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Internal Assessment Test – 3 June 2024

Sub:	Mathematics II for CSE Stream							Code:	BMATS201
Date:	24/06/2024	Duration:	90 mins	Max Marks:	50	Sem:	II	Section:	I,J,K,L

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE CO	OBE RBT
1	Given $\frac{dy}{dx} = xy + y^2$, y (0) = 1, y (0.1) = 1.1169, y (0.2) = 1.2773, y (0.3) = 1.5049. Compute y (0.4) using Milne's Predictor-Corrector method.	[8]	CO4	L3
2	Employ Taylor's series method to obtain the approximate value of y at $x = 0.2$ considering terms upto the third degree for the differential equation $\frac{dy}{dx} = 2y - 3e^x$, y(0) = 0.	[7]	CO4	L3
3	Using the Runge-Kutta method of fourth order, find y (0.2) given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0.1) = 1.0912, taking h = 0.1.	[7]	CO4	L3
4	Given $\frac{dy}{dx} = -xy^2$, y(0) = 2. Compute y (0.2) with h = 0.1 using Euler's modified method. Perform two modifications in each stage.	[7]	CO4	L3

for HoD

5	Prove that the subset $\{W = (x, y, z) \| ax + by + cz = 0; x, y, z \in \mathbb{R}\}$ of the vector space $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$.	[7]	CO3	L3
6	Determine whether the following vectors are linearly independent or not. $x_1 = (1, 2, 4)$, $x_2 = (1, 0, 0)$, $x_3 = (0, 1, 0)$, and $x_4 = (0, 0, 1)$ in $\mathbb{R}^3$.	[7]	CO3	L2
7	Verify the rank-nullity theorem for the linear transformation $T: V_3(R) \to V_3(R)$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Also, find range space and null space.	[7]	CO3	L3
8	Define an Inner Product Space. Consider $f(t) = 4t + 3$ and $g(t) = t^2$, the inner product $< f, g > = \int_0^1 f(t)g(t)dt$. Find $< f, g >$.	[7]	CO3	L2

1) Given:

$\frac{dy}{dx} = xy + y^2$ $\quad h = 0.1$

x	y	$y' = xy + y^2$
0	1	$y'_0 = 1.$
0.1	1.1169	$y'_1 = 1.3592$
0.2	1.2773	$y'_2 = 1.8870$
0.3	1.5049	$y'_3 = 2.7162$
0.4		

$$y_4^{(P)} = y_0 + \frac{4}{3}h\,[2y'_1 - y'_2 + 2y'_3]$$

$$= 1 + \frac{4}{3}(0.1)\,[2 \times 1.3592 - 1.8870 + 2 \times 2.7162]$$

$$y_4^{(P)} = 1.8352$$

$$y'_4 = 4.1020.$$

$$y_4^{(c)} = y_2 + \frac{h}{3}\,[y'_2 + 4y'_3 + y'_4]$$

$$= 1.2773 + \frac{0.1}{3}\,[1.8870 + 4 \times 2.7162 + 4.1020]$$

$$y_4^{(c)} = 1.8391$$

$$y'_4 = 4.1179.$$

$$y_4^{(c)} = y_2 + \frac{h}{3}\,[y'_2 + 4y'_3 + y'_4]$$

$$= 1.2773 + \frac{0.1}{3}\,[1.8870 + 4 \times 2.7162 + 4.1179]$$

$$= 1.8396$$

$y_4' = 4.1200$.

$$y_4^{(c)} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$$

$$= 1.2773 + \frac{0.1}{3}[1.8870 + 4 \times 2.7162 + 4.1200]$$

$$= 1.8397.$$

$$y_4' = 4.1204$$

$$y_4^{(c)} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$$

$$= 1.2773 + \frac{0.1}{3}[1.8870 + 4 \times 2.7162 + 4.1204]$$

$$y_4^{(c)} = 1.8397$$

The two consecutive value of y_4 is equal.

$\therefore\ y(0.4) = 1.8397$

Taylor's series.

$y(x) = y_0(x_0) + (x - x_0)\, y_1(x_0) + \frac{(x-x_0)^2}{2!} y_2(x_0) + \frac{(x-x_0)^3}{3!} y_3(x_0)$

$y(0.2) = y_0(0) + (0.2)(-9) + 0.0200(-21) + 0.0013(-45)$.

$y(0.2) = -3 - 1.8 - 0.4200 - 0.0585$

$y(0.2) = \underline{\underline{-5.2785}}$ |x

3) Given:-

$\frac{dy}{dx} = \frac{y-x}{y+x}$

$f(x,y) = \frac{y-x}{y+x}$

$y(0.1) = 1.0912$

$x_0 = 0.1 \quad y_0 = 1.0912$

$h = 0.1$

$y(0.2)$

$K_1 = h f(x_0, y_0) = 0.1 f(0.1, 1.0912) = 0.1 \times 0.8321 = 0.0832$.

