CMR INSTITUTE OF TECHNOLOGY	USN										

Internal Assessment Test – II July 2024

Sub:	Discrete Mathematical Structures							Code:	BCS405A
Date:	08/07/2024	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	AIML/AIDS

Question 1 is compulsory and answer any 6 from the remaining questions.

		Marks	OBE	
			CO	RBT
1	Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by "aRb if and only if a is a multiple of b". Write down the relation R, relation matrix of R and draw its digraph. List out its in-degree and out-degree.	[8]	CO3	L3
2	How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000?	[7]	CO2	L3
3	A Question paper contains 10 questions of which 7 are to be answered. In how many ways a student can select the 7 questions (i) if he can choose any 7? (ii) if he would select three questions from the first five and four questions from the last five? (iii) if he should select at least three questions from the first five?	[7]	CO2	L3
4	Determine the coefficient of (i) x^0 in the expansion of $\left(3x^2 - \left(\frac{2}{x}\right)\right)^{15}$. (ii) xyz^2 in the expansion of $(2x - y - z)^4$.	[7]	CO2	L3

5	In how many ways can one distribute eight identical balls into four distinct containers so that (i) no container is left empty (ii) the fourth container gets an odd number of balls.	[7]	CO2	L3
6	For any non-empty sets A, B, C, prove that (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$	[7]	CO3	L3
7	Draw the Hasse diagram representing the positive divisors of 36.	[7]	CO3	L3
8	Let A = $\{1, 2, 3, 4, 5\}$. Define a relation R on A X A by $(x_1, y_1)R(x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$. Find the partition of $A\times A$ induced by R.	[7]	CO3	L3

IAT II - July 2024

DMS - BCS405A

Q3) (i) No of ways of choosing 7 questions out of 10 is given by $^{10}C_7 =$

(ii) No of ways of selecting 3 Qs from first five = $^5C_3$

" 4 Qs " last five = $^5C_4$

Required answer = $^5C_3 \times {}^5C_4 = 10 \times 5 = 50$

(iii)	First 5	Last 5
case(a)	3	4
case(b)	4	3
case(c)	5	2

Case (a): 3 from first five & 4 from last five

$^5C_3 \times {}^5C_4 = 50$

Case (b): 4 from first five and 3 from last five

$^5C_4 \times {}^5C_3 = 50$

Case (c): 5 from first five and 2 from last five

$^5C_5 \times {}^5C_2 = 1 \times 10 = 10$

Required answer = 50 + 50 + 10 = 110

Q.1 ▶ From the definition of the given R, we note that

$$R = \{(1,1), (2,1), (2,2), (3,1), (3,3), (4,1), (4,2), (4,4), (6,1), (6,2), (6,3), (6,6)\}$$

By examining the elements of R, we find that the matrix of R is

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

The diagraph of R is as shown below:

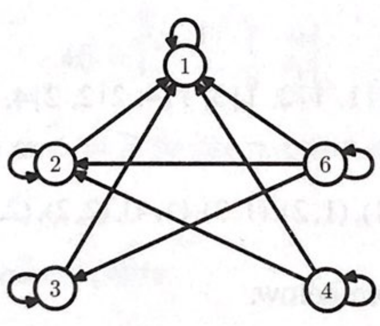

Figure 6.5

■

Example 11 *How many positive integers n can we form using the digits* 3, 4, 4, 5, 5, 6, 7 *if we want n to exceed* 5,000,000*?*

▸ Here n must be of the form

$$n = x_1x_2x_3x_4x_5x_6x_7$$

where $x_1, x_2, \ldots, x_7$ are the given digits with $x_1 = 5, 6$ or 7. Suppose we take $x_1 = 5$. Then $x_2x_3x_4x_5x_6x_7$ is an arrangement of the remaining 6 digits which contains two 4's and one each of 3, 5, 6, 7. The number of such arrangements is

$$\frac{6!}{2!\,1!\,1!\,1!\,1!} = 360.$$

Next, suppose we take $x_1 = 6$. Then, $x_2x_3x_4x_5x_6x_7$ is an arrangement of 6 digits which contains two each of 4 and 5 and one each of 3 and 7. The number of such arrangements is

