

① * Hash function:

→ Hash function is a mathematical function which transforms the input (or "message") into a fixed-sized output string of bytes, typically a value called as hash value.

→ A general hash function has the following 3 properties:

1) Input flexibility: Input can be of any size and any string.

2) Fixed-output size: Output has a pre-determined fixed length. For example, a hash function gives a 256-bit output hash value.

3) Computational efficiency: The hash function should find the hash value in a reasonable amount of time, i.e., it should be computationally efficient.

* Cryptographic hash function:

→ To make the hash value cryptographically secure, three additional hash properties have to be followed:

1) Collision resistant:

→ A hash function is said to be collision resistant if it is infeasible to find two distinct inputs x and y such that $x \neq y$, but $H(x) = H(y)$.

→ Since the input is infinite and the output is finite, it is ensured that multiple inputs have the same output.

→ For example, for a 256-bit output string, there

are 2^{256} possible outputs. Hence, if we take $2^{256} + 1$ input strings, it is guaranteed to have a pair of inputs mapping to the same output.

→ It will require a computation of 2^{256} but according to birthday paradox, only the square root of computations, approximately 2^{130} are required and it is guaranteed to have a collision with 99.8% probability.

2) Hiding:

→ A hash function is hiding for $y = H(x)$, if it is infeasible to find the input x .

→ It is required that the x is spread out, that it is not easily predictable.

→ If x is not spread out, choose a random r with high min-entropy distribution such that $H(r || x) = y$, makes it difficult to predict x .

→ min-entropy refers to the difficulty in predicting the value & hence makes the hash function secure. Example has $1/2^{256}$ probability.

3) Puzzle friendliness:

→ A hash function is said to be puzzle friendly for a n -bit output y , and a random variable k with high min-entropy such that $H(k || x) = y$ it is infeasible to determine the input x in significantly less than 2^n times.

- For a search puzzle, there are multiple possible values of x , which requires exhaustive search.
- Here, the puzzle ID id is chosen at the random entropy value and concatenated with x .
 $H(id || x) = y$ is the hash function value & the target set of outputs.
- If y is the output set which is small, it's determining x difficult; if y is large as much as n , determining x becomes trivial.

Example:

SHA-256 combines all these properties to make hash function secure, provide data integrity & security.

② * Hash pointers:

- Hash pointers combine 2 things: a pointer to previous block and a cryptographic hash of its contents.
- Hash pointer can be used ~~at~~ instead of regular pointer to detect tamper to data and data integrity.
- There are 2 data structures that can be implemented using hash pointers:
 - 1) Blockchain: (linked list with hash pointers)
 - Instead of using a regular pointer, a hash is used in a blockchain.

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Example:

SHA-256 combines all these properties to make the hash function secure, provide data integrity & reliability.

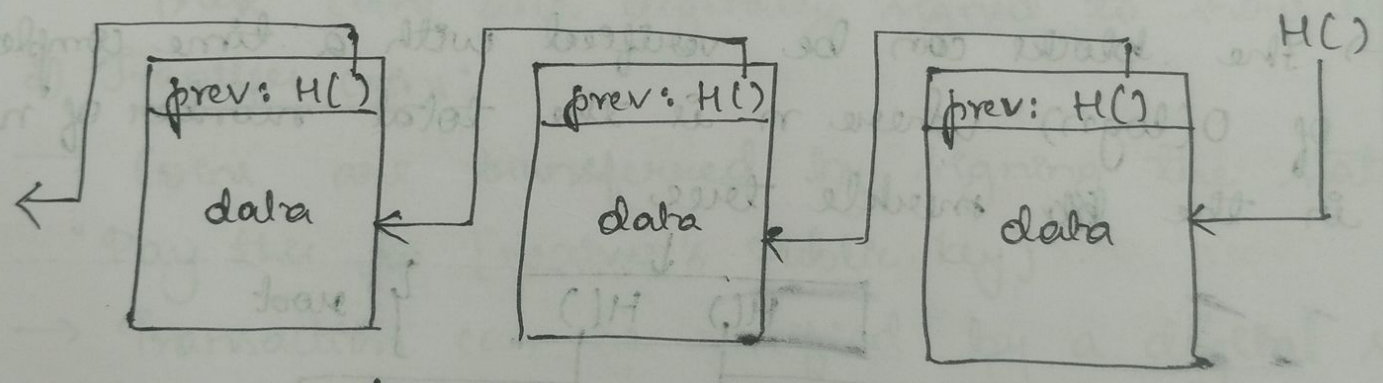
② Hash pointers:

- Hash pointers combine 2 things: a pointer to the previous block and a cryptographic hash of its contents.
- Hash pointer can be used ~~as a~~ instead of a regular pointer to detect tamper to data and maintain data integrity.
- There are 2 data structures that can be implemented using hash pointers:

1) Blockchain: (linked list with hash pointers)

- Instead of using a regular pointer, a hash pointer is used in a blockchain.

