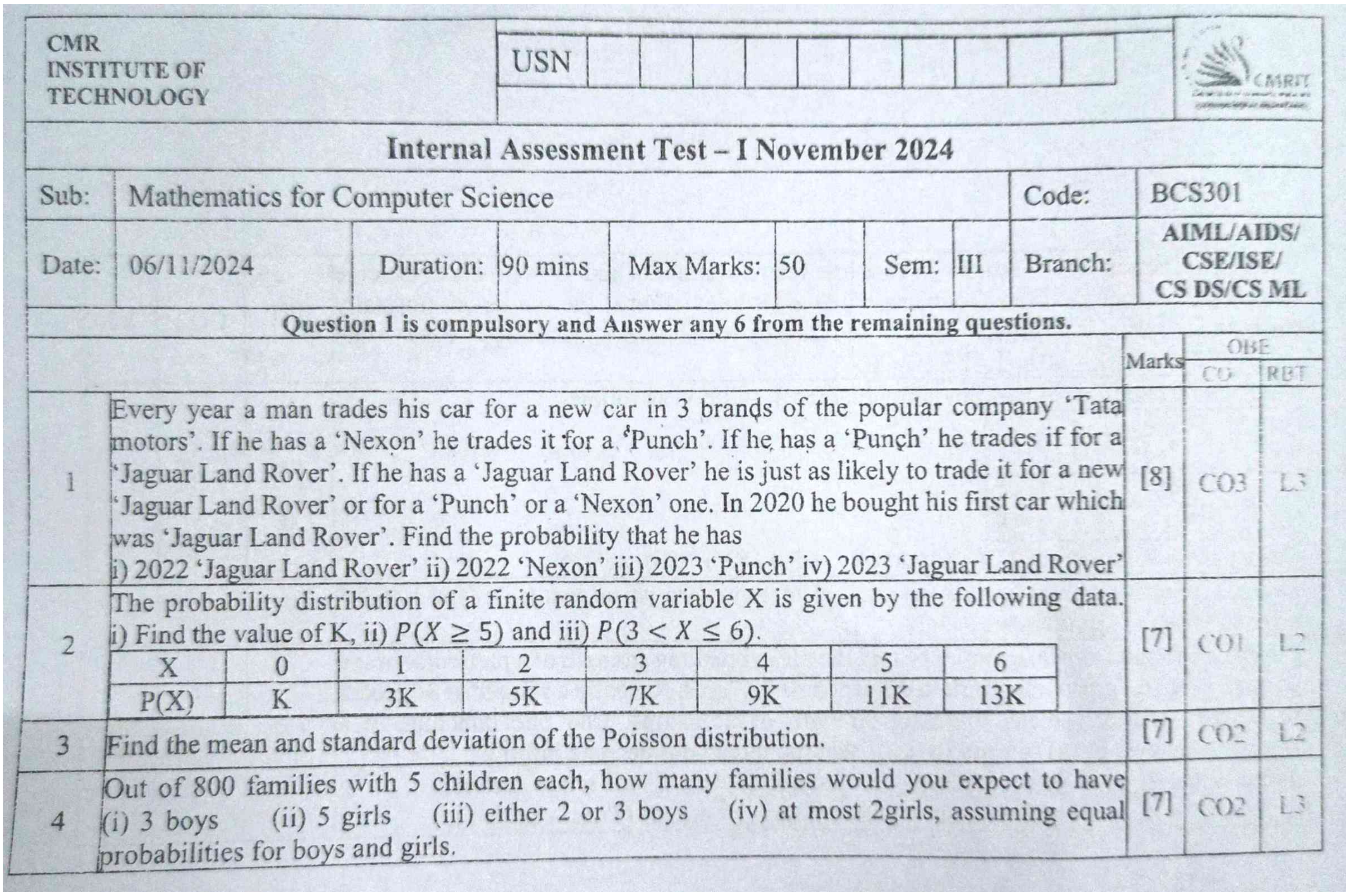

CMR
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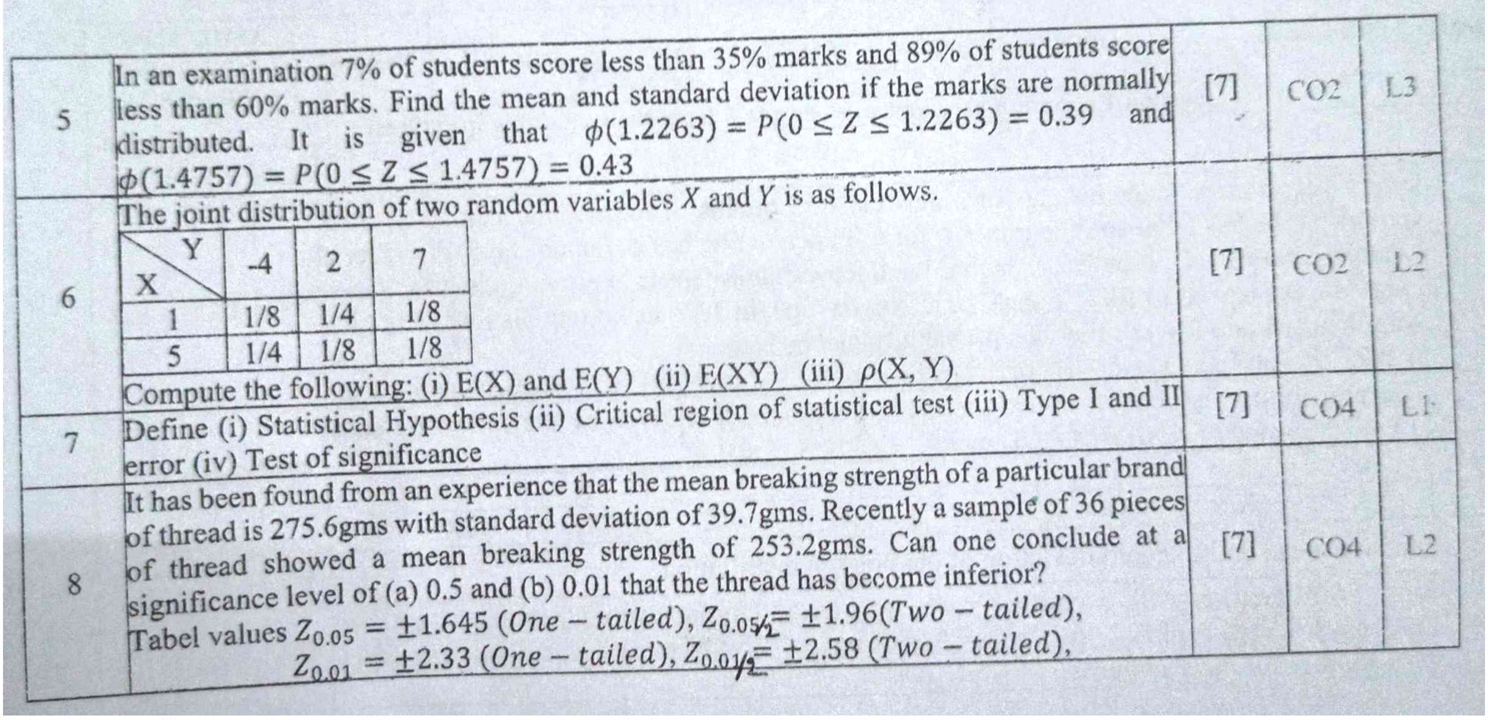
Internal Assessment Test – I November 2024

Sub:	Mathematics for Computer Science							Code:	BCS301
Date:	06/11/2024	Duration:	90 mins	Max Marks:	50	Sem:	III	Branch:	AIML/AIDS/ CSE/ISE/ CS DS/CS ML

Question 1 is compulsory and Answer any 6 from the remaining questions.

