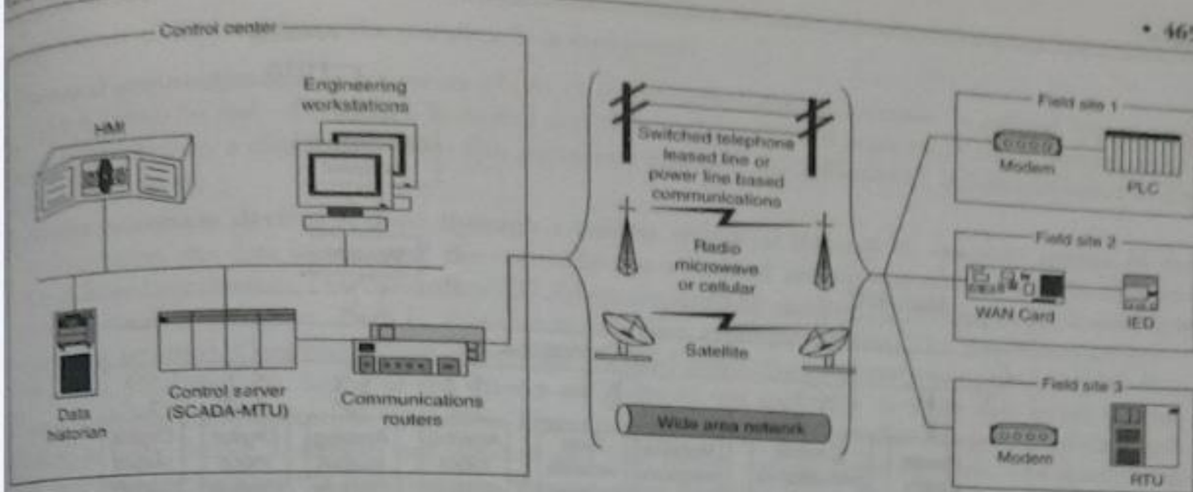


Figure 12.1 General SCADA configuration.

control settings if necessary, and permits the operator to override any automatic control previously set, should an emergency arise. The HMI is also responsible for displays, reports, historical information, status information, etc.

The major components of a SCADA system are thus classified as:

1. Field instrumentation,
2. Remote stations,
3. Communication network,
4. Central monitoring station and
5. Software.

1B

12.7.1 Transducers for Data Acquisition

Transducers are used for data acquisition. They take the measurements and convert it into a convenient form to be used by the system. The main transducers are power transducers, voltage transducers, current transducers and frequency transducers.

1. ***Power transducers:*** They are used for measurement of active and reactive power, both under balanced and un-balanced loading conditions. They are suitable for both incoming and outgoing power. The output of the transducer is load-independent DC, which is proportional to input power. It should have low internal power consumption, should be compact in construction and have galvanic isolation between input and output. These require a low voltage auxiliary power supply.
2. ***Voltage transducers:*** They are used to convert the sinusoidal voltage into a load independent direct current which is proportional to the measured voltage within a prescribed range.

   The output current can be connected directly to the indicating instruments or to the data transmission system. They should have low internal consumption, galvanic isolation between input and output. They do not require auxiliary power.

3. ***Frequency transducers:*** They measure power frequency over a specified range and convert it into an industry standard output signal which is directly proportional to the measured input. These transducers provide an output which is load independent and isolated from the input. The output can be directly connected to display units, recorders or data loggers.
4. ***Current transducers:*** Many types are available. They are meant to accept AC current input and provide a proportional DC current output. Current transducers use current transformers with a ratio from 100:5 to 6000:5. They have a high degree of accuracy.

1C

12.9 Common Communication Channels for SCADA in Power Systems

SCADA in power systems use different communication channels. These are briefly discussed in the following subsections.

12.9.1 Power Line Carrier Communication

The power lines are used to carry the communication. This channel is common, has a simple technique, is easy to maintain and cheap. It is used for speech and data transmission. The speed of data transmission is limited and long distance, point-to-point communication is not easy. Modems are used at the sending as well as at the receiving end to modulate and demodulate power and data, respectively. In this system, since the conductor acts as a medium of transmission there is attenuation in the transmitted signal.

12.9.2 Microwave Communication

The frequency range is from 1 GHz to 1,000 GHz. A choice of 10 GHz would limit the transmission distance to 5 miles. The main advantage is that the data carrying capacity is high due to the large bandwidth and the data are totally protected from noise.

12.9.3 Fiber-Optic Communication

This is becoming very popular in the power sector because of the wide bandwidth and high transmission rate over long distances. It produces no emission outside the cable and is not affected by external radiation, and is hence preferred where security is an issue. Further, it is totally immune to electromagnetic interferences, corrosion and noise.

12.9.4 Satellite Communication

Satellite communications for SCADA networks form a reliable alternative to traditional methods. The benefits include broadcast networks (wherein multiple stations can receive a single message), cost effectiveness when compared to landlines or radio towers, highly reliable with world-wide coverage and easy to integrate with RTUs.

2A

4.4 Priority List Method

This is the simplest unit commitment solution method. It consists of creating a priority list of units. In Example 4.1, we saw that shutting down a unit is more advantageous under certain conditions. When a unit is shut down, the load it carried must be transferred to the other units. By shutting down an inefficient unit, the no-load running cost of the unit is eliminated. Once a unit is shut down, it will have to be started at a later hour when the next peak load approaches (see Fig. 4.3). The saving gained by shutting down the inefficient unit may, at times, be offset by the cost of starting up the unit again when needed.

To determine the optimum shutdown rule for a group of generating units, the units are ordered according to a *priority* rule. A simple method is to prepare the priority list based on the full-load average production cost of each unit. The total production cost for a period is the hourly production costs plus the cost of

Example 4.4 Construct a priority list for three units whose data are given below:

Unit	Full-Load Average Production Cost Rs./MWh	$P_{G_{min}}$ (MW)	$P_{G_{max}}$ (MW)
1	1,000	150	500
2	850	100	500
3	1,040	125	200

Solution

A priority order for these units prepared based on their average production costs is as follows:

Priority	Unit	Rs./MWh
1	2	850
2	1	1,000
3	3	1,040

It is important to note that the unit *with the least priority number is the most economical one*. Hence, it is always committed first. Higher number units are inefficient and uneconomical.

Ignoring start-up cost, the commitment scheme for different combinations is as follows:

Unit Combination	Min MW from Combination	Max MW from Combination
2 (priority 1)	100	500
2 + 1 (priority 1 and 2)	250	1,000
2 + 1 + 3	375	1,200

The commitment rule is simple:

1. $P_{DT} \leq 500$ MW, run unit 2 only (Priority 1)
2. $500 < P_{DT} < 1{,}000$ MW, run units 2 and 1 (Priorities1 and 2)
3. $1{,}000 < P_{DT} < 1{,}200$ MW, run units 2,1 and 3

P_{DT} includes load demand + spinning reserve.

Let us now consider the reverse case. Assume the demand is 1,200 MW. To meet this demand, all the three units need to be committed. Now, unit 3 is shut down only when the demand drops to 1,000 MW. Similarly, when the demand falls below 500 MW, unit 1 is next shutdown. A simple algorithm to implement the priority list method is given as follows:

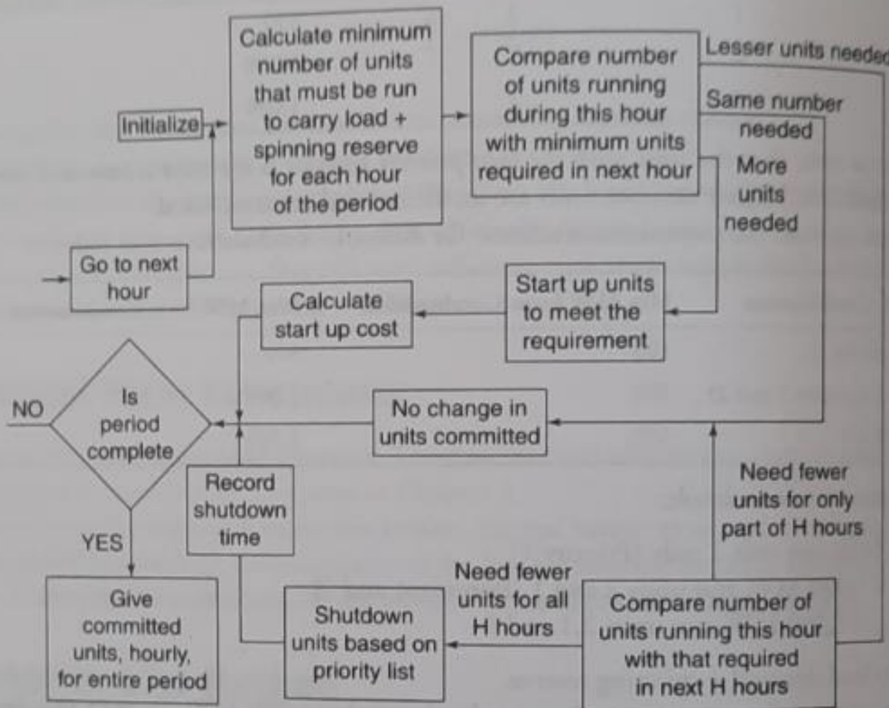
Algorithm Priority List Method

1. Determine the hourly load forecast for next 24 h (or any other period).
2. Prioritize the units based on their production costs and prepare a table based on unit combination to meet required load.
3. For the first hour, determine the minimum number of units necessary to carry the maximum predicted load and the spinning reserve.

4. Compare the number of units running in the present hour with the minimum number required for the next hour.
5. If the number required in the next hour is greater than the number of units in the present hour, start the units according to the priority list.
6. If the minimum number of units required in the next hour is lesser than those running in the present hour, then determine whether dropping the unit with the highest priority number (least efficient) in the present group will leave sufficient generation to supply the load + spinning reserve. If not, do not shut down the unit.
7. Else, determine the number of H hours, before which the unit would be needed again.
8. If H is less than the minimum downtime of the unit, continue with the present commitment.
9. Else, calculate two costs.
   - Sum of the hourly production costs for the next H hours with unit up.
   - Hourly production costs with unit shut down + the start-up cost of the unit (which is the minimum of cooling or banking cost).

   If there is significant saving from shutting down the unit, shut it down.
10. Repeat the procedure hour by hour for the next 24 h.

The flowchart for the algorithm is given in Fig. 4.5.



* The load is dispatched to commited units using economic dispatch algorithm.

** The period is usually between 24 and 120 hours (1–5 days).

Figure 4.5 Flowchart for priority list method for UC.

The priority list may be reordered as and when necessary if

1. Some units are unavailable due to breakdowns/maintenance.
2. Spinning reserve requirement is changed.
3. Running of some units for area protection to improve reliability is mandatory.

2B

4.5.3 Forward DP Approach

The DP-SC algorithm described can be used to develop a forward DP algorithm for h time periods from the present time period. The algorithm takes into account the start-up cost also. We use the following nomenclature in the algorithm:

1. **K:** Time period (we assume the total time is divided into periods for scheduling. A 24 h period could be divided into 24 periods each of 1 h duration).
2. **C:** Combination of units.
3. **X:** Number of states which have to be searched in each time period. If we enumerate all combinations then with n units $X = 2^n - 1$. If we use priority listing $X = n$.
4. **Y:** Number of paths or strategies saved at each step (time period) from the X states which have been searched. In forward dynamic programming, a *strategy is the transition or path from one state at a given time period (or hour) to a state at the next time period.* In determining Y, we retain only some of the feasible states.
5. ***(K, C):*** A state at K^{th} hour with combination C.
6. ***F_{cost}(K, C):*** Production cost for state (K, C).
7. ***S_{cost}(K–1, L:K, C):*** Transition cost from state $(K-1, L)$ to state (K, C).

This would include start-up and shutdown costs. The forward DP algorithm is recursive and computes the minimum cost at hour K with combination C using the formula

With the above nomenclature, we arrive at a recursive formula to calculate the cost at a period K with a combination C as follows:

$$F_{cost}(K,C) = \min_{\{L\}}[P_{cost}(K,C) + S_{cost}(K-1, L : K, C) + F_{cost}(K-1, L)] \tag{4.9}$$

Here $\{L\}$ is the set of Y states retained in the step. As mentioned earlier, with complete enumeration both X and Y will be $2^n - 1$. Reducing X means, we restrict our search space and do not search all the feasible solutions. Reducing Y means, out of the X states searched, we discard schedules with higher costs and in each time period retain only Y strategies or paths. Hence, we are not assured of an optimal solution by reducing the search range (X value) or strategies (Y value). Different values of X and Y will obviously lead to different schedules since the search space is different.

The flowchart for forward DP is shown in Fig. 4.7.

To illustrate the method, let us consider a simple problem. Assume that the units of Example 4.7 have to be scheduled for next 4 h. The load pattern is as shown in Fig. 4.8.