$K_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right) = 0.1 f\left(0.1 + \frac{0.1}{2}, 1.0912 + \frac{0.0832}{2}\right)$

$= 0.1 f(0.15, 1.1328)$

$= 0.1 \times 0.7661$

$K_2 = 0.0766$.

$K_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right) = 0.1 f(0.15, 1.1295)$

$= 0.1 \times 0.7655$

$K_3 = 0.0766$.

$K_4 = h f(x_0 + h, y_0 + K_3) = 0.1 f(0.2, 1.1678)$

$K_4 = 0.0708$.

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$y(x) = y_0 + \frac{h}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$y(0.2) = 1.0912 + \frac{1}{6}[0.0832 + 2 \times 0.07664 + 2 \times 0.0766 + 0.0708]$

$y(0.2) = 1.01679$

5) Given:

$\{W = (x, y, z) \mid ax + by + cz = 0;\ x, y, z \in R\}$

1) Let

$c_1\alpha + c_2\beta \in W$

$c_1\alpha \in W$

$c_1, c_2 \in F \qquad \alpha, \beta \in W$

$\alpha = (x_1, y_1, z_1) \Rightarrow ax_1 + by_1 + cz_1 = 0$

$\beta = (x_2, y_2, z_2) \Rightarrow ax_2 + by_2 + cz_2 = 0$

$c_1\alpha + c_2\beta = c_1(x_1, y_1, z_1) + c_2(x_2, y_2, z_2)$

$= (c_1x_1, c_1y_1, c_1z_1) + (c_2x_2, c_2y_2, c_2z_2)$

$= (c_1x_1 + c_2x_1, c_1y_1 + c_2y_2, c_1z_1 + c_2z_2)$

$ax + by + cz = 0$

$a(c_1x_1 + c_2x_2) + b(c_1y_1 + c_2y_2) + c(c_1z_1 + c_2z_2) = 0$

$c_1(ax_1 + by_1 + cz_1) + c_2(ax_2 + by_2 + cz_2) = 0$

$c_1(0) + c_2(0) = 0$

$0 = 0$

$\therefore c_1\alpha + c_2\alpha \in W$

2) $c_1 \alpha \in W$

Let

$c_1 \alpha = c_1 (x_1, y_1, z_1)$

$c_1 \alpha = (c_1 x_1, c_1 y_1, c_1 z_1)$

$ax + by + cz = 0$

$a(c_1 x_1) + b(c_1 y_1) + c(c_1 z_1) = 0$

$c_1 (ax_1 + by_1 + cz_1) = 0$

$c_1 (0) = 0.$

$\therefore c_1 \alpha \in W.$

$\Rightarrow$ The vector space R^3 is a subspace of R^3.

6) Given :-

$x_1 = (1, 2, 4)$

$x_2 = (1, 0, 0)$

$x_3 = (0, 1, 0)$

$x_4 = (0, 0, 1)$

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$$= \begin{bmatrix} 1 & 2 & 4 \\ 0 & -2 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \rightarrow 2R_3 + R_2$

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -2 & -4 \\ 0 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_4 \rightarrow 4R_4 + R_3$

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -2 & -4 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

zero rows.

$\therefore$ The vector is not linearly independent.

7) Given:-

$T : V_3(R) \rightarrow V_3(R)$

$T(x, y, z) = (x + 2y - z,\ y + z,\ x + y - 2z)$

$T(1,0,0) = (1, 0, 1)$

$T(0,1,0) = (2, 1, 1)$

$T(0,0,1) = (-1, 1, -2)$

Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 + R_1$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$r(A) = 2$

$n(A) = 1$

$r(A) + n(A) = \dim(V_3(R))$

$2 + 1 = 3$

$3 = 3$

∴ The rank - nullity theorem for the linear transformation verified.

Range:-

$R(T) = L(T)$

i.e $S = \{(1,0,1), (0,1,-1)\}$

$= \{x_1(1,0,1), x_2(0,1,-1)\}$

$= \{x_1(1,0,1) + x_2(0,1,-1)\}$

$= \{(x_1, 0, x_1) + (0, x_2, -x_2)\}$

$= \{(x_1, x_2, x_1 - x_2)\}$

$R(T) = \{(x_1, x_2, x_1 - x_2) \mid x_1, x_2 \in R\}$.

Null space:-

$N(T)$

$T(x, y, z) = 0$

$(x + 2y - z, y + z, 2x + y - 2z) = 0$

$x+2y-z=0$; $y+z=0$; $2x+y-2z=0$.

$x+2y+y=0$; $y=-z$; $2x+y+2y=0$

$x+3y=0 \rightarrow (1)$; $2x+3y=0 \rightarrow (2)$

$x+3y=0$
$2x+3y=0$
(-) (-) (-)

$-3x=0$

$x=0$

$y=-z$

$x+3y=0$

$x=-3y$

$x=3z$

$z=0$.