<table>
<tr><th rowspan="2"></th><th rowspan="2"></th><th rowspan="2">Marks</th><th colspan="2">OBE</th></tr>
<tr><th>CO</th><th>RBT</th></tr>
<tr><td>1</td><td>Every year a man trades his car for a new car in 3 brands of the popular company 'Tata motors'. If he has a 'Nexon' he trades it for a 'Punch'. If he has a 'Punch' he trades if for a 'Jaguar Land Rover'. If he has a 'Jaguar Land Rover' he is just as likely to trade it for a new 'Jaguar Land Rover' or for a 'Punch' or a 'Nexon' one. In 2020 he bought his first car which was 'Jaguar Land Rover'. Find the probability that he has
i) 2022 'Jaguar Land Rover' ii) 2022 'Nexon' iii) 2023 'Punch' iv) 2023 'Jaguar Land Rover'</td><td>[8]</td><td>CO3</td><td>L3</td></tr>
<tr><td>2</td><td>The probability distribution of a finite random variable X is given by the following data.
i) Find the value of K, ii) $P(X \geq 5)$ and iii) $P(3 < X \leq 6)$.
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>P(X)</td><td>K</td><td>3K</td><td>5K</td><td>7K</td><td>9K</td><td>11K</td><td>13K</td></tr>
</table></td><td>[7]</td><td>CO1</td><td>L2</td></tr>
<tr><td>3</td><td>Find the mean and standard deviation of the Poisson distribution.</td><td>[7]</td><td>CO2</td><td>L2</td></tr>
<tr><td>4</td><td>Out of 800 families with 5 children each, how many families would you expect to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) at most 2girls, assuming equal probabilities for boys and girls.</td><td>[7]</td><td>CO2</td><td>L3</td></tr>
</table>



<table>
<tr><td>5</td><td>In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed. It is given that $\phi(1.2263) = P(0 \leq Z \leq 1.2263) = 0.39$ and $\phi(1.4757) = P(0 \leq Z \leq 1.4757) = 0.43$</td><td>[7]</td><td>CO2</td><td>L3</td></tr>
<tr><td>6</td><td>The joint distribution of two random variables X and Y is as follows.
<table>
<tr><td>Y
X</td><td>-4</td><td>2</td><td>7</td></tr>
<tr><td>1</td><td>1/8</td><td>1/4</td><td>1/8</td></tr>
<tr><td>5</td><td>1/4</td><td>1/8</td><td>1/8</td></tr>
</table>
Compute the following: (i) E(X) and E(Y) (ii) E(XY) (iii) ρ(X, Y)</td><td>[7]</td><td>CO2</td><td>L2</td></tr>
<tr><td>7</td><td>Define (i) Statistical Hypothesis (ii) Critical region of statistical test (iii) Type I and II error (iv) Test of significance</td><td>[7]</td><td>CO4</td><td>L1</td></tr>
<tr><td>8</td><td>It has been found from an experience that the mean breaking strength of a particular brand of thread is 275.6gms with standard deviation of 39.7gms. Recently a sample of 36 pieces of thread showed a mean breaking strength of 253.2gms. Can one conclude at a significance level of (a) 0.5 and (b) 0.01 that the thread has become inferior?
Tabel values $Z_{0.05} = \pm 1.645\ (One - tailed),\ Z_{0.05/2} = \pm 1.96 (Two - tailed),$
$Z_{0.01} = \pm 2.33\ (One - tailed),\ Z_{0.01/2} = \pm 2.58\ (Two - tailed),$</td><td>[7]</td><td>CO4</td><td>L2</td></tr>
</table>

Q1. • Let the three states be N-Nexon,
P-Punch,
J-Jaguar land rover.