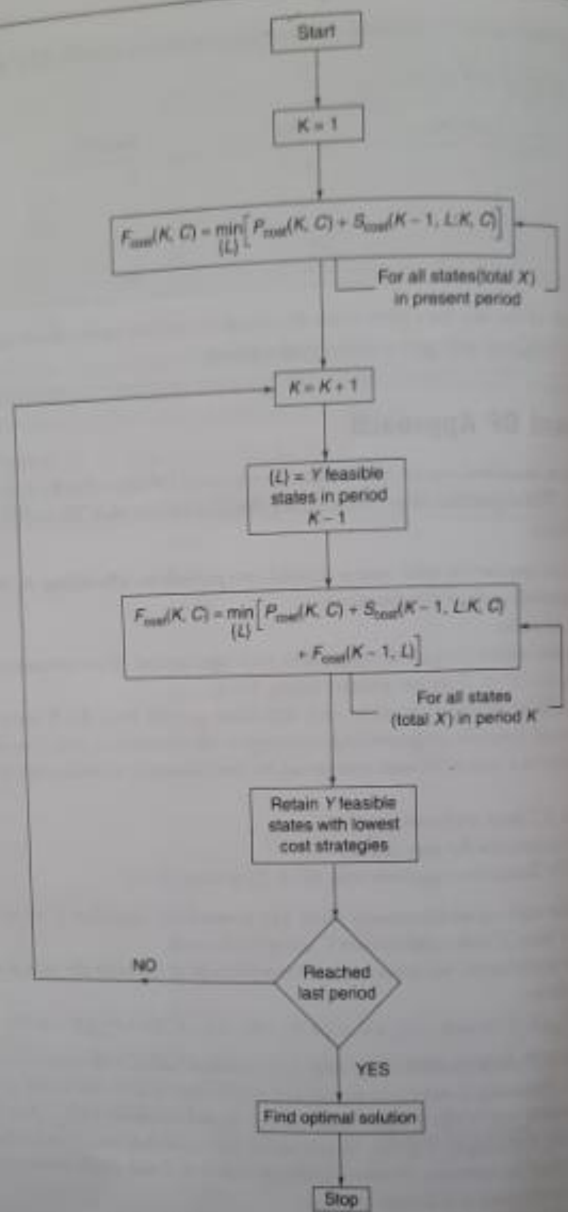


Figure 4.7 Flowchart for forward DP method.

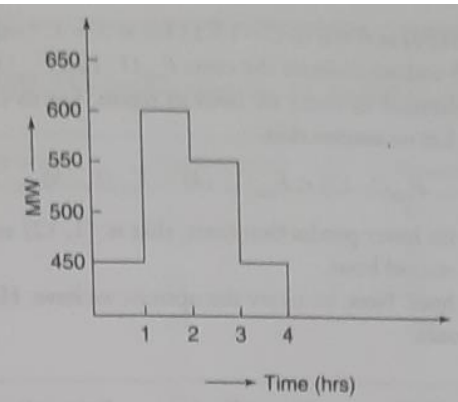


Figure 4.8 Power demand for 4 h.

We will use the data of Tables 4.9 and 4.10. Let us assume that currently the system is in state 12, that is, units 3, 2 are on ($C = 0110$). Therefore, initial state = = 0110. The algorithm in determining the schedule using forward DP works as follows:

Step 1: At $K = 0$ (present hour) $C = 12$ (0110; units 3, 2 are ON).

Step 2: The load in the next hour is 450 MW. According to priority ordering we have four states: 5, 12, 14, 15 (Table 4.11). State 5 is not feasible since the capacity in this state is only 300 MW. So the feasible states are 12, 14 and 15. Note that with complete enumeration we have 15 states and this has been reduced to a search space of only three states. For this load, X(number of states to be searched) = 3. So to move from $K = 0$ to $K = 1$, the paths are as follows:

State at $K = 0$	State at $K = 1$
12	12
12	14
12	15

Using Eq. (4.9), we now evaluate the cost at $K = 1$ as follows:

1. For **path 12–12,** we have
   $F_{cost}(1, 12) = \min\{12\}[P_{cost}(1, 12) + S_{cost}(0, 12\text{: }1, 12)]$
   Where $\boldsymbol{F_{cost}(1, 12)}$ is the cost to arrive at state $K = 1$ with combination 12(0110)
   $\boldsymbol{P_{cost}(1, 12)}$ is the production cost for state (1, 12). This is calculated using Economic dispatch coordination equations to allocate 450 MW between units 3 and 2.
   $\boldsymbol{S_{cost}(0, 12; 1, 12)}$ is the cost to move from state (0, 12) to (1, 12).
   In this case, since no new units are committed or decommitted this cost is zero.
2. For **path 12–14** we calculate

$$F_{cost}(1, 14) = \min_{\{12\}}[P_{cost}(1, 14) + S_{cost}(0, 12\text{: }1, 14)]$$

   Here we move from $C = 12(0110)$ at $K = 0$ to 14(1110) at $K = 1$. Since unit 1 is newly committed we add the start-up cost of unit 1 in calculating $S_{cost}(0, 12\text{: }1, 14)$
3. For **path 12–15** we calculate

$$F_{cost}(1, 15) = \min_{\{12\}}[P_{cost}(1, 15) + S_{cost}(0, 12\text{: }1, 15)]$$

Here we move from $C = 12(0110)$ at $K = 0$ to $C = 15(1111)$ at $K = 1$. Two units (1 and 4) are comm...
Thus in step 1 we have $X = 3$ and we evaluate the costs $F_{cost}(1, 12)$, $F_{cost}(1, 14)$ and $F_{cost}(1, 15)$.
We now decide what is the number of states we need to retain. Let us choose to retain two states ...
least costs. Therefore, $Y = 2$. Let us assume that

$$F_{cost}(1, 12) < F_{cost}(1, 14) < F_{cost}(1, 15)$$

So we retain the two states with lower production costs, that is, (1, 12) and (1, 14) and discard the ...
(1,15). We now move to the second hour.
Step 3: We move to the next hour. Now, let us see the options we have. Here the load is 600 MW. ...
feasible states are 14 and 15 only.

State at $K = 0$	State at $K = 1$	State at $K = 2$
12	12	14
12	12	15
12	14	14
12	14	15

A graphical representation may be drawn, as shown in Fig. 4.9, to depict the paths.
For each of the paths F_{cost} is calculated. We again retain the two paths with lowest cost. Let us assume ...
paths 12–12–15 and 12–14–15 are with lowest costs. We now move to the next hour.
Step 4: $K = 3$. Here the load is 550 MW. So the feasible states are 12, 14 and 15. The options we ...
are as follows.

For each of the paths F_{cost} is calculated. We again retain the two paths with lowest cost. Let us assume paths 12–12–15 and 12–14–15 are with lowest costs. We now move to the next hour.

Step 4: $K = 3$. Here the load is 550 MW. So the feasible states are 12, 14 and 15. The options we have are as follows.

State at $K = 0$	State at $K = 1$	State at $K = 2$	State at $K = 3$
12	12	15	12
12	12	15	14
12	12	15	15
12	14	15	12
12	14	15	14
12	14	15	15

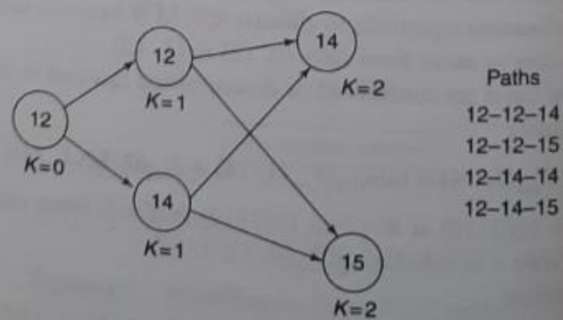


Figure 4.9 Transition paths.

We evaluate the F_{cost} for each path and retain the 2 with lowest costs. Assume that the two paths are 12–12–15–14 and 12–14–15–12.

Step 5: $K = 4$ and load is 450 MW. The feasible states are 12, 14 and 15. So now we have the paths,

State at $K = 0$	State at $K = 1$	State at $K = 2$	State at $K = 3$	State at $K = 4$
12	12	15	14	12
12	12	15	14	14
12	12	15	14	15
12	14	15	12	12
12	14	15	12	14
12	14	15	12	15

Now we pick up the one with lowest cost. Assume it to be 12–12–15–14–12. This is the transition path from $K = 0$ to $K = 4$. We can tabulate the UC schedule.

K	Load (MW)	State	UC	Units to be Committed
0	–	12	–	3, 2
1	450	12	0110	3, 2
2	600	15	1111	3, 2, 1, 4
3	550	14	1110	3, 2, 1
4	450	12	0110	3, 2

If we compare this approach of scheduling to the previous method described where the schedule depended on the load (Table 4.8), we see that here the schedule is based on the minimum cost transition path from the present time to the end of schedule. It *depends on the load curve and not just on the load.* In Table 4.8, the unit commitment is based on the load at that particular hour.

There are many other ways of applying the DP technique to unit commitment problem!!

3A

5.4 Discrete Time Interval Method – A General Algorithm for Hydrothermal Scheduling

Let us consider the system of Fig. 5.2. The problem here is to determine the water discharge at various intervals, to minimize the cost of thermal generation and to meet the operating constraints of the hydro plant.

5.4.1 Mathematical Formulation of Objective Function

Let us consider a time period of operation, T_{max}. We make the following assumptions:

1. Storage of reservoir is specified at the beginning and end of the period.
2. Water inflow is known.
3. Load demand is known for the entire period.

We now have to determine the rate of water discharge to minimize the cost of thermal generation. Mathematically

$$\text{Minimize } F_T = \int_0^{T_{max}} F'(P_{GT}(t))dt \tag{5.20}$$

where F_T is the total cost.

5.4.2 Operational Constraints

1. ***Constraint of power balance:*** At any time t, the active power has to be balanced between generation and demand. Mathematically

$$P_{GT}(t) + P_{GH}(t) - P_{LOSS}(t) - P_D(t) = 0, \quad t \in (0, T_{max}) \tag{5.21}$$

2. ***Constraint of water availability:***

$$S(T)_{max} - S(0) - \int_0^{T_{max}} Q_m(t)dt + \int_0^{T_{max}} Q_0(t)\, dt = 0 \tag{5.22}$$

where $S(T_{max})$ is storage at the end of period (m^3), $S(0)$ is storage at the beginning of period (m^3).

3. ***Constraint on hydropower generated:***

$$P_{GH}(t) = f(S(t), Q_0(t)) \tag{5.23}$$

5.4.3 Discretization

Let us divide the total time period T_{max} into N sub-intervals, each of time ΔT. Let all variables remain fixe[d] within the subintervals. We now discretize Eqs. (5.20) to (5.23) as follows:

1. Minimize $\sum_{k=1}^{N} F(P_{GT}^{k})$ (5.24)

   under the constraints,

2. $P_{GT}^{k} + P_{GH}^{k} - P_{LOSS}^{k} - P_{D}^{k} = 0$ (5.25)

   $P_{LOSS}^{k} = B_{TT}(P_{GT}^{k})^2 + 2B_{TH}P_{GT}^{k}P_{GH}^{k} + 2B_{HH}(P_{GH}^{k})^2$

   where B_{TT}, B_{HH} and B_{TH} are the loss coefficients as discussed in Chapter 3.

3. $S^{k} - S^{k-1} - Q_{in}^{k}\Delta T + Q_{o}^{k}\Delta T = 0$ Dividing by ΔT we get

   $$S^{*k} - S^{*k-1} - Q_{in}^{k} + Q_{o}^{k} = 0 \quad (5.26)$$

   where $S^{*k} = S^{k}/\Delta T$ (unit would be m^3/s or m^3/h)

4. The hydropower generated is given by $P_{GH}^{k} = 9.81 \times 10^{-3} h_{av}(Q_{o}^{k} - \rho)$ MW (5.27)

   where h_{av}^{k} = average head in k^{th} interval

   $$= h_{o}[1 + 0.5e\,(S^{*k} + S^{*k-1})] \quad (5.28)$$

   h_{o} = water head corresponding to dead storage

   e = head correction factor = $\dfrac{\Delta T}{Ah_{o}}$

   ρ = dead storage

   A = area of cross section of reservoir.

   Substituting Eq. (5.28) into Eq. (5.27), we get

   $$P_{GH}^{k} = 9.81 \times 10^{-3} h_{o}[1 + 0.5e(S^{*k} + S^{*k-1})][Q_{o}^{k} - \rho]$$

   $$= h_{o}^{*}[1 + 0.5e(S^{*k} + S^{*k-1})][Q_{o}^{k} - \rho] \quad (5.29)$$

   where $h_{o}^{*} = 9.81 \times 10^{-3} h_{o}$.

5.4.4 Dependent Variables

From Eq. (5.26) we can obtain the sum of all equations for values of $k = 1, \ldots\ldots, N$. We get

$$S^{*N} - S^{*O} - \sum_{k=1}^{N} Q_{in}^{k} + \sum_{k=1}^{N} Q_{o}^{k} = 0$$

From Eq. (5.30) we can see that only $(N - 1)$ independent values of Q_o can be specified and one of discharges becomes a dependent variable. Let us specify $Q_o^2, Q_o^3, \ldots, Q_o^N$ (independent variables) and Q_o^1 be the dependent variable. From Eq. (5.30)

$$Q_O^1 = S^{*O} - S^{*N} + \sum_{k=1}^{N} Q_{in}^{k} - \sum_{k=2}^{N} Q_{o}^{k}$$

In the second summation term in Eq. (5.31) note that the summation is from 2 to N. The other dependent variables are P_{GT}^{k}, $k = 1, \ldots\ldots, N$, P_{GH}^{k}, $k = 1, \ldots\ldots, N$ and S^{*k}.