$\Rightarrow 2x+3y=0$

$2x+y-2z=0$

$2(3z)-z-2z=0$

$6z-3z=0$

$3z=0$, $z=0$

$2y-z=0$

$2y+y=0$

$3y=0$

$y=0$

$x=3z$, $y=-z$, $z=0$

~~$N(T)=\{(0,0,0)/0\in R\}$~~. $\therefore N(T)=\{(3k,-k,0)/k\in R\}$

8) Inner Product space:-

Let V be a real vector space suppose each pair of vector $u,v\in V$ there is number assign to real number denoted by $\langle u,v\rangle$ this function is called real inner product space on V if it satisfies following conditions.

1) linear property.

$\langle au+bv, w\rangle = a\langle u,w\rangle + b\langle v,w\rangle$

2) Symmetric property

$\langle u,v\rangle = \langle v,u\rangle$.

Given:-

$f(t) = 4t + 3$

$g(t) = t^2$

$\langle f, g \rangle = \int_0^1 f(t)(g(t))\, dt.$

$= \int_0^1 (4t+3)\, t^2\, dt$

$= \int_0^1 (4t^3 + 3t^2)\, dt$

$= \left[\frac{4t^4}{4}\right]_0^1 + \left[\frac{3t^3}{3}\right]_0^1$

$= 1 + 1$

$\langle f, g \rangle = 2$

4) Given:-

$\frac{dy}{dx} = -xy^2 \qquad y(0) = 2$

$y(0.2) \qquad h = 0.2.$

Euler's modified method.

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$y_2^{(0)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$y_3^{(0)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})].$

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$f(x, y) = -xy^2$

$y(0.2) = ?$

$y(0) = 2$

$x_0 = 0 \quad y_0 = 2$

$x_1 = x_0 + h = 0 + 0.1$

$x_1 = 0.1$

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$= 2 + 0.1 f(0, 2)$

$= 2 + 0.1 (0)$

$= 2$

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 2 + \frac{0.1}{2}[f(0, 2) + f(0.1, 2)]$

$= 2 + 0.05[0 + (-0.4)]$

$= 1.9800$

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$= 2 + \frac{0.1}{2}[f(0, 2) + f(0.1, 1.98)]$

$= 2 + 0.05[0 - 0.3920]$

$= \cancel{2.9804} \; \cancel{2.4420} \; = 1.9804.$

$y_1^{(3)} = \cancel{2} + \frac{0.1}{2} [\cancel{0 - 0.3840}] \; [0 - 0.5963]$

$y_1^{(3)} = 2 + 0.05[0 - 0.3922]$

$= 1.9804$

$y_3(0.1) = 1.9804.$

$x_0 = 0.1 \qquad y_0 = 1.9804.$

$x_1 = x_0 + h$

$x_1 = 0.1 + 0.1$

$x_1 = 0.2.$

$$y_1^{(0)} = y_0 + h\, f(x_0, y_0)$$

$$= 2 + 0.1\, f(0.1, 1.9804)$$

$= ~~2 + 0.1 (-0.3922)~~ \quad 2 + (0.1)(-0.3922)$

$= ~~1.6078~~ \qquad = \underline{1.9608}.$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right].$$

-0.3922

$= 2 + \frac{0.1}{2}\left[~~1.9608~~ + f(0.2, 1.9608)\right]$

$= 2 * 0.05 \left[-0.3922 - 0.7689\right]$

$= 1.9419$

$$y_1^{(2)} = 2 + 0.05\left[-0.3922 - 0.7542\right]$$

$= 1.9427$

$$y_1^{(3)} = 2 + 0.05\left[-0.3922 - 0.7548\right]$$

$= \underline{1.9426}$

$\underline{\underline{y(0.2) = 1.9426}}$

2) Given:-

$\frac{dy}{dx} = 2y - 3e^x$

$y = 2y - 3e^x$

$y(0) = 0$

$x_0 = 0, \quad y_0 = 0$

$y_1 = 2y_0 - 3e^x$

$y_2 = 2y_1 - 3e^x$

$y_3 = 2y_2 - 3e^x$.

$y(x) = y_0(x_0) + (x - x_0) y_1(x_0) + \frac{(x-x_0)^2}{2!} y_2(x_0) + \frac{(x-x_0)^3}{3!} y_3(x_0)$

$y(0.2) = y_0(0) + (0.2) y_1(0) + \frac{0.2^2}{2} y_2(0) + \frac{0.2^3}{6} y_3(0)$

$y(0.2) = y_0(0) + 0.2 y_1(0) + 0.02 y_2(0) + 0.0013 y_3(0) \longrightarrow (1)$

$y_1(0) = 2y_0 - 3e^x = 2(0) - 3e^0 = -3$

$y_2(0) = -9$

$y_3(0) = -21$

$y(0.2) = 0 + 0.2(-3) + 0.02(-9) + 0.0013(-21)$

$= -0.6 - 0.18 - 0.0280$

$y(0.2) = -0.8080$