• From the given data, the transition probability matrix P can be written as

$$P = \begin{array}{c|ccc} & N & P & J \\ \hline N & 0 & 1 & 0 \\ P & 0 & 0 & 1 \\ J & 1/3 & 1/3 & 1/3 \end{array}$$

• In 2020, initial state, the probability vector $p^{(0)}$ can be written as $p^{(0)} = [0 \quad 0 \quad 1]$

• In 2021, first step probability vector $p^{(1)} = p^{(0)} P$

$$\Rightarrow p^{(1)} = [0 \quad 0 \quad 1] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} = [1/3 \quad 1/3 \quad 1/3]$$

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• In 2022, second step probability vector $p^{(2)} = p^{(1)} P$

$$\Rightarrow p^{(2)} = [1/3 \quad 1/3 \quad 1/3] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} = [1/9 \quad 4/9 \quad 4/9]$$

★ ∴ i) In 2022, the probability that he has a Jaguar land Rover is $\boxed{4/9}$

★ ii) In 2022, the probability that he has a Nexon is $\boxed{1/9}$

• In 2023, third step probability vector $p^{(3)} = p^{(2)} P$

$$\Rightarrow p^{(3)} = [1/9 \quad 4/9 \quad 4/9] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} = [4/27 \quad 7/27 \quad 16/27]$$

★ ∴ iii) In 2023, the probability that he has a Punch is $\boxed{7/27}$

★ iv) In 2023, the probability that he has a Jaguar land Rover is $\boxed{16/27}$

2. wkt (given), the probability distribution of a finite random variable (X) is

X	0	1	2	3	4	5	6
P(X)	k	3k	5k	7k	9k	11k	13k

i) wkt as P(X) is a probability distribution it has to satisfy

a) $P(X) \geq 0 \quad \forall X \Rightarrow k \geq 0$

b) $\sum P(X) = 1 \Rightarrow k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$$\Rightarrow \boxed{k = 1/49}$$

4

~~on substituting~~

ii) $P(X \geq 5) = P(5) + P(6) = 11k + 13k$

$$= 24k = \frac{\cancel{24}}{\cancel{49}}$$

$$\therefore \boxed{P(X \geq 5) = \frac{24}{49}}$$

iii) $P(3 < X \leq 6) = P(4) + P(5) + P(6)$

$$= 9k + 11k + 13k$$

$$= 33k.$$

$$\boxed{P(3 < X \leq 6) = \frac{33}{49}}$$

3). wkt the probability function for a poisson's distributio is given by $P(X) = \left.\frac{m^{+x} e^{-m}}{x!}\right] \longrightarrow (1)$

wkt the mean $\mu = \sum_{i=0}^{\infty} x P(x)$

$$\mu = \sum_{i=0}^{\infty} \frac{x\, m^{+x} e^{-m}}{x!}$$

$$\mu = e^{-m} \sum_{i=0}^{\infty} \frac{x\, m^{+x}}{x!}$$

$$\mu = e^{-m} \sum_{i=1}^{\infty} \frac{\cancel{x}\, m^{+x}}{\cancel{x}(x-1)!} \cdot \frac{m}{m}$$ $\therefore n! = n(n-1)!$

④

$$\mu = e^{-m} \sum_{i=1}^{\infty} \frac{m^{+x-1} \cdot m^{+1}}{(x-1)!}$$

$$\mu = me^{-m} \sum_{i=1}^{\infty} \frac{m^{+x-1}}{(x-1)!}$$

$$\mu = me^{-m} \left[\frac{m^0}{0!} + \frac{m^1}{1!} + \dots \right] \rightarrow e^{-m}$$

$$\mu = me^{-m} \cdot e^{m}$$

$$\therefore \underline{\underline{\mu = m}}$$

wkt the variance $\sigma^2 = \sum_{i=0}^{\infty} (x-\mu)^2 P(X) = \sum_{i=0}^{\infty} x^2 P(X) - \mu^2$

$$\sigma^2 = \sum_{i=0}^{\infty} (x^2 + x - x) P(X) - \mu^2$$

$$\sigma^2 = \sum_{i=0}^{\infty} [x(x-1) + x]\, P(X) - \mu^2$$

$$\sigma^2 = \sum_{i=0}^{\infty} x(x-1) P(X) + \sum_{i=0}^{\infty} x P(X) - \mu^2 \longrightarrow (2)$$