5.4.5 Lagrange Function

A Lagrangian function £ is formulated by augmenting the cost function with the constraints using Lagrange multipliers

$$£ = \sum_{k} [F(P_{GT}^{k}) - \lambda_1^k (P_{GT}^{k} + P_{GH}^{k} - P_{LOSS}^{k} - P_{D}^{k})$$

$$+ \lambda_2^k (S^{*k} - S^{*k-1} - Q_{in}^{k} + Q_{o}^{k})$$

$$+ \lambda_3^k [P_{GH}^{k} - h_o^{*}(1 + 0.5e(S^{*k} + S^{*k-1}))][Q_o^k - \rho]]$$

We equate the partial derivatives of the Lagrangian with respect to the dependent variables to zero and compute the Lagrange multipliers

$$\frac{\partial £}{\partial P_{GT}^{k}} = \frac{dF(P_{GT}^{k})}{dP_{GT}^{k}} - \lambda_1^k \left[1 - \frac{\partial P_{LOSS}^{k}}{\partial P_{GT}^{k}}\right] = 0$$

$$\frac{\partial £}{\partial P_{GH}^{k}} = \lambda_3^k - \lambda_1^k \left[1 - \frac{\partial P_{LOSS}^{k}}{\partial P_{GH}^{k}}\right] = 0$$

$$\left(\frac{\partial £}{\partial S^{*k}}\right)_{\substack{k \neq 0 \\ k \neq N}} = \lambda_2^k - \lambda_2^{k+1} - \lambda_3^k [h_o^{*} 0.5e(Q_o^k - \rho)] - \lambda_3^{k+1} [h_o^{*} 0.5e(Q_o^{k+1} - \rho)]$$

We need to compute $\frac{\partial £}{\partial Q_o^1}$. From Eq. (5.26)

$$S^{*1} = S^{*o} + Q_{in}^1 - Q_o^1$$

In Eq. (5.32), we consider the terms on RHS with $k = 1$ and substitute for S^{*1}. We obtain

$$£ = F(P_{GT}^1) - \lambda_1^1 (P_{GT}^1 + P_{GH}^1 - P_{LOSS}^1 - P_D^1) + \lambda_2^1 (S^{*1} - S^{*o} - Q_{in}^1 + Q_o^1)$$

$$+ \lambda_3^1 (P_{GH}^1 - h_o^{*} (1 + 0.5e(S^{*1} + S^{*o}))[Q_o^1 - \rho]]$$

Substituting for S^{*1} we obtain

$$\frac{\partial £}{\partial Q_o^1} = \lambda_2^1 - \lambda_3^1 - h_o^{*}[1 + 0.5e(2S^{*o} + Q_{in}^1 - 2Q_o^1 + \rho)] = 0$$

The gradient vector is the partial derivative of the Lagrangian with respect to the independent variables, which in this case are Q_o^k.

$$\left[\frac{\partial £}{\partial Q_o^k}\right]_{k\neq 1} = \lambda_2^1 - \lambda_3^k h_o^* [1 + 0.5e(2S^{*k-1} + Q_m^k - 2Q_o^k - \rho)] \qquad (5.38)$$

The optimal solution is reached when the magnitude of the gradient vector is $\leq \epsilon_1$.

5.4.6 Algorithm

1. Assume Q_o^k, $k = 2, \ldots\ldots, N$.
2. Calculate S^{*k}, P_{GH}^k, P_{GT}^k and Q_o^1 from Eqs. (5.26), (5.27), (5.25) and (5.31), respectively.
3. Obtain λ_1^k, λ_3^k, λ_2^k $(k \neq 1)$ and λ_2^1 from Eqs. (5.33), (5.34), (5.35) and (5.37), respectively.
4. Obtain the gradient vector from Eq. (5.38). If it is $\leq \epsilon$, stop.
5. $Q_o^k\,(\text{new}) = Q_o^k\,(\text{old}) - \alpha\left(\dfrac{\partial £}{\partial Q_o^k}\right) k \neq 1.$ (5.39)

where α is a positive scalar. Go to step 2.

During any iteration, if any of the control variables violates their limits, then the variables are set to the violated limit.

3B

5.5 Short-Term Hydrothermal Scheduling Using γ–λ Iterations

In this method, the mathematical model is built slightly differently in terms of constraints. The problem is defined as follows:

Minimize
$$F_T = \sum_{k=1}^{N} h^k F(P_{GT}^k) \qquad (5.4$$

such that
$$\sum_{k=1}^{N} Q_o^k h^k = Q_{total} \qquad (5.4$$

where Q_{total} is the total volume of water available for discharge

and
$$P_D^k + P_{LOSS}^k - P_{GT}^k - P_{GH}^k = 0 \qquad (5.4$$

Again
$$\sum_{k=1}^{N} h^k = T_{max} \qquad (5.4$$

We assume a constant head operation. The discharge Q_o^k depends on P_{GH}^k. The Lagrangian function is given by

$$£ = \sum_{k=1}^{N} h^k F(P_{GT}^k) + \lambda^k (P_D^k + P_{LOSS}^k - P_{GT}^k - P_{GH}^k) + \gamma\left(\sum_{k=1}^{N} h^k Q_o^k (P_{GH}^k) - Q_{total}\right) \qquad (5.4$$

The coordination equations are given by

$$\frac{\partial £}{\partial P_{GT}^k} = h^k \frac{dF(P_{GT}^k)}{dP_{GT}^k} + \lambda^k \frac{\partial P_{LOSS}^k}{\partial P_{GT}^k} = \lambda^k \qquad (5.4$$

$$\frac{\partial \pounds}{\partial P_{GH}^k} = \gamma h^k \frac{dQ_o^k(P_{GH}^k)}{dP_{GH}^k} + \lambda^k \frac{\partial P_{LOSS}^k}{\partial P_{GH}^k} = \lambda^k \tag{5.45b}$$

It's noteworthy to mention that γ converts the incremental water rate to the incremental plant cost. It is constant over the iterations, unless the reservoir storage limits are violated. If losses are neglected, the coordination equation become

$$h^k \frac{dF(P_{GT}^k)}{dP_{GT}^k} = \lambda^k \tag{5.46}$$

$$\gamma h^k \frac{dQ_o^k(P_{GH}^k)}{dP_{GH}^k} = \lambda^k \tag{5.47}$$

5.5.1 γ–λ Iterative Algorithm for Short-Term Hydrothermal Scheduling

We need to solve the coordination equations. For this reason, we need to compute the following:

1. $\frac{dF(P_{GT}^k)}{dP_{GT}^k}$: The general cost function of a thermal plant is given by

$$F(P_{GT}^k) = a_T + b_T P_{GT} + c_T P_{GT}^2 \tag{5.48}$$

This is the quadratic cost function we have used in Chapter 3.

$$\frac{dF(P_{GT}^k)}{dP_{GT}^k} = b_T + 2c_T P_{GT} \tag{5.49}$$

2. $\frac{dQ_o^k(P_{GH}^k)}{dP_{GH}^k}$: The discharge Q_o^k is a function of P_{GH}^k. It is given by

$$Q_o^k = a_H + b_H P_{GH} \text{ (for generation below some limit)}$$

$$Q_o^k = a_H + b_H P_{GH} + c_H P_{HG}^2 \text{ (for generation above a limit)}$$

The characteristic is as shown in Fig. 5.4.

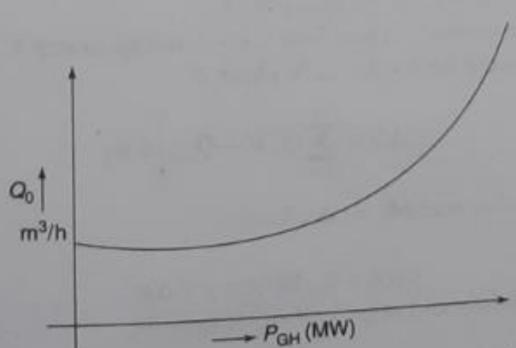


Figure 5.4 Input–output characteristics of hydel plant.

$$\frac{dQ_o^k(P_{GH}^k)}{dP_{GH}^k} = b_H \text{ (if 5.50 is used)}$$

$$\frac{dQ_o^k(P_{GH}^k)}{dP_{GH}^k} = b_H + 2c_H\, P_{GH} \text{ (if 5.51 is used)}$$

3. The loss is given by

$$P_{LOSS}^k = B_{TT}(P_{GT}^k)^2 + 2B_{TH}P_{GT}^k P_{GH}^k + B_{HH}(P_{GH}^k)^2$$

$$\frac{\partial P_{LOSS}^k}{\partial P_{GT}^k} = 2B_{TT}P_{GT}^k + 2B_{TH}P_{GH}$$

$$\frac{\partial P_{LOSS}^k}{\partial P_{GH}^k} = 2B_{HH}P_{GH}^k + 2B_{TH}P_{GT}$$

Algorithm

1. Choose $\in_1$, tolerance for power mismatch, and $\in_2$, tolerance for water discharge.
2. Select initial values of γ and λ^k. Assume initially all generations are zero (or some arbitrary value
3. Set $k = 1$, for the first time interval.
4. Calculate P_{GT}^k using the coordination Eqs. (5.44). Substitute Eqs. (5.49) and (5.53a) into Eq. to get

$$h^k(b_T + 2c_T P_{GT}^k) + \lambda^k(2B_{TT}P_{GT}^k + 2B_{TH}P_{GH}^k) = \lambda^k$$

$$P_{GT}^k = \frac{\lambda^k - h^k b_T - 2\lambda^k B_{TH} P_{GH}^k}{2h^k c_T + 2\lambda^k B_{TT}}$$

5. Calculate P_{GH}^k from Eq. (5.45). We substitute Eqs. (5.50) and (5.53b) to get

$$\gamma h^k (b_H) + \lambda^k (2B_{HH} P_{GH}^k + 2B_{TH} P_{GT}^k) = \lambda^k$$

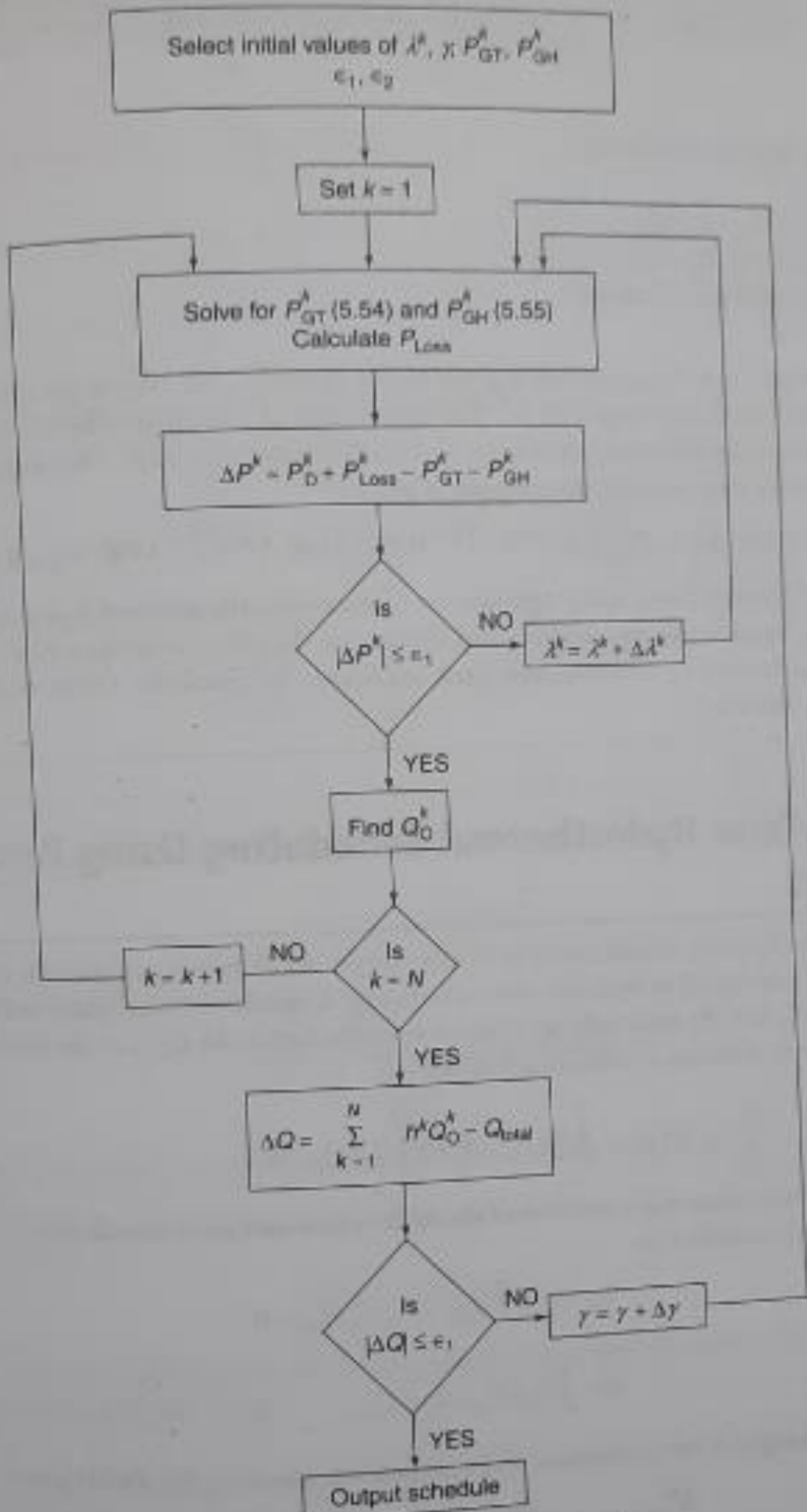
$$P_{GH}^k = \frac{\lambda^k - \gamma h^k b_H - 2\lambda^k B_{TH} P_{GT}^k}{2\lambda^k B_{HH}}$$