$$\frac{6!}{1!\,2!\,2!\,1!} = 180.$$

Similarly, if we take $x_1 = 7$, the number of arrangements is

$$\frac{6!}{1!\,2!\,2!\,1!} = 180.$$

Accordingly, by the Sum Rule, the number of n's of the desired type is

$$360 + 180 + 180 = 720.$$

■

4(i) (ii) By Binomial theorem, we have

$$\left(3x^2 - \frac{2}{x}\right)^{15} = \sum_{r=0}^{15} \binom{15}{r} (3x^2)^r \left(-\frac{2}{x}\right)^{(15-r)}$$

$$= \sum_{r=0}^{15} \binom{15}{r} 3^r (-2)^{(15-r)} x^{3r-15}$$

The coefficient of x^0 (namely the constant term) which corresponds to $r = 5$ in this is

$$\binom{15}{5} \times 3^5 \times (-2)^{10} = \frac{15!}{10!\,5!} \times 3^5 \times 2^{10}.$$

4(i)

(i) By the Multinomial theorem, we note that the general term in the expansion of $(2x-y-z)^4$ is

$$\binom{4}{n_1,\ n_2,\ n_3}(2x)^{n_1}(-y)^{n_2}(-z)^{n_3}.$$

For $n_1 = 1$, $n_2 = 1$ and $n_3 = 2$, this becomes

$$\binom{4}{1,\ 1,\ 2}(2x)(-y)(-z)^2 = \binom{4}{1,\ 1,\ 2} \times 2 \times (-1) \times (-1)^2 xyz^2$$

This shows that the required coefficient is

$$\binom{4}{1, 1, 2} \times 2 \times (-1) \times (-1)^2 = \frac{4}{1!\,1!\,2!} \times (-2)$$

$$= -12.$$

5)

Example 10 *In how many ways can one distribute eight identical balls into four distinct containers so that* (i) *no container is left empty?* (ii) *the fourth container gets an odd number of balls?*

► (i) First we distribute one ball into each container. Then we distribute the remaining 4 balls into 4 containers. The number of ways of doing this is the required number. This number is

$$C(4+4-1,4) = C(7,4) = \frac{7!}{4!\,3!} = 35.$$

(ii) If the fourth container has to get an odd number of balls, we have to put one or three or five or seven balls into it.

Suppose we put one ball into it (the fourth container). Then the remaining 7 balls can be distributed into the remaining three containers in

$$C(3+7-1,7) = C(9,7) \quad \text{ways.}$$

Similarly, putting 3 balls into the fourth container and the remaining 5 into the remaining 3 containers can be done in

$$C(3+5-1,5) = C(7,5) \quad \text{ways}$$

Next, putting 5 balls into the fourth container and the remaining 3 into the remaining 3 containers can be done in

$$C(3+3-1,3) = C(5,3) \quad \text{ways}$$

Lastly, putting 7 balls into the fourth container and the remaining 1 into the remaining 3 container can be done in

$$C(3+1-1,1) = C(3,1) = 3 \quad \text{ways.}$$

Thus, the total number ways of distributing the given balls so that the fourth container gets an odd number of balls is

$$C(9,7) + C(7,5) + C(5,3) + 3 = \frac{9!}{7!\,2!} + \frac{7!}{5!\,2!} + \frac{5!}{3!\,2!} + 3$$
$$= 36 + 21 + 10 + 3 = 70$$

■

(6) ▶ (a) Take *any* $(x, y) \in A \times (B \cup C)$. Then, (i) $x \in A$ and (ii) $y \in B \cup C$; that is, (i) $x \in A$ and (ii) $y \in B$ or $y \in C$. This means that (i) $x \in A$ and $y \in B$, or (ii) $x \in A$ and $y \in C$. Hence $(x, y) \in A \times B$ or $(x, y) \in A \times C$; that is, $(x, y) \in (A \times B) \cup (A \times C)$. This proves that

$$A \times (B \cup C) \subseteq (A \times B) \cup (A \times C) \tag{1}$$

Conversely, take *any* $(x, y) \in (A \times B) \cup (A \times C)$. Then $(x, y) \in A \times B$ or $(x, y) \in A \times C$; that is, (i) $x \in A$ and $y \in B$, or (ii) $x \in A$ and $y \in C$. This means that (i) $x \in A$ and (ii) $y \in B$ or $y \in C$; that is, $x \in A$ and $y \in B \cup C$. Hence $(x, y) \in A \times (B \cup C)$. This proves that