on expanding $\sum_{i=0}^{\infty} x(x-1) P(X) = \sum_{i=0}^{\infty} \frac{x(x-1)\, m^x e^{-m}}{x!}$

$$\Rightarrow \sum \frac{x(x-1)\, e^{-m}\, m^{x}}{x!} = e^{-m} \sum_{i=0}^{\infty} \frac{x(x-1)\, m^{x}}{x!} \cdot \frac{m}{m}$$

$$= e^{-m} \sum_{i=1}^{\infty} \frac{\cancel{x}(x-1)\, m^{x-1} \cdot m}{\cancel{x}(x-1)!} \cdot \frac{m}{m}$$

$$= me^{-m} \sum_{i=2}^{\infty} \frac{\cancel{(x-1)}\, m^{x-2} \cdot m}{\cancel{(x-1)}(x-2)!}$$

$$= m^2 e^{-m} \sum_{i=2}^{\infty} \frac{m^{x-2}}{(x-2)!}$$

$$= m^2 e^{-m} \left[\frac{m^0}{0!} + \frac{m^1}{1!} + \frac{m^2}{2!} \cdots \right] \rightarrow e^{m}$$

$$= m^2 e^{-m} . e^{m}$$

$$\sum \frac{x(x-1)e^{-m} m^{x}}{x!} = m^2$$

on substituting the value in equation 2,

$$\sigma^2 = m^2 + \overset{m}{\sum_{i=0}^{\infty} xP(x)} - \mu^2$$

$$\sigma^2 = m^2 + m - m^2$$

$\therefore \boxed{\sigma^2 = v = m}$ $\quad\therefore$ variance of Poisson's dist^n = m

$\boxed{\therefore \text{ std dev } \sigma = \sqrt{m}}$ → standard deviation of Poisson's Distribution

4. from the given data,

for 1 family $n = 5$

$p = 1/2$ (probability of success - having a ~~girl~~ boy)

$q = 1/2$ (probability of failure - having a ~~boy~~ girl)

$x \rightarrow$ no of favourable outcomes = ~~the~~ no of boys.

wrt the probability function $P(X) = {}^nC_x\, p^x\, q^{n-x}$ as per the binomial distribution => $P(X) = {}^5C_x\, {\frac{1}{2}}^x\, {\frac{1}{2}}^{5-x}$

i) $P(X=3) = {}^5C_3\, {\frac{1}{2}}^3\, {\frac{1}{2}}^2 = \frac{5!}{3!\,2!} \cdot \frac{1}{2}^5 = 10 \times \frac{1}{32}$

$\boxed{P(X=3) = 5/16}$ → for 1 family ∴ for 800 families, $\boxed{P(X=3) = 250.}$ ★★

↳ ×800

ii) $P(X=0) \Rightarrow$ 0 boys and 5 girls

$\Rightarrow 800 \times \left(\frac{1}{2}\right)^5 \times {}^nC_0 = 25$

∴ for 800 families $\boxed{P(X=0) = 25}$ ★★

iii) $\overset{\text{3 boys}}{P(X=3)}$ or $\overset{\text{2 boys}}{P(X=2)} = P(X=3) + P(X=2)$

$= {}^5C_3\, {\frac{1}{2}}^3\, {\frac{1}{2}}^2 + {}^5C_2\, {\frac{1}{2}}^2\, {\frac{1}{2}}^3$

$= \frac{1}{32} \times 800 \times \left[\frac{5!}{3!\,2!} + \frac{5!}{2!\,3!}\right]$

$\boxed{P(X=3 \text{ or } \cancel{P}(X=2)) = 500}$ ~~★~~ ★★

7

iv) ~~P(X≥3)~~ At most two girls $= P(X=3) + P(X=4) + P(X=5)$

0 girls = 5 boys
1 girl = 4 boys
2 girl = 3 boys

$= \left[{}^5C_3 (1/2)^5 + {}^5C_4 (1/2)^5 + {}^5C_5 (1/2)^5\right] \times 800$

$= \frac{800}{32}\left[{}^5C_3 + {}^5C_4 + {}^5C_5\right] = 25\,[10+5+1]$

$\boxed{\therefore \text{probability of having at most 2 girls} = 400}$

5). $P(x < 0.35) = 7\% = 0.07$ $P(x < 0.6) = 89\% = 0.89$.