6. Calculate
$$\Delta P^k = P_D^k + P_{LOSS}^k - P_{GT}^k - P_{GH}^k$$
This gives the power deviation. If $|\Delta P^k| > \epsilon_1$, then let $\lambda^k = \lambda^k + \Delta\lambda^k$. Choose a positive value of $\Delta\lambda^k$ if $\Delta P^k > 0$ and a negative value if $\Delta P^k < 0$. Go to step 4.
7. If $|\Delta P^k| \le \epsilon_1$, calculate Q_o^k using Eqs. (5.50) or (5.51).
8. Check if $k = N$ (last time interval). If not, let $k = k + 1$ and go to step 4.
9. Once Q_o^k has been computed for $k = 1, \ldots, N$, check if
$$\Delta Q = \left| \sum_{k=1}^{N} Q_o^k h^k - Q_{total} \right| \le \epsilon_2$$
where Q_{total} is the total value available for discharge.
10. If yes stop.
11. If
$$|\Delta Q| > \epsilon_2, \text{ let } \gamma = \gamma + \Delta\gamma$$
$\Delta\gamma$ is positive if ΔQ is positive and negative if ΔQ is negative.
12. Go to step 4.

We continue the loop iterations for $\gamma - \lambda$, until ΔP and ΔQ have converged.

5.5.2 Flowchart for γ–λ Iterations

The flowchart for the γ–λ iterative technique for hydrothermal scheduling is given in Fig. 5.5.



4A

6.4 Functions of AGC

In a power system the loads and losses are sensitive to frequency. If a generating unit is tripped or the load on the system is increased, the power mismatch is initially compensated by extracting the kinetic energy from system inertial storage causing a decline in system frequency. As the frequency decreases, the power taken by the loads also decreases. Equilibrium in larger systems is generally obtained when the reduction in frequency sensitive load balances the output of the tripped generator or the load increase at the new frequency. If equilibrium is reached it is in less than 2 s.

If the mismatch is large, then the governor action has to increase the generation of the units such that equilibrium is reached, when the reduction in the power taken by the loads plus the increase in generation makes up for the mismatch. Such equilibrium is reached in 10–15 s after tripping of a unit or connection of additional load. The main requirement of the AGC is to ensure the following:

1. The frequency of the various bus voltages are maintained at the scheduled frequency.
2. The tie-line power flows are maintained at the scheduled levels.
3. The total power is shared by all generators economically (economic dispatch).

The first two functions are realized using the ALFC, whereas the third has been extensively dealt with in Chapter 3. Apart from this, modern AGC strategies[2] include many more functions. Some of them are listed here.

1. Yield a generation trend acceptably matching the trend required to serve the changing load at the scheduled frequency, over the selected time frame.
2. Schedule generation to accumulate lower fuel cost over the selected time frame, which includes recognizing undesirable generation ranges in different units and avoiding sustained operation in these ranges.
3. Maintain a sufficient level of reserved control range and sufficient level of control rate.
4. Operate the system with higher security margins.
5. Provide timely recommendations for changing of outputs of units which are manually controlled.
6. Provide meaningful alarms such as display in control center for deviation from desired generation, unit not responding to AGC control signal, anticipated future generation, etc.

The design of AGC system depends on the way the units respond to AGC signals. The response characteristics of units vary widely and depend on many factors such as:

1. Type of generating unit: fossil-fired, nuclear hydro, combined cycle, etc.
2. Type of fuel used: coal, oil, uranium, gas, etc.
3. Type of plant control.
4. Type of plant: once-through boiler, drum-type boiler, pressurized-water nuclear reactor, pumped storage hydro, etc.
5. Operating point of units.
6. Manual control by operators.

In multi-area control, tie-line power deviation dictates the AGC control. This is dealt with in Chapter 7. The speed governors play a vital role in the primary control of the frequency. This is discussed in detail in the next section.

4B

Steam Turbine Governing System

Consider the governing system for a steam turbine as shown in Fig. 6.2.

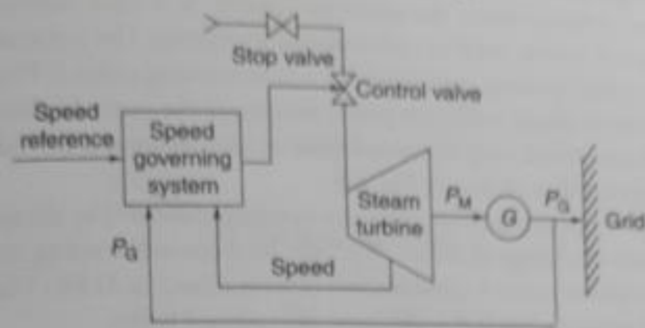


Figure 6.2 Steam turbine governing scheme.

In the operating range, the steam flow through the control valve is proportional to the valve opening. When the valve opening changes, the steam flow to the turbine changes, changing the mechanical power output of the turbine and hence the electrical power of the generator. The rate of speed change depends on the inertia of the entire rotor system. When the turbine-generator unit is being started, the governing system controls the speed by regulating the steam flow. After the unit has been synchronized to the grid, the governor increases the output to load the unit. Referring to the figure, we can see that the valve output can be changed by changing the reference input or by a change in the speed (reflected in the change in frequency) with the reference speed remaining the same. This is the primary regulation or simply governor control. The *secondary regulation* changes the reference setting by using the load-frequency control.

Conventional Governor

The conventional governor is shown in Fig. 6.3.

The major components are discussed below.

Fly ball speed governor: This is a mechanical device, which is speed sensitive and directly adjusts the valve opening via the linkage mechanism. It senses the change in speed or power output and appropriately initiates valve opening or closing.

Linkage mechanism: This transforms the fly ball movement to the turbine valve, through a hydraulic amplifier and provides a feedback from the turbine valve movement.

Hydraulic amplifiers: It is a hydraulic servomotor interposed between the governor and the valve to build mechanical forces strong enough to operate the steam valves or water gates.

Speed changer: It is used to provide a steady-state power output setting for the turbines.

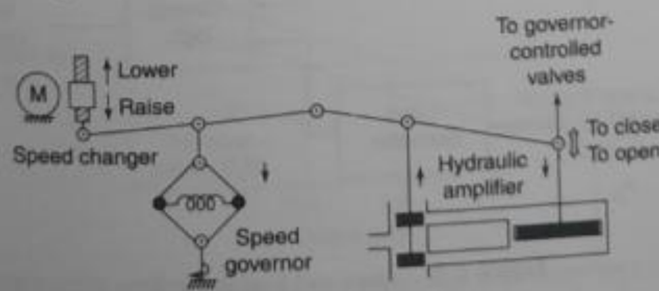


Figure 6.3 Conventional governor.

Electronic Hydraulic Governing System

The electronic hydraulic governing system is shown in Fig. 6.4.

The electronic sensing element senses the speed and power. It is used instead of the fly ball gover... which is a mechanical speed sensor, used in conventional governors. The processing element processes the speed error to obtain the valve opening command. This is done using either P, PI or PID controllers. A pri... mary amplification is done to obtain sufficient power to operate the control valves.

With a pure governor controller, only the speed error is used to control the valve opening. With ALF... load control is also incorporated as shown in Fig. 6.5.

With the speed controller operating alone, the valve opening is decided by the speed error and the deriva... tive of the speed error (rate of change of error). With the load controller acting alone, the valve opening... determined by the output power error. A combination of both is used in ALFC. The function of the hydrau... lic amplifier and speed changer in Fig. 6.4 is the same as explained before.

4C

6.5.2 Modes of Governor Operation

In defining the modes of the governor operation, the normal speed is considered as 100% speed and the
load as 100% load. There are two modes of operation.

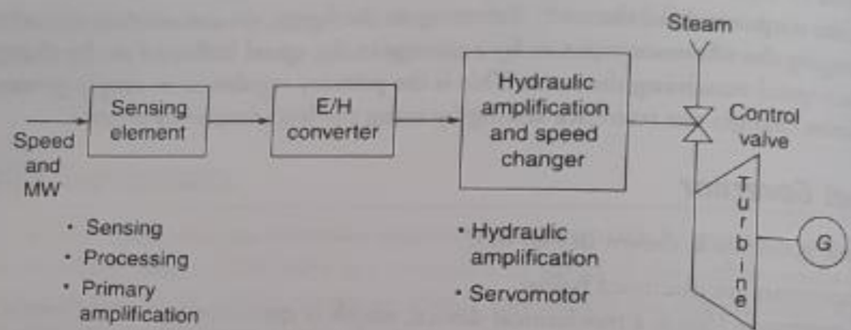
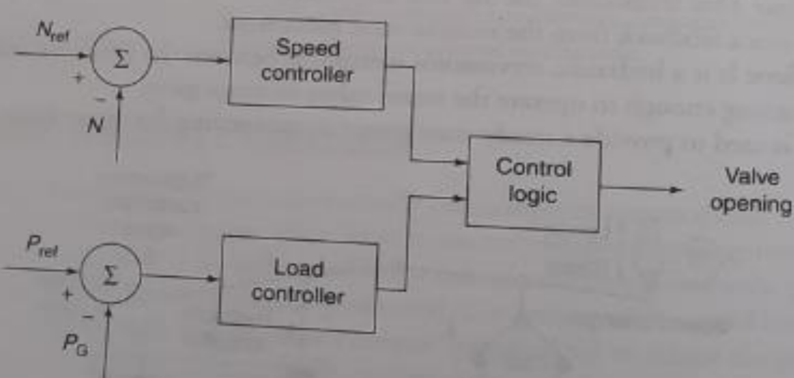


Figure 6.4 Electronic hydraulic governing system.



Isochronous Operation

The governor maintains a constant speed from no load to full load as shown in Fig. 6.6.

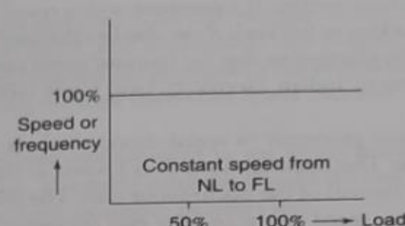


Figure 6.6 Isochronous governor curve.

A prime mover and a generator, operating in the isochronous mode can maintain the desired output frequency, regardless of load changes as long as the prime mover capacity is not exceeded. This mode is normally used in isolated systems or when one generator is required to respond to the load changes.

Droop Mode Operation

Speed droop is a decrease in speed or frequency proportional to the load as shown in Fig. 6.7(a).

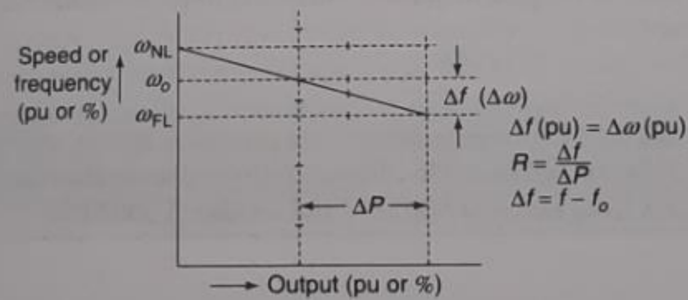


Figure 6.7(a) Speed droop curve.

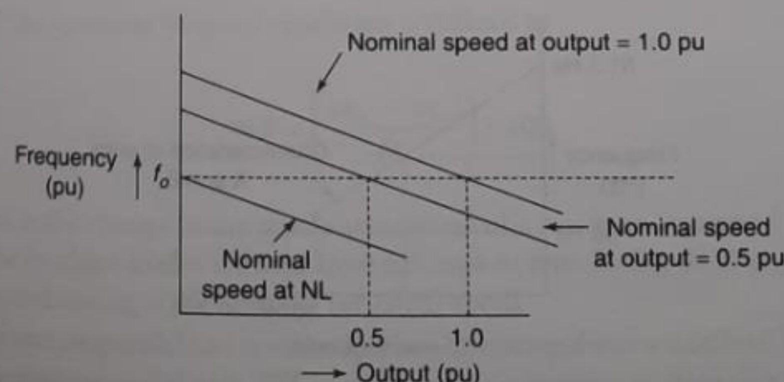


Figure 6.7(b) Speed droop curve with change in governor set point.