$$(A \times B) \cup (A \times C) \subseteq A \times (B \cup C) \tag{2}$$

From results (1) and (2), it follows that

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

(b) Take *any* $(x, y) \in A \times (B \cap C)$. Then $x \in A$ and $y \in B \cap C$; that is, (i) $x \in A$ and (ii) $y \in B$ and $y \in C$. This means that (i) $x \in A$ and $y \in B$, and (ii) $x \in A$ and $y \in C$; that is, $(x, y) \in A \times B$ and $(x, y) \in A \times C$ so that $(x, y) \in (A \times B) \cap (A \times C)$. This proves that

$$A \times (B \cap C) \subseteq (A \times B) \cap (A \times C) \tag{3}$$

Conversely, take *any* $(x, y) \in (A \times B) \cap (A \times C)$. Then $(x, y) \in A \times B$ and $(x, y) \in A \times C$. Hence (i) $x \in A$ and $y \in B$, and (ii) $x \in A$ and $y \in C$. This implies that (i) $x \in A$, and (ii) $y \in B$ and $y \in C$; that is, $x \in A$ and $y \in B \cap C$. Thus, $(x, y) \in A \times (B \cap C)$. This proves that

$$(A \times B) \cap (A \times C) \subseteq A \times (B \cap C) \tag{4}$$

From results (3) and (4), it follows that

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

Q7

Example *Draw the Hasse diagram representing the positive divisors of* 36.

▶ The set of all positive divisors of 36 is

$$D_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\} \qquad (*)$$

The relation R of divisibility (that is, aRb if and only if a divides b) is a partial order on this set. The Hasse diagram for this partial order is required here.

We note that, under R,

1 is related to all elements of D_{36},
2 is related to 2, 4, 6, 12, 18, 36;
3 is related to 3, 6, 9, 12, 18, 36;
4 is related to 4, 12, 36;
6 is related to 6, 12, 18, 36;
9 is related to 9, 18, 36;
12 is related to 12 and 36;
18 is related to 18 and 36;
36 is related to 36.

The Hasse diagram for R must exhibit all of the above facts. The diagram is as shown below:

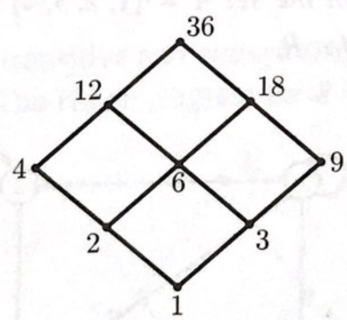

Figure 6.28 ■

Q8 (iii) To determine the partition induced by R, we have to find the equivalence classes of all elements (x, y), of $A \times A$, w.r.t. R. From what has been found above, we note that

$$[(1,1)] = \{(1,1)\},$$
$$[(1,3)] = [(2,2)] = [(3,1)],$$
$$[(2,4)] = [(1,5)] = [(3,3)] = [(4,2)] = [(5,1)].$$

The other equivalence classes are

$$[(1,2)] = \{(1,2),(2,1)\} = [(2,1)]$$
$$[(1,4)] = \{(1,4),(2,3),(3,2),(4,1)\} = [(2,3)] = [(3,2)] = [(4,1)]$$
$$[(2,5)] = \{(2,5),(3,4),(4,3),(5,2)\} = [(3,4)] = [(4,3)] = [(5,2)]$$
$$[(3,5)] = \{(3,5),(4,4),(5,3)\} = [(4,4)] = [(5,3)]$$

$$[(4,5)] = \{(4,5),(5,4)\} = [(5,4)]$$
$$[(5,5)] = \{(5,5)\}$$

Thus, $[(1,1)]$, $[(1,2)]$, $[(1,3)]$, $[(1,4)]$, $[(1,5)]$, $[(2,5)]$, $[(3,5)]$, $[(4,5)]$ and $[(5,5)]$ are the only distinct equivalence classes of $A \times A$ w.r.t. R. Hence the partition of $A \times A$ induced by R is represented by

$$A \times A = [(1,1)] \cup [(1,2)] \cup [(1,3)] \cup [(1,4)] \cup [(1,5)] \cup [(2,5)] \cup [(3,5)] \cup [(4,5)] \cup [(5,5)]. \blacksquare$$