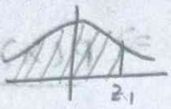
$P(x < 0.35) = 0.07$ $P(x < 0.6) = 0.89$.

wkt ~~[illegible]~~

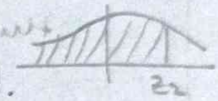
the std normal variate $\boxed{z = \frac{x-\mu}{\sigma}}$

when $x = 0.35$, $z_1 = \frac{0.35-\mu}{\sigma} \rightarrow (1)$ when $x = 0.6$, $z_2 = \frac{0.6-\mu}{\sigma} \rightarrow (2)$

$\Rightarrow P(z < z_1) = 0.07$



$P(z < z_2) = 0.89$.



$\Rightarrow P(z \geq 0) + P(0 \leq z \leq z_1) = 0.07$

$0.5 + \phi(z_1) = 0.07$.

$\phi(z_1) = 0.07 - 0.5$

$\phi(z_1) = -0.43$

given $\phi(1.4757) \simeq 0.43$.

$\therefore \underline{z_1 = -1.4757}$

4

$P(z \leq 0) + P(0 \leq z \leq z_2) = 0.89$

$0.5 + \phi(z_2) = 0.89$

$\phi(z_2) = 0.89 - 0.5$

$\phi(z_2) = 0.39$.

given, $\phi(1.2263) \simeq 0.39$.

$\therefore \underline{z_2 = 1.2263}$

on substituting value of z_1 and z_2 in eqn 1 and 2,

$\Rightarrow -1.4757 = \frac{0.35-\mu}{\sigma}$ $\Rightarrow 1.2263 = \frac{0.6-\mu}{\sigma}$.

$\mu - 1.4757\sigma = 0.35$ $\mu + 1.2263\sigma = 0.6$

$\therefore \mu = 0.4865$ $y = 0.0925$

$\Rightarrow \mu \times 100\%$

$\boxed{\mu = 48.65}$ $\boxed{\sigma = 0.92}$

6. given, the joint probability distribution table of two random variables X and Y.

x \ y	-4	2	7	sum	
1	1/8	1/4	1/8	1/2	f(1)
5	1/4	1/8	1/8	1/2	f(5)
sum	3/8	3/8	2/8	1	
	g(-4)	g(2)	g(7)		