The steady-state speed regulation R is given by

$$R = \frac{\Delta\omega}{\Delta P}$$

It is the slope of the speed droop characteristic. If a governor with a speed droop is connected to an isolated load, the speed would drop from no load to full load. R can also be calculated as $\Delta f / \Delta P$, since in pu, $\Delta f = \Delta\omega$. The set point of the governor can be changed so that the nominal speed can be obtained at different outputs as shown in Fig. 6.7(b). According to Indian Electricity Grid Code (IEGC), the droop of the governors should be set between 3 and 6%.

Assume now that a prime mover generator in speed droop mode is connected in parallel to another in isochronous mode in operation. The isochronous unit is known as the *swing machine*. Now the droop machine will run at the frequency (speed) of the isochronous unit. The power output of the droop machine is determined by its speed set point, speed droop and the grid frequency. If now the load is increased, the power of the swing machine will increase to meet the increased load while the power of the droop machine would remain unchanged (since its speed set point is not changed). The system can be loaded until the system load is equal to the combined maximum output of the swing machine and the set power output of the droop machine. If the system load exceeds this, then the frequency will drop. In this setup, if we wish to change the portion of the load met by the droop machine, we need to change its speed setting.

Instead if the load were to be decreased continuously, the isochronous machine's output will decrease eventually to zero, when the load is equal to the set point load of the droop machine. If the load is further decreased, then the frequency will increase (determined by the droop) and the swing machine will be motorized and trip on reverse power.

Suppose a generator whose governor is acting in the isochronous mode is connected to the utility, its set point will not be exactly same as the grid frequency. Therefore, its prime mover may be driven either to full power or into reverse power!

5A

6.6.6 Complete ALFC Model

We now connect the block diagrams of Figs. 6.18, 6.21 and 6.28 to obtain the block diagram of the complete ALFC as shown in Fig. 6.29.

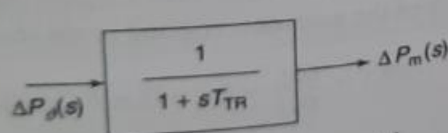


Figure 6.28 Turbine model.

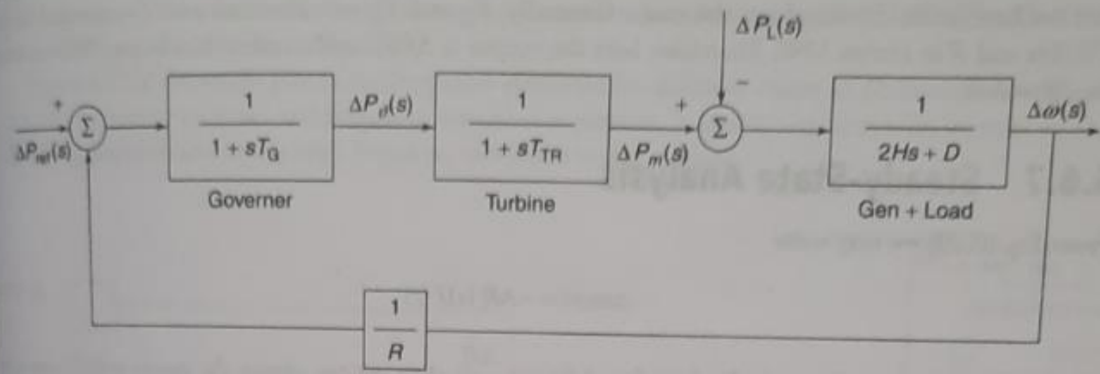


Figure 6.29 Block diagram of complete ALFC.

We are now interested in deriving the effect of change in load on the frequency without change in the reference set point. By changing the reference set point, we can set the system to give specified frequency at any load point as explained in Fig. 6.7(b). This is a secondary control to be discussed later. Here, we assume P_{ref} is kept at a constant value so that $\Delta P_{ref} = 0$. We now find the transfer function $\frac{\Delta\omega(s)}{-\Delta P_L(s)}$. From the block diagram of Fig. 6.29,

$$\Delta\omega(s) = -\Delta P_L(s)\left[\frac{\frac{1}{2Hs+D}}{1+\frac{1}{R}\left(\frac{1}{2Hs+D}\right)\left(\frac{1}{1+sT_G}\right)\left(\frac{1}{1+sT_{TR}}\right)}\right] \tag{6.29a}$$

$$= -\Delta P_L(s)\left[\frac{(1+sT_G)(1+sT_{TR})}{(2Hs+D)(1+sT_G)(1+sT_{TR})+\frac{1}{R}}\right] \tag{6.29b}$$

The transfer function is given by

$$\Delta\omega(s) = -\Delta P_L(s)\left[\frac{\frac{1}{2Hs+D}}{1+\frac{1}{R}\left(\frac{1}{2Hs+D}\right)\left(\frac{1}{1+sT_G}\right)\left(\frac{1}{1+sT_{TR}}\right)}\right]$$

$$= -\Delta P_L(s)\left[\frac{(1+sT_G)(1+sT_{TR})}{(2Hs+D)(1+sT_G)(1+sT_{TR})+\frac{1}{R}}\right]$$

The transfer function is given by

$$T(s) = \left[\frac{(1+sT_G)(1+sT_{TR})}{(2Hs+D)(1+sT_G)(1+sT_{TR})+\frac{1}{R}}\right]$$

An alternate expression for the transfer function commonly used is

$$T(s) = \left[\frac{\frac{K_{ps}}{1+sT_{ps}}}{1+\left(\frac{K_{ps}}{1+sT_{ps}}\right)\left(\frac{1}{1+sT_G}\right)\left(\frac{1}{1+sT_{TR}}\right)\frac{1}{R}}\right]$$

$$= \left[\frac{K_{ps}(1+sT_G)(1+sT_{TR})}{(1+sT_{ps})(1+sT_G)(1+sT_{TR})+\frac{K_{ps}}{R}}\right]$$

5B

Solution

Let the load taken by unit 1 be x MW and that by unit 2 be $800 - x$ MW.

$$R_1 = 5\% = \frac{0.05\times 50}{1000} = 2.5\times 10^{-3}\ \text{Hz/MW} = \frac{\Delta f}{x}$$

$$R_2 = 4\% = \frac{0.04\times 50}{500} = 4\times 10^{-3}\ \text{Hz/MW} = \frac{\Delta f}{800-x}$$

Equating Δf from the two equations we have

$$2.5\times 10^{-3}(x) = 4\times 10^{-3}(800-x)$$

$$x = 492.3\ \text{MW}$$

$$\Delta f = 2.5\times 10^{-3}\times 492.3 = 1.23\ \text{Hz}$$

Unit 1 supplies 492.3 MW
Unit 2 supplies 307.7 MW
Frequency = 50 − 1.23 = 48.77 Hz

6A

7.2 Tie-Line Control wi...

Let us consider a two-area system as shown in Fig. 7.1.

Let us take the positive power flow to be P_{12}, to be the power flow from area 1 to area 2. The power on the tie-line from area 1 to area 2 is

$$P_{12} = \frac{E_1 E_2}{X_{12}} \sin(\delta_1 - \delta_2)$$

where

$$X_{12} = X_1 + X_{tie} + X_2$$

Equation (7.1) can be linearized about an initial operating point $\delta_1 = \delta_{10}$ and $\delta_2 = \delta_{20}$ as

$$\Delta P_{12} = \frac{E_1 E_2}{X_{12}} \cos(\delta_{10} - \delta_{20}) \Delta\delta_{12}$$

$$\Delta\delta_{12} = \Delta\delta_1 - \Delta\delta_2$$

$$\text{Let } T = \frac{E_1 E_2}{X_{12}} \cos(\delta_{10} - \delta_{20}) = P_{max} \cos(\delta_{10} - \delta_{20})$$

where T is called the *synchronizing torque coefficient* (often designated as P_s).

Substituting Eq. (7.4) into Eq. (7.2), we get

$$\Delta P_{12} = T(\Delta\delta_1 - \Delta\delta_2)$$

The block diagram representation of the two-area system with only primary control is shown in Fig... A positive ΔP_{12} means an increase in power flow from area 1 to 2. This is equivalent to a load increase in area 1 and/or decreasing load in area 2. Therefore, the feedback from ΔP_{12} has a *negative sign for area 1* and positive sign for area 2. We will now see how the system behaves for a change in the load.

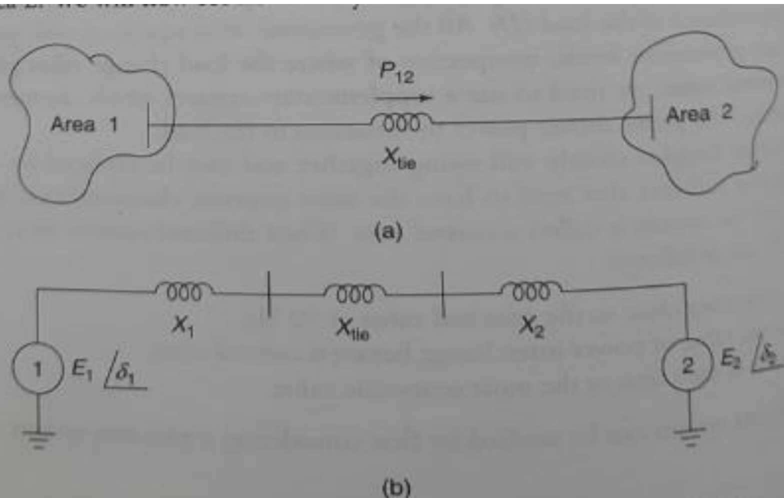


Figure 7.1 (a) Two-area system and (b) Electrical equivalent.

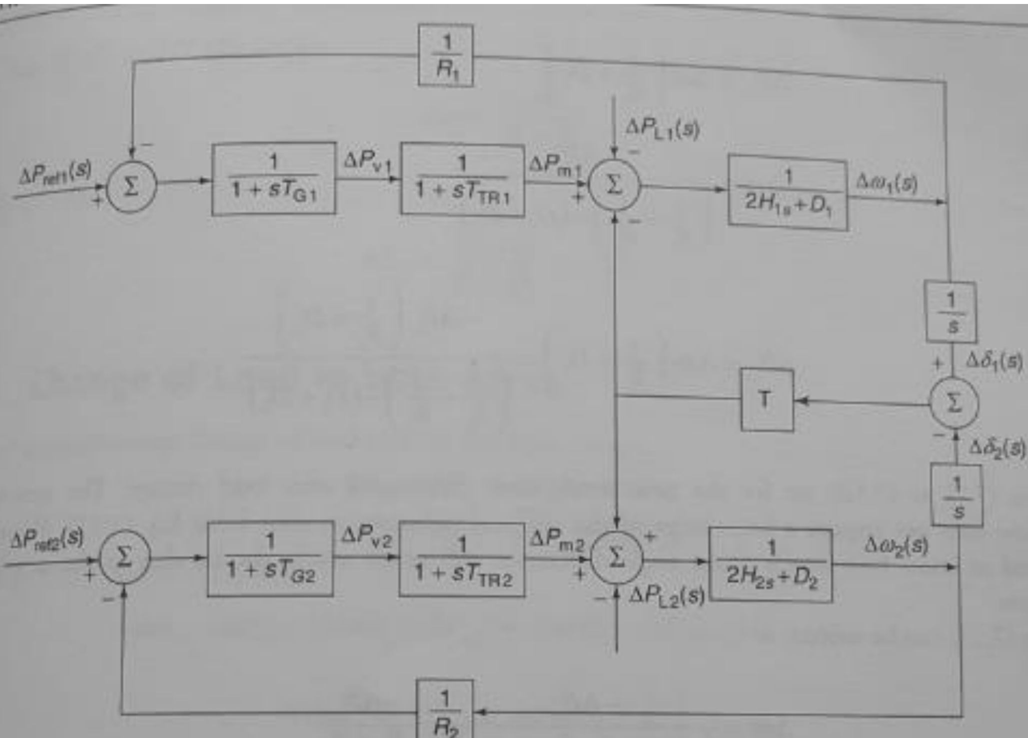


Figure 7.2 Two-area system with primary loop.

6B

$$\Delta P_{m1} = D_1\Delta\omega_1 + \Delta P_{L1} + \Delta P_{12} = \Delta P_{L1} + \Delta P_{12}$$
$$\Delta P_{m2} = D_2\Delta\omega_2 + \Delta P_{L2} + \Delta P_{21} = \Delta P_{21} = \Delta P_{12}$$

$$\Delta\omega = \Delta\omega_1 = \Delta\omega_2$$

$$\Delta P_{m1} = -\frac{1}{R_1}\Delta\omega_1 \text{ and } \Delta P_{m2} = -\frac{1}{R_2}\Delta\omega_2$$

$$\Delta\omega = \frac{-\Delta P_{L1}}{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)} = \frac{-10}{\left(\frac{1}{0.015}+\frac{1}{0.0015}\right)} = -0.0136 \text{ rad/s}$$

or

$$\Delta P_{m1} = \frac{-(-0.0136)}{0.015} = 0.9091 \text{ MW}$$

$$\Delta P_{12} = \Delta P_{m1} - \Delta P_{L1} = 0.9091 - 10 = -9.091 \text{ MW}$$

$$\Delta P_{21} = -\Delta P_{12} = 9.091 \text{ MW}$$

$$\text{ACE}_1 = \Delta P_{12} + \frac{1}{R_1}\Delta\omega = -9.091 + \frac{1}{0.015}(-0.0136) = -10 \text{ MW}$$

$$\text{ACE}_2 = \Delta P_{21} + \frac{1}{R_2}\Delta\omega = 9.091 + \frac{1}{0.015}(-0.0136) = 0 \text{ MW}$$

The ACEs indicate the action to be taken in each area. $\text{ACE}_2 = 0$ means that area 2 need not take any action. $\text{ACE}_1 = -10$ means that area 1 should increase its load power reference point to $-(-10) = 10$ MW and increase generation to meet the increased load.