4

i) $E(X) = \sum_{i=1}^{n} x_i f(x_i)$ $= 1(1/2) + 5(1/2) = 1/2 + 5/2$

$\boxed{E(X) = 3}$

$E(Y) = \sum_{i=1}^{n} y_j g(y_j) = -4(3/8) + 2(3/8) + 7(2/8) = \frac{20-12}{8}$

$\boxed{E(Y) = 1}$

ii) $E(XY) = \sum x y f(x, y) = \sum x_i y_j J_{ij}$

$= 1(-4)(1/8) + 1(2)(1/4) + 1(7)(1/8)$
$+ 5(-4)(1/4) + 5(2)(1/8) + 5(7)(1/8)$

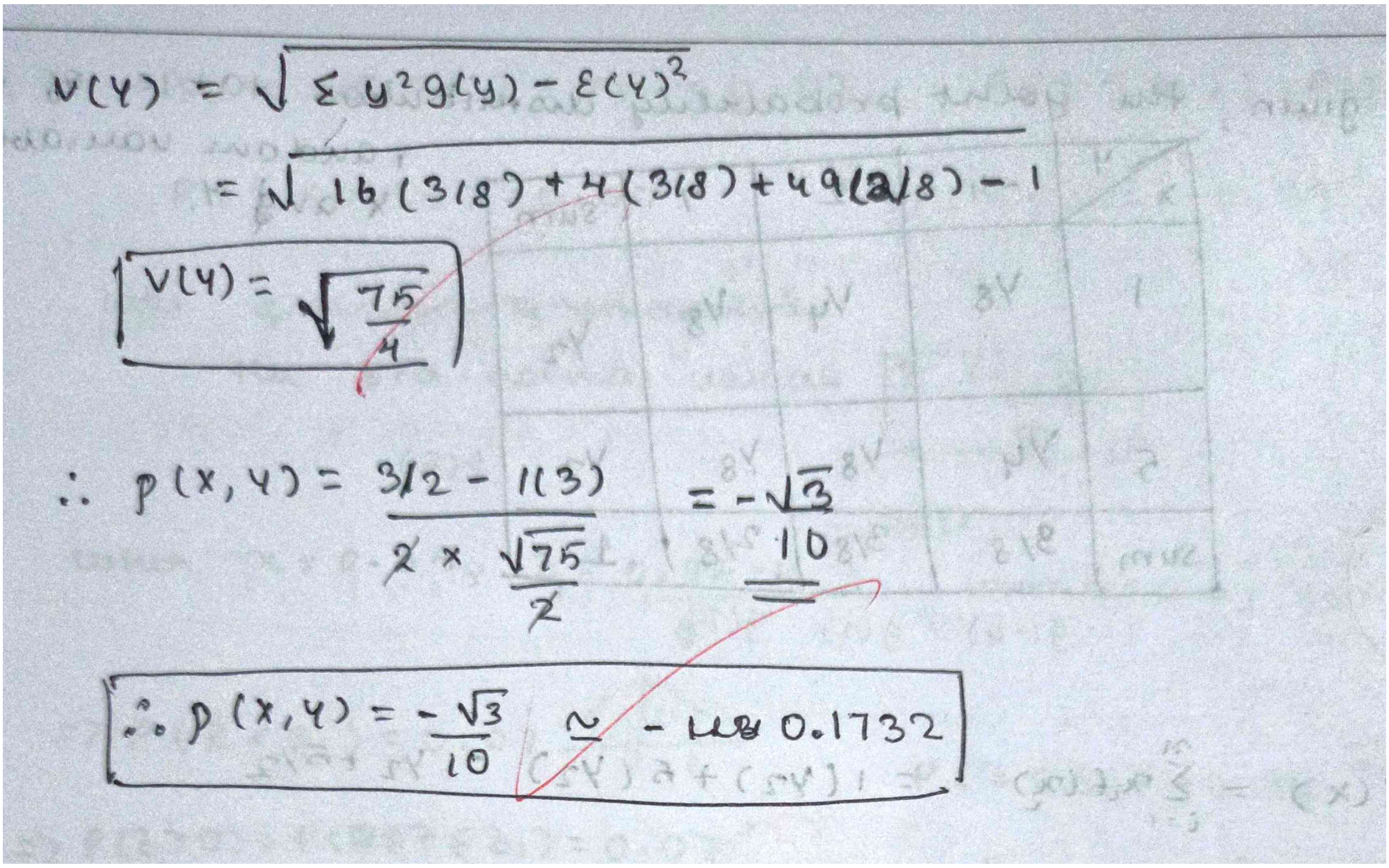
$= \frac{-4}{8} + \frac{2}{4} + \frac{7}{8} - \frac{20}{4} + \frac{10}{8} + \frac{35}{8} = \frac{3}{2}$

$\boxed{\therefore E(XY) = 3/2}$

iii) $\rho(X, Y) = \frac{\mathrm{cov}(X Y)}{\sigma_x \sigma_y} = \frac{E(XY) - E(X)(E(Y))}{\sqrt{V(X)}\ \sqrt{V(Y)}}$

$V(X) = \sqrt{\sum x^2 f(x) - \mu x^2} = \sqrt{1(1/2) + 25(1/2) - 9}$
$= \sqrt{1/2 + 25/2 - 9} = \sqrt{13 - 9} = 2$

$\underline{V(X) = 2}$



$$V(Y) = \sqrt{\Sigma\, y^2 g(y) - E(Y)^2}$$

$$= \sqrt{16(3/8) + 4(3/8) + 49(2/8) - 1}$$

$$\boxed{V(Y) = \sqrt{\frac{75}{4}}}$$

$$\therefore \rho(X,Y) = \frac{3/2 - 1(3)}{\cancel{2} \times \frac{\sqrt{75}}{\cancel{2}}} = \underline{\underline{-\frac{\sqrt{3}}{10}}}$$

$$\boxed{\therefore \rho(X,Y) = -\frac{\sqrt{3}}{10} \simeq -0.1732}$$

7. (i) Statistical hypothesis:-

A hypothesis is a statement about a population parameter that can be tested using statistical methods.
There are 2 hypothesis like Null Hypothesis (H_0) and Alternative Hypothesis (H_1).

(ii) Critical region of statistical test: The range of values where the null hypothesis is rejected is called critical region of statistical test ~~If~~
If ~~we~~ ~~fail to~~ test statistic fails in that regio
then we conclude that the result is statistically significant

(iii) Type I and Type II error:

Type I error: If we reject the null hypothesis when it is actually true, then it is called Type I error. (false positive)

Level of significance denoted by 'α'.

~~If we reje~~

Type II error: If we fail to reject the hypothesis when it is actually false (false negative) is called Type II error.

Level of significance denoted by 'β'.

(iv) **Test of significance:** The method used to determine that the observed data provides enough evidence to ~~prove~~ reject the null hypothesis.

There are various test statistics like z test, t test etc.

expected

8. $H_0 : \overset{\swarrow}{\mu} \leq 275.6$ gms, $\sigma = 39.7$ gms

$H_1 : \mu > 275.6$.

$n = 36$, $x = 253.2$ gms

2

wkt $Z = \dfrac{x-\mu}{\sigma\sqrt{1/n}} = \dfrac{253.2 - 275.6}{39.7\sqrt{1/36}}$

$Z = \dfrac{-22.4}{6.5166} \simeq -3.4373$

$|Z| \simeq 3.4373$.

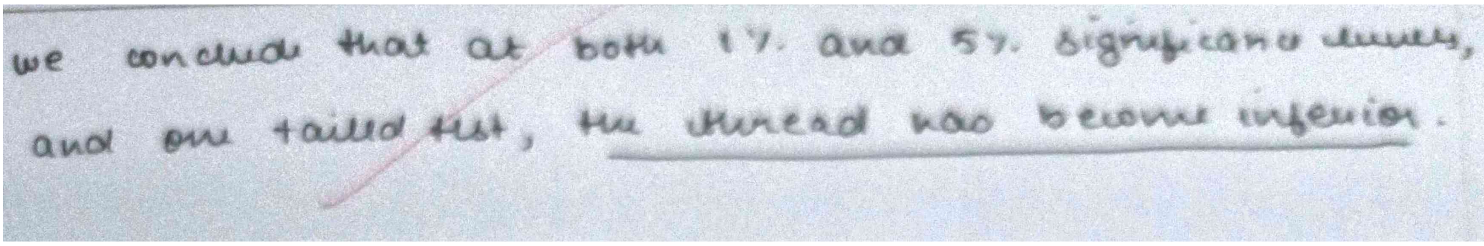
a). wrt one tail test, $Z > Z_{0.05}$

$\Rightarrow 3.473 > 1.65$

⇒ Thus, wrt 5% significance level, we must reject the hypothesis

b). wrt one tail test, $Z > Z_{0.01} \Rightarrow 3.473 > 2.33$

∴ Thus wrt 1% significance level we must reject the hypothesis.



we conclude that at both 1% and 5% significance levels, and one tailed test, the thread has become inferior.