7A

We saw in the previous section that the system state matrix is a 9×9 matrix for a two-area system. With some simplification, we can get a fairly good idea of the effect of the system parameters on the dynamic response. Let us make the following assumptions:

1. Neglect turbine and governor time constants.
2. Neglect damping constants D_1 and D_2.
3. Both areas are identical.

With these assumptions, the two area equations reduce to

$$\Delta P_{m1}(s) = \frac{-\Delta\omega_1(s)}{R} \tag{7.70}$$

$$\Delta P_{m2}(s) = \frac{-\Delta\omega_2(s)}{R}$$

$$\Delta\omega_1(s) = \frac{1}{2Hs}[\Delta P_{m1}(s) - \Delta P_{L1}(s) - \Delta P_{12}(s)]$$

Substituting for $\Delta P_{m1}(s)$, we get

$$\Delta\omega_1(s) = \frac{1}{1 + \frac{1}{2RHs}}\left[\frac{-\Delta P_{L1}(s) - \Delta P_{12}(s)}{2Hs}\right]$$

Similarly we get

$$\Delta\omega_2(s) = \frac{1}{1 + \frac{1}{2RHs}}\left[\frac{-\Delta P_{L2}(s) + \Delta P_{12}(s)}{2Hs}\right]$$

$$\Delta P_{12}(s) = \frac{T}{s}[\Delta\omega_1(s) - \Delta\omega_2(s)]$$

$$= \frac{T}{s}\frac{1}{\left(1 + \frac{1}{2RHs}\right)}\left[\frac{1}{2Hs}(\Delta P_{L2}(s) - \Delta P_{L1}(s) - 2\,\Delta P_{12}(s))\right]$$

or

$$\Delta P_{12}(s)\left[1 + \frac{2T}{2Hs^2 + \frac{s}{R}}\right] = \frac{T}{2Hs^2 + \frac{s}{R}}[\Delta P_{L2}(s) - \Delta P_{L1}(s)]$$

or

$$\Delta P_{12}(s) = \frac{\frac{T}{2H}}{s^2 + \frac{s}{2RH} + \frac{T}{H}}\quad[\Delta P_{L2}(s) - \Delta P_{L1}(s)]$$

or

$$\Delta P_{12}(s) = \frac{\frac{T}{2H}}{s^2 + \frac{s}{2RH} + \frac{T}{H}} \quad [\Delta P_{L2}(s) - \Delta P_{L1}(s)]$$

The poles of the denominator determine the oscillations in ΔP_{12}. We compare the denominator with the standard second-order characteristic equation.

$$s^2 + 2\alpha s + \omega_n^2 = s^2 + 2\xi\omega_n s + \omega_n^2$$

We can see that

$$\alpha = \frac{1}{4RH}$$

And

$$\omega_n = \sqrt{\frac{T}{H}} \text{ pu or } \sqrt{\frac{2\Pi f_0 T}{H}} \text{ rad/s}$$

The damping is determined by the relative values of α and ω_n and the roots of Eq. (7.75). The roots of Eq. (7.75) are

$$s_1, s_2 = -\xi\omega_n \pm j\,\omega_n\sqrt{1-\xi^2}$$
$$= -\alpha \pm j\,\omega_d$$

where $\alpha = \xi\omega_n$

and
$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

α is called the *damping factor* or *damping constant*. ω_n is the *natural undamped frequency* of oscillations, ... is called the *damped or conditional frequency* and ξ is called the *damping ratio*. We now have the following cases:

1. When $\xi = 1$ or $\alpha = \omega_n$ This condition is called "critical damping."
2. When $\xi = 0$ or $\alpha = 0$, the roots are purely imaginary and we get purely sinusoidal oscillations for a step change in input. In Eq. (7.74), the input is $\Delta P_{L2}(s) - \Delta P_{L1}(s)$. Therefore, for a step change in the load, we would get sustained sinusoidal oscillations of tie-line power at a frequency of ... From Eq. (7.76), we can see that $\alpha = 0$ when $R = \infty$. This means that there is no governor speed control.
3. When $\alpha < \omega_n$, $\xi < 1$, we get a pair of complex conjugate roots. The system is "under damped" and we have oscillations in tie-line power flow which have a frequency ω_d as in Eq. (7.79). The time constant of the system is $1/\alpha$.
4. When $\alpha > \omega_n$, we have an "over damped" system. The roots are both real.

The above analysis is only approximate, but is helpful in knowing the effect of the choice of parameters on the stability of the system. If we consider the damping constants of the load, then α is modified as

$$\alpha = \frac{1}{4H}\left[D + \frac{1}{R}\right]$$

Example 7.1...

7B

Solution

We convert all parameters into a common base of 10,000 MW.

$$R_1 = 1 \times \frac{10{,}000}{1{,}500} = 6.667 \text{ Hz/pu MW}$$

$$D_1 = 0.02 \times \frac{1{,}500}{10{,}000} = 0.003 \text{ pu MW/Hz}$$

$$\beta_1 = \frac{1}{R_1} + D_1 = 0.153 \text{ pu MW/Hz}$$

$$\beta_2 = \frac{1}{R_2} + D_2 = 1.02 \text{ pu MW/Hz}$$

$$\Delta P_{L2} = \frac{200}{10{,}000} = 0.02 \text{ pu}$$

$$\Delta f = \frac{-\Delta P_{L2}}{\beta_1 + \beta_2} = \frac{-0.02}{0.153 + 1.02} = -0.017 \text{ Hz}$$

Steady-state frequency = 50 − 0.017 = 49.983 Hz

$$\Delta P_{12} = \frac{+\Delta P_{12}\,\beta_1}{\beta_1 + \beta_2} = \frac{0.02 \times 0.153}{0.153 + 1.02} = 2.6 \times 10^{-3}\text{ pu} = 26.08\text{ MW}$$

8A

the net reactive p...

8.2 Production and Absorption of Reactive Power

Various elements in a network absorb or generate reactive power. Let us consider these.

1. ***Synchronous generators:*** They can absorb or generate reactive power depending on the excitation. When the generator is over excited, its generated emf is greater than the terminal voltage, and the generator current injected at the terminal bus lags the terminal voltage. An over-excited generator generates reactive power. Similarly, an under-excited generator absorbs reactive power. The generator conditions are shown in Fig. 8.2 and the phasor diagrams in Fig. 8.3.

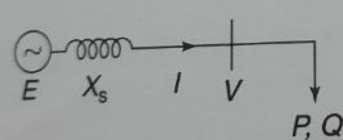


Figure 8.2 Generator model.

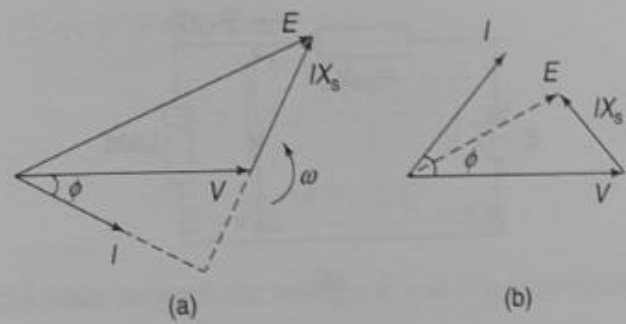


Figure 8.3 Excitation of generators: (a) Over excitation and (b) under excitation.

The capability of a synchronous generator to continuously supply or absorb reactive power is limited by the field current, armature current and end-region heating limits.

2. ***Overhead lines:*** The *surge impedance* or *characteristic impedance* of an overhead line is given by

$$Z_c = \sqrt{\frac{L}{C}} \tag{8.3}$$

where L is the inductance of the line and C is the shunt capacitance. The natural load or surge impedance load (SIL) is given by

$$\text{SIL} = \frac{V_o^2}{Z_c}\ \text{W} \tag{8.4}$$

where V_o is the rated voltage of the line. At loads below SIL, transmission lines generate reactive power and at loads above SIL, overhead lines absorb reactive power.

3. ***Underground cables:*** They have high capacitance, owing to which they have high natural loads or SIL. Hence, they are always loaded below their SIL and generate reactive power.
4. ***Transformers:*** They always absorb reactive power, irrespective of their loads.
5. ***Loads:*** They normally absorb reactive power. Heavy lagging loads cause severe voltage drops.
6. ***Compensating devices:*** These are added to either generator absorb reactive power. They are controlled to balance the reactive power as desired.

8B

Solution

The base value of Z in the 132 kV circuit is

$$\frac{\text{kV}^2}{\text{MVA}} = \frac{132^2}{500} = 34.848\ \Omega$$

The line ...

The single-phase equivalent circuit is shown in Fig...

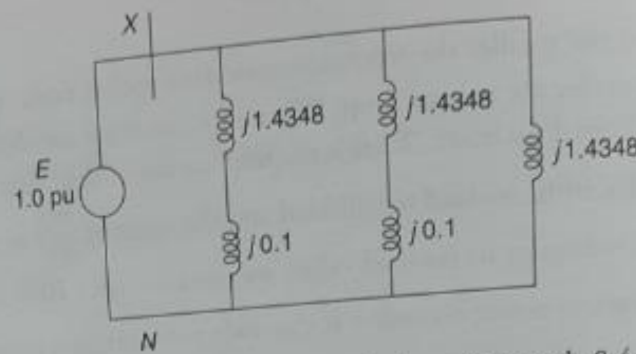


Figure 8.9 Single-phase equivalent of Example 8.4.

The reactance from X to N is given by

$$\frac{1}{X_{eq}} = \frac{1}{1.5348} + \frac{1}{1.5348} + \frac{1}{1.4348}$$

$$X_{eq} = 0.5\,\text{pu}$$

The short-circuit current $= \dfrac{E}{X_{eq}} = \dfrac{1}{0.5} = 2\,\text{pu}$

Base current $I_B = \dfrac{500 \times 10^6}{\sqrt{3} \times 132 \times 10^3} = 2187\text{ A}$

$$I_{sc} = 2 \times 2187 = 4373\,\text{A}$$

When three-phase Q is used, and V is the line-to-line voltage then

$$\left|\frac{\partial Q}{\partial V}\right| = \left|\sqrt{3} I_{sc}\right| = 7574.2\,\text{Var/V}$$

$$= 7.574\,\text{MVAr/kV}$$

For a voltage drop of 2 kV, the reactive power to be injected is $7.579 \times 2 = 15.148$ MVAr (th

9A

9.5.4 Factors Affecting Security

We have seen that there are two major objectives to be met:

1. Operate the system reliably.
2. Within the security constraints operate the system economically.

In Chapters 3, 4 and 5, we have seen the operation of the power system from an economic stand-point. What factors affect it from a reliability point? If unexpected events, unpredictable failures do not occur, then we can build a 100% reliable system by proper planning and design. However, the occurrences of unpredictable events, sometimes more than one by sheer coincidence, have known to cause catastrophic blackouts. Consider the case of the (in) famous blackout in London, on 14 August 2003, which left thousands of commuters underground

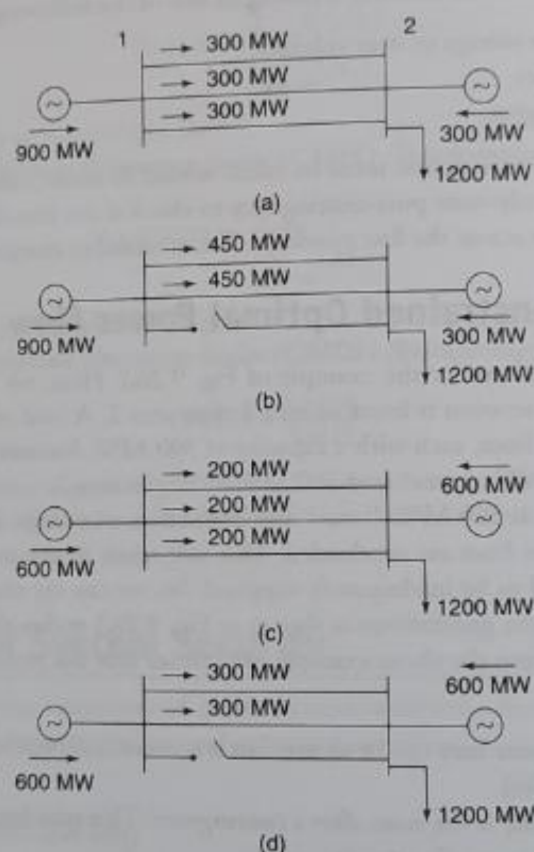


Figure 9.2 Illustration of security (a) Non-secure, (b) Post contingency: Power flows with one line outage in non-secure state, (c) N−1 secure state, and (d) Post contingency secure state.

cutting off supply to London underground metro. The problem began when an alarm indicated a transformer fault (due to oil leak) at a substation in Hurst, forcing the transformer to be switched off. Exactly seven seconds after the transformer was switched off, a 275 kV line outage occurred, stopping power flow between the two substations in New Cross and Wimbledon, over that line. The relay setting of a parallel 275 kV line was wrongly set at a lower value, (www.newscientist.com) causing the parallel line also to trip, and cutting off supply to the metro. This is a typical case, where, in spite of built-in redundancies, catastrophic blackouts are still possible, making reliability and security issues a real challenge and making 100% reliability a myth. Systems are designed to make load-shedding acceptably small, rather than zero. Two major factors affecting reliability are:

1. Generator outages.
2. Transmission line outages.

We need to consider the impact of these on voltages and line flows. This would require an AC power flow. However, considering the large number of contingencies, we need to screen them first and decide on those contingencies which would require a rigorous AC power flow.

To select the critical contingency, there is a tendency to select the most important ones. Other strategies for contingency selection are also present. One such strategy is to simulate all single element contingencies (loss of one generator or of one line) and several multiple contingencies using a fast, approximate technique, like the DC power flow. Those contingencies are deemed critical, which lead to system insecurity. A second strategy is to compute a *severity index* for each strategy and select those which have a severity index above a threshold value. Each method has its own pros and cons. While transmission equipment failure leads to voltage and line flow changes, generator loss, in addition, also involves changes in system frequency.

9B

9.6 Contingency Analysis

Automatic contingency analysis is an increasingly valuable analytical tool in EMS. It is used to predict the steady-state conditions following branch or generation outages. The analysis consists of simulating the outages, one after the other and clicking the results for possible violation of voltage magnitude limits, line power flow limits and MVAr limits. In a real-time tool, it would not be possible to simulate for all contingencies. So various techniques are used to speed up the process. Important amongst these is "*contingency selection*" which selects a subset of the exhaustive contingency set, which is severe for a detailed analysis. A common method used for selection is called the *contingency ranking method*. Here, the contingencies are ranked for relative severity based on a performance index (PI), which measures some stress on the system. An alternative screening approach is to find local solutions for each outage, under the assumption that areas remote from the contingency are rigid and unaffected. Two major considerations for an on-line security assessment are the speed and accuracy of the contingency selection algorithm.

The changes in bus voltages and line flows depend on how lost generation is picked up by the remaining units. The network model chosen will also affect the speed of solution in case of integrated systems. We know, if the system being modelled is part of a large interconnected network, the generation lost is made up by other units in the system outside the immediate control area. This causes an increase in tie-line flows to the neighbouring systems. This case is often modelled by representing the external system as a single swing bus (one generator equivalent). In such a model, the lost generation is picked up by the swing bus. In an isolated system, the lost generation will be picked up by generators within the system.

It is important for the operators to know which contingencies (generator or transmission line) will cause line-flow or voltage violations. Contingency analysis procedures model all events one after the other until all possible and probable outages have been covered. For each outage, all voltages and line-flows are tested for limit violations. Such a technique is shown in the flowchart of Fig. 9.3.

In the procedure of Fig. 9.3, speed of solution for the chosen model is a key issue. The search space can be reduced if we select only "*credible outages*". Even if a case takes a second to solve, the task is horrendous as we have thousands of cases to solve. In the time taken to run the cases once, the system would already have changed, making the studies irrelevant!!

One way to speed up matters is to use an approximate load flow model. DC load flow models provide adequate accuracy for this analysis, since voltage accuracy is not very important and DC load flow models provide sufficient accuracy for active power flows with 5% accuracy. Where voltage magnitude is a concern, AC power flow models can be used. An alternative would be to run DC load flow, and for a selected set of critical contingencies, we can run the AC power flow. Modern computing facilities with parallel processors have speeded up the analysis.

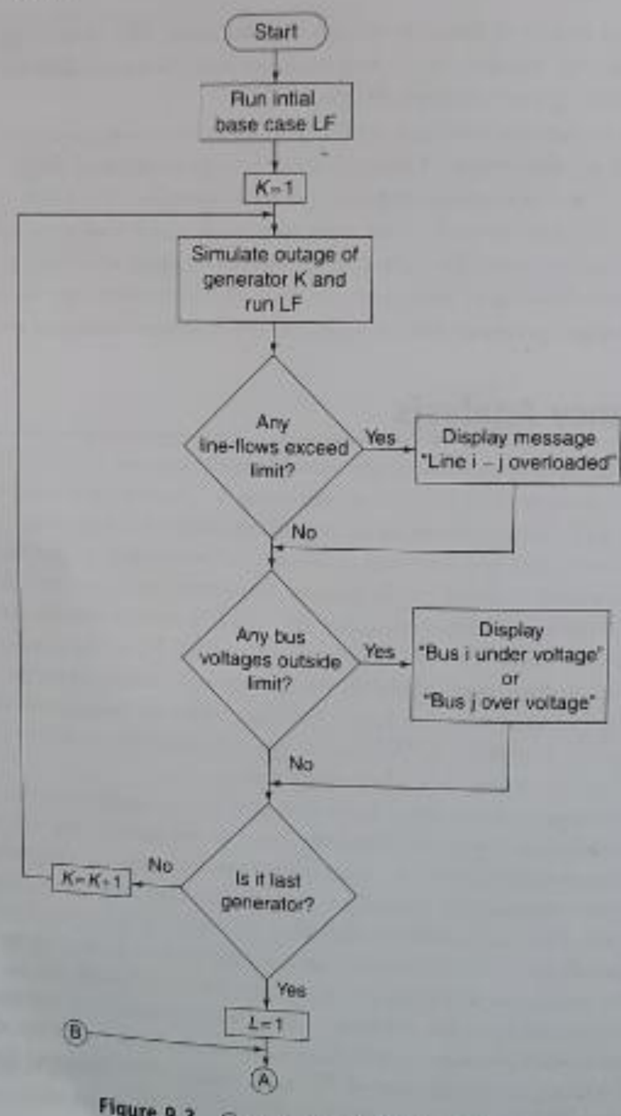


Figure 9.3 Contingency analysis procedure

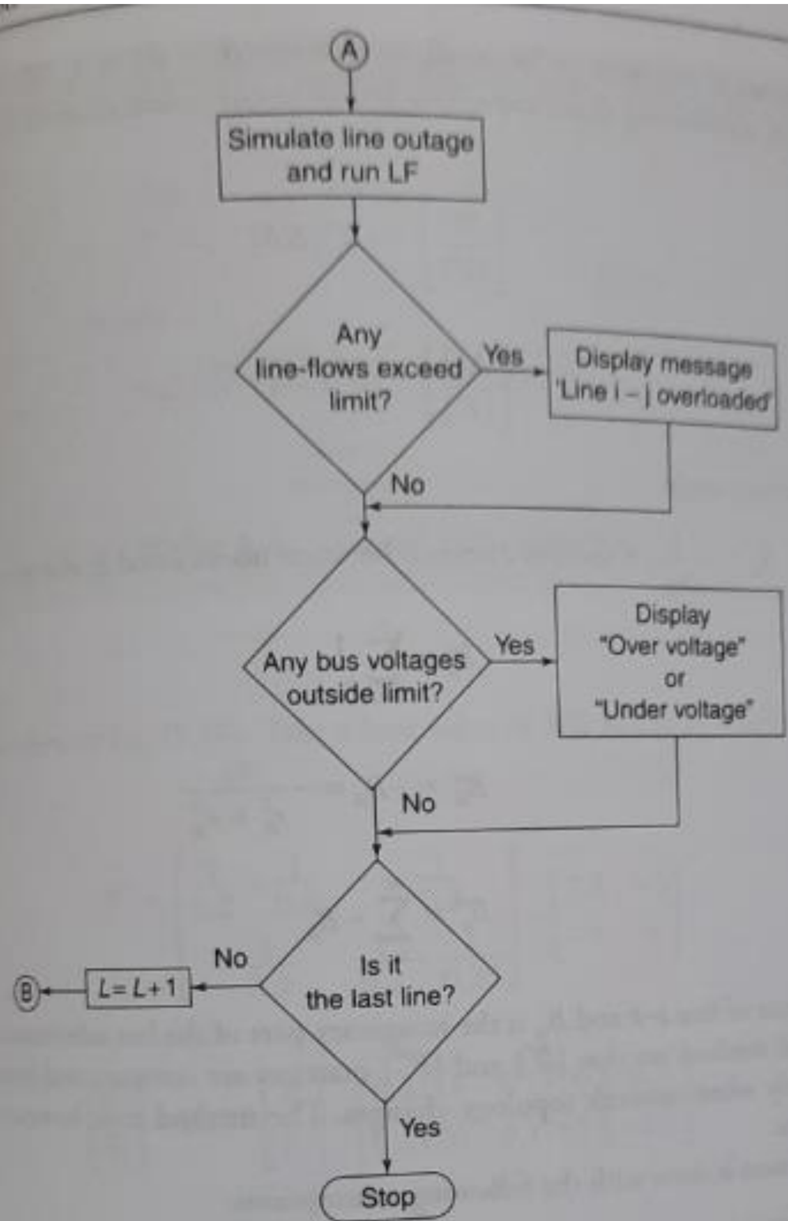


Figure 9.3 (Continued).

10A

9.7.2 Generation-Shift Sensitivity Factors

From the DC load flow equations, we have

$$[\delta] = [X][P] = [B']^{-1}[P] \tag{9.20}$$

This is a linear model. We can easily calculate perturbation about a given set of system conditions. Let us assume the bus power injections change by an amount ΔP. Then the change in bus phase angles, $\Delta\delta$, is given by

$$[\Delta\delta] = [X][\Delta P] \tag{9.21}$$

In Eq. (9.20), the swing bus power is the sum of the injections of all other buses. Likewise, in Eq. (9.21) the net perturbation of the swing bus is the sum of perturbations on all other buses.

The generation-shift factor is defined as

$$a_i^l = \frac{\Delta f_l^i}{\Delta P_i} \tag{9.22}$$

where a_i^l = generator shift factor

l = line index

i = generator index

Δf_l^i = change in megawatt power flow on-line l when a change in generation, ΔP_i, occurs at bus i

ΔP_i = change in generation at bus i

Now let us consider that we cause a perturbation of +1 pu power on bus i, which means an increase of 1 pu in power at bus i. We further assume that this is compensated by a decrease of 1 pu power

$$[\Delta\delta_p^i] = [X_p]\begin{bmatrix}-1\\ \vdots\\ 1\\ \vdots\\ 0\end{bmatrix} \quad (9.23)$$

where $[\Delta\delta_p^i]$ is the change in phase angles for a perturbation (p) at bus i. In Eq. (9.23), we have considered bus 1 to be the swing bus (hence −1). Note the following while programming to solve for Eq. (9.23).

1. From the bus admittance matrix, remove the row and column corresponding to swing bus. Generally, for ease of programming, either the first bus or the last bus is taken as swing bus. Let us assume that the first bus is swing bus.
2. The negative of the imaginary part of the reduced $[Y]$ bus matrix is the B' matrix.
3. $X=[B']^{-1}$
4. $X_p = X$ appended with a row and column of zeroes, for swing bus.

$$= \begin{bmatrix}0 & 0 & \cdots & 0\\ 0 & & & \\ \vdots & & X & \\ 0 & & & \end{bmatrix}$$

5. $[\Delta\delta_p]$ is vector of phase angle changes of all buses. However, because of the nature of X_p, the phase angle change of swing bus will be zero. In other words, the first element of $\Delta\delta_p^i$ (assuming bus 1 is the swing bus) is zero for all i.
6. $[\Delta\delta_p]$ as obtained from Eq. (9.23) is nothing but the i^{th} column of X_p.

The value of $[\Delta\delta_p^i]$ obtained in Eq. (9.23) is nothing but the derivative of the bus phase angles with respect to a change in power injection at bus i. If we now consider a line l, connected between buses m and n, then from Eq. (9.19a) we get

$$f_l^i = \frac{1}{X_l}\left[\delta_m^i - \delta_n^i\right] \quad (9.24)$$

X_l = the reactance of line l.

Therefore,

$$a_l^i = \frac{\Delta f_l^i}{\Delta P_i} = \frac{df_l^i}{dP_i} = \frac{d}{dP_i}\left[\frac{1}{X_l}(\delta_m^i - \delta_n^i)\right]$$

$$= \frac{1}{X_l}\left[\frac{d\delta_m^i}{dP_i} - \frac{d\delta_n^i}{dP_i}\right] = \frac{1}{X_l}\left[\Delta\delta_{Pm}^i - \Delta\delta_{Pn}^i\right] \tag{9.25}$$

In Eq. (9.25), $\Delta\delta_{Pm}^i$ and $\Delta\delta_{Pn}^i$ are the m^{th} and n^{th} elements corresponding to the vector $\left[\Delta\delta_P^i\right]$ obtained in Eq. (9.23). In the above derivation, we have assumed that the change in generation at bus i, ΔP_i is exactly compensated by an opposite change in the generation of the swing bus. Therefore, a_l^i represents the sensitivity of the power flow in line l to a power change at bus i.

Now consider the case of a generator outage. If the generator was generating P_{i0} MW, then an outage of the generator would mean a change in generation of $-P_{i0}$. Further, according to our assumption, the complete generation lost would be made up by an increase in generation of the swing bus. We have

$$\Delta P_i = -P_{i0} \tag{9.26}$$

The new power flow in each line can be calculated using the sensitivity factors, a_l^i, as follows:

$$f_l^{new} = f_l^{old} + a_l^i \, \Delta P_i; \quad l = 1, 2, \ldots\ldots\ldots, L \tag{9.27}$$

f_l^{new} = power flow in line l after generation outage

f_l^{old} = power flow before outage

L = Total number of lines.

The value of f_l^{new} can be compared to their limits, and any violations can be used to set off alarms to the operator for suitable action.

Instead of just the swing bus compensating for generation lost, we can consider the more general case where each of the remaining generators picks up generation in proportion to its MW rating. Thus, the proportion of generation picked up by a unit $j (j \neq i)$ is given by

$$\gamma_{ji} = \frac{P_j^{max}}{\sum_{k, k \neq i}(P_k^{max})} \tag{9.28}$$

where γ_{ji} = proportionality factor for generation pick up by unit j, when generator i faces an outage

P_j^{max} = maximum MW rating of generator j.

Since the generation-shift sensitivity factors are linear estimates, the effect of simultaneous changes in generations of several units can be obtained by superposition. Thus, the flow on line l, with changes in generations given by Eq. (9.28) is

$$f_l^{new} = f_l^{old} + a_l^i \, \Delta P_i - \sum_{j \neq i}\left[a_l^j \gamma_{ji} \, \Delta P_i\right] \tag{9.29}$$

In Eq. (9.29) we have not considered generators hitting their limits. If these have to be included more detailed algorithms have to be considered, beyond the scope of presentation considered in this chapter. The

7.3 Line-Outage Distribution Factors

ne-outage distribution factors indicate how power flows change in lines, when there is outage of another e. Consider a line k, connected between m and n as shown in Fig. 9.8(a).

The power flowing from bus m to bus n is P_{mn}. If there is an outage, the breakers open (Fig. 9.8(b)) and wer flow through the line is zero. Obviously, this will change the line flows through the other lines in the stem. To calculate the change, we model the line outage, as shown in Fig. 9.8(c). Here, the line connec- on is as before, and power ΔP_m and ΔP_n are injected as shown. If $\Delta P_m = \tilde{P}_{mn} = -\Delta P_n$, then the net current owing through the circuit breakers is zero, even though they are closed. Since no current flows through he circuit breakers, this model is equivalent to the line being disconnected as far as rest of the network is oncerned.

We know

$$\Delta\delta = [X]\Delta P \tag{9.30}$$

Here, use $[X]$ including swing bus elements which is actually X_r defined earlier in Eq. (9.23). If we con- ider Fig. 9.8(c), we have

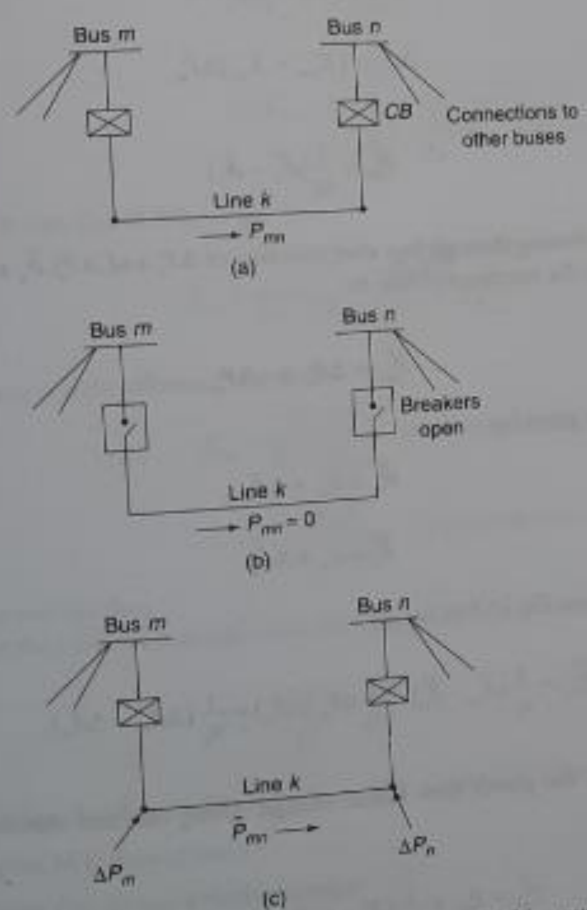


Figure 9.8 Line voltage condition and model: (a) Before outage, (b) Line outage, and (c) Model of outage

$$\Delta P = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \Delta P_m \\ \vdots \\ \Delta P_n \\ \vdots \\ 0 \end{bmatrix} \quad (9.31)$$

In Eq. (9.31), we have accounted for change in injections at bus m and n, without any change in injections of other buses. Substituting Eq. (9.31) into Eq. (9.30), we get

$$\Delta\delta_m = X_{mm}\Delta P_m + X_{mn}\Delta P_n$$

$$= [X_{mm} - X_{mn}]\Delta P_m \quad (9.32)$$

$$\Delta\delta_n = [X_{nm} - X_{nn}]\Delta P_m \quad (9.33)$$

Further,

$$\overline{P_{mn}} = \frac{1}{x_k}[\overline{\delta_m} - \overline{\delta_n}] \quad (9.33)$$

Here, $\overline{P_{mn}}$ is the power flowing through line after injection of ΔP_m and ΔP_n; $\overline{\delta_m}$ and $\overline{\delta_n}$ are bus angles after power injections and x_k is the reactance of line k.

Further,

$$\overline{P_{mn}} = \Delta P_m = -\Delta P_n$$

Now, the new angles are given by

$$\overline{\delta_m} = \delta_m + \Delta\delta_m$$

$$\overline{\delta_n} = \delta_n + \Delta\delta_n \quad (9.34)$$

Substituting Eq. (9.34) into Eq. (9.33), we get

$$\overline{P_{mn}} = \frac{1}{x_k}(\overline{\delta_m} - \overline{\delta_n}) = \frac{1}{x_k}(\delta_m - \delta_n) + \frac{1}{x_k}(\Delta\delta_m - \Delta\delta_n) \quad (9.35)$$

Now, $\frac{1}{x_k}(\delta_m - \delta_n)$ is P_{mn}, the power flow before outage. Using this and substituting Eq. (9.32) and Eq. (9.35), we get

$$\overline{P_{mn}} = P_{mn} + \frac{1}{x_k}(X_{mm} + X_{nn} - 2X_{mn})\Delta P_m \quad (9.36)$$

Now $\tilde{P}_m = \Delta P_m$. Hence, from Eq. (9.36) we get

$$\Delta P_m = \left[\frac{1}{1 - \frac{1}{x_k}(X_{mm} + X_{nn} - 2X_{mn})} \right] P_{mn} \tag{9.37}$$

In Eq. (9.37), we get the injection ΔP_m in terms of the pre-contingency power flow P_{mn} and elements of X matrix. A *sensitivity factor* β is defined as the ratio of the changes in the bus phase angles δ, anywhere in the system, to the pre-contingency power flow P_{mn}.

$$\beta^i_{mn} = \frac{\Delta \delta_i}{P_{mn}} \tag{9.38}$$

Here, β^i_{mn} is the sensitivity factor of bus i to line k between buses m–n. From Eqs. (9.30) and (9.31), we have

$$\begin{aligned} \Delta \delta_i &= X_{im} \Delta P_m + X_{in} \Delta P_n \\ &= (X_{im} - X_{in}) \Delta P_m \\ &= (X_{im} - X_{in}) \left[\frac{1}{1 - \frac{1}{x_k}(X_{mm} + X_{nn} - 2X_{mn})} \right] P_{mn} \end{aligned} \tag{9.39}$$

Substituting Eq. (9.39) into Eq. (9.38), we get

$$\beta^i_{mn} = \frac{(X_{im} - X_{in}) X_k}{X_k - (X_{mm} + X_{nn} - 2X_{mn})} \tag{9.40}$$

If one of the buses m or n is the reference bus then

$$\begin{aligned} \beta^i_{mn} &= \frac{X_{im} X_k}{X_k - X_{mm}} \quad \text{if } n \text{ is reference.} \\ &= -\frac{X_{in} X_k}{X_k - X_{nn}} \quad \text{if } m \text{ is reference.} \end{aligned} \tag{9.41}$$

If bus i itself is reference bus then

$\beta^i_{mn} = 0$, since the reference bus angle does not change. Now the line-outage distribution factors are defined as

$$d_{l,k} = \frac{\Delta f_l}{f_k^0} \tag{9.42}$$

where $d_{l,k}$ = line-outage distribution factor for line l after an outage of line k.

Δf_l = change in MW flow of line l

Using Eq. (9.43) in Eq. (9.42), we get

$$d_{l,k} = \frac{\frac{1}{x_l}(\Delta\delta_i - \Delta\delta_j)}{P_{mn}}$$

$$= \frac{1}{x_l}\left[\frac{\Delta\delta_i}{P_{mn}} - \frac{\Delta\delta_j}{P_{mn}}\right]$$

$$= \frac{1}{x_l}\left[\beta^i_{mn} - \beta^j_{mn}\right]$$

If i, j are not the reference bus, then

$$d_{l,k} = \frac{1}{x_l}\left[\frac{(X_{im} - X_{in})x_k - (X_{jm} - X_{jn})x_k}{x_k - (X_{mm} + X_{nn} - 2X_{mn})}\right]$$

$$= \frac{\frac{x_k}{x_l}(X_{im} - X_{jm} - X_{in} + X_{jn})}{x_k - (X_{mm} + X_{nn} - 2X_{mn})} \qquad (9.44)$$

To clarify again, in Eq. (9.44),

$d_{l,k}$ = distribution factor for line l due to outage of line k.

line l is connected from bus i to bus j

line k is connected from bus m to bus n.

In Eq. (9.44) use the elements of X, (X augmented with zeroes). If we know the original power flow on line l, then we can find the power flow on line l, with line k opened using

$$f_l^{new} = f_l^{old} + d_{l,k} f_k^{old} \qquad (9.45)$$

where f_l^{old}, f_k^{old} are power flows in lines l and k before outage.

f_l^{new} is power flow in line l after outage of line k.

The distribution factors for line outages can be calculated off-line, since it depends only on the network parameters. Using these, we can set up a very fast procedure for evaluating overloads on-lines, due to various outages. Overloads can be reported to the operator as alarm messages. In Fig. 9.7, we developed the flowchart to study generator outages. The flowchart to study the effect of line outages has been given in Fig. 9.9.

The two flowcharts can be appended to a single flowchart for contingency analysis using both generation shift factors and line-outage distribution factors. The sensitivity factors remain constant as long as the network topology does not change. If the topology changes because of any switching, then the factors have to be recalculated. Contingency analysis tool is a major tool in the EMS.

11.3 DC State Estimator[7]

The DC state estimator is an over-determined system of linear equations which is solved using the weighted least-squares (WLS) problem. The model is given by

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{e} \tag{11.9}$$

where

$\mathbf{z}$ = m vector of measurements
$\mathbf{x}$ = n vector of true states (unknown)
$\mathbf{H}$ = $m \times n$ Jacobian matrix
$\mathbf{e}$ = m vector of random errors
$n < m$ (over-determined).

The residual vector of measurement is given by

$$\mathbf{r} = \mathbf{z} - \mathbf{H}\,\mathbf{x} \tag{11.10}$$

The estimated value of the measurements is $\hat{\mathbf{z}} = \mathbf{H}\,\hat{\mathbf{x}}$. An estimate of the residue is given by $\hat{\mathbf{r}} = \mathbf{z} - \mathbf{H}\hat{\mathbf{x}}$.

The problem is to find an n-vector x that minimizes $\mathbf{J}\,(\mathbf{x})$ defined as

$$\mathrm{J}\,(\mathbf{x}) = (\mathbf{z} - \mathbf{H}\,\mathbf{x})^{T}\mathbf{W}\,(\mathbf{z} - \mathbf{H}\,\mathbf{x}) \tag{11.11}$$

Matrix $\mathbf{W}$ is a diagonal matrix whose elements are the measurement weights, to represent meter accuracy, reliability, importance of measurement, etc. A common practice is to define W based on the reciprocal of the variance of measurement error,

$$\mathbf{W} = \mathbf{R}_z^{-1} = \begin{bmatrix} \sigma_1^{-2} & & & \\ & \sigma_2^{-2} & & \\ & & \ddots & \\ & & & \sigma_m^{-2} \end{bmatrix} \tag{11.12}$$

where $\mathbf{R}_z$ is the measurement covariance matrix. Differentiating $\mathrm{J}(\mathbf{x})$ we get the first- order optimal condition

$$\mathbf{G}\,\hat{\mathbf{x}} = \mathbf{H}^{T}\mathbf{W}\mathbf{z} \tag{11.13}$$

where $\mathbf{G}$ is $\mathbf{H}^{T}\mathbf{W}\mathbf{H}$. From Eq. (11.13) we get

$$\hat{\mathbf{x}} = \mathbf{G}^{-1}\,\mathbf{H}^{T}\mathbf{W}\mathbf{z} \tag{11.14}$$

Equation (11.14) is the *state estimator equation* and the process of obtaining the state variables is called *state estimation.*