- It consists of 2 informations:
 - 1) Pointer to previous block.
 - 2) ^{Cryptographic} Hash of its contents.
- Each block in the blockchain contains data and a hash pointer.
- If an adversary tampers the data in block k , it will not match with the hash in block $k+1$, indicating tampering.
- This tampering will be detected at the head of the list following the chained connections.
- The head of the list points to the most recent block.



* Blockchain structure

2) Merkle tree: (Binary tree with hash pointers)

- Here the regular pointers are replaced with hash pointers.
- The data block forms the leaves.
- Each parent consists of 2 hash pointers to its child nodes.
- This pattern continues until a single root node is reached.

→ Proof of membership:

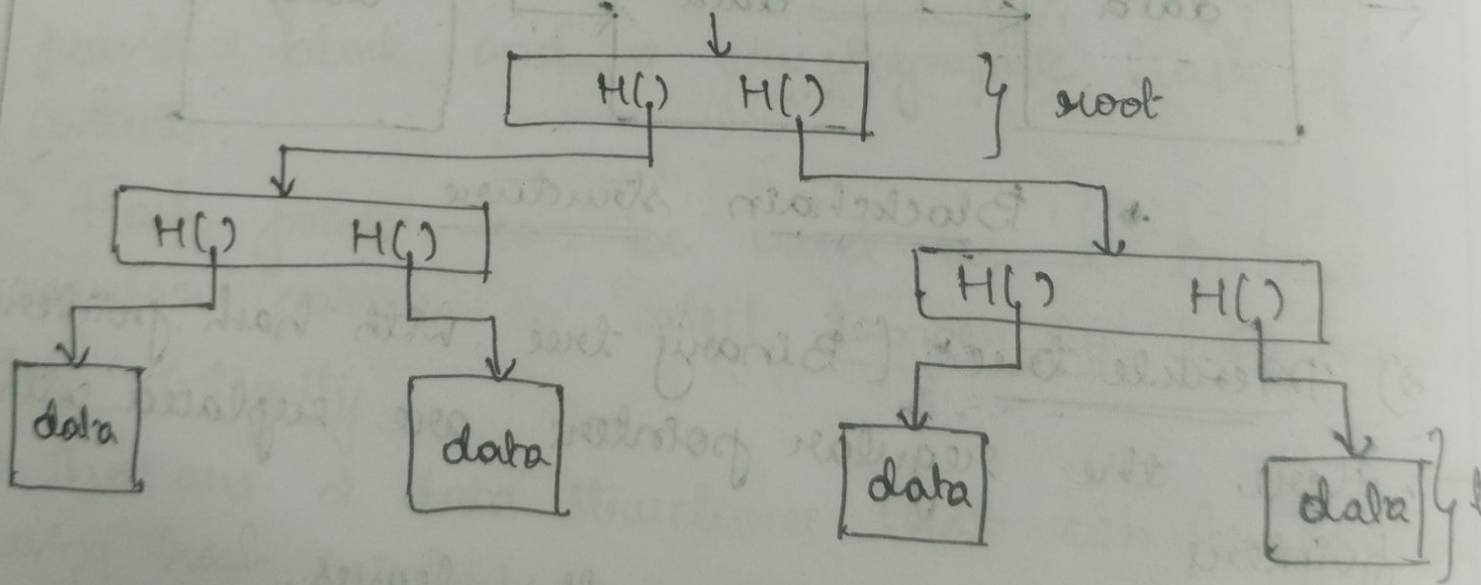
A block can be verified if it is a member of the merkle tree by following the hashes from that block to the root.

→ Proof of non-membership:

Using a sorted merkle tree, we can verify if a block is a part of the tree by verifying the block before and after that block.

If both the blocks are continuous, the queried block is not present.

→ The blocks can be verified with a time complexity of $O(\log n)$ where n is the total number of nodes in the merkle tree.



* Merkle tree structure

③. * Cryptocurrency types:

→ There are 2 types of cryptocurrencies!

- 1) Goofy Coin
- 2) Scrooge Coin

* Goofy coin:

→ It is the simplest cryptocurrency which consists of 2 types of transactions:

1) Create coin:

→ Coins are created by signing the statement `createCoin[uniqueCoinID]`.

→ These coins are digitally signed to show its validity.

2) Transfer coin:

→ Coins are transferred by signing the statement "Pay this to [receiver's Public key]".

→ Transaction can be verified by a digital signature.

→ Double spending attack:

During the transfer, it is possible to transfer the same coin to more than one recipient.

For example:

→ If Alice has 25 coins initially.

→ Then, Alice may sign 2 statements transferring the same coin to Bob and Chuck.

→ Now both Bob and Chuck have a valid claim to this coin.

→ This creates distrust and undermines the currency system.

signed by sk_{Alice}
pay to $pk_{Bob}: H(\cdot)$

signed by sk_{Goofy}
pay to $pk_{Alice}: H(\cdot)$

signed by sk_{Goofy}
Create Coin [unique coin]

* Example for Double-spending in Goofy coins

* Serogee coin:

→ It is an implementation of Goofy coin with a prevention mechanism for double spending attacks

→ These are the features of Serogee coin that overcome the drawback of goofy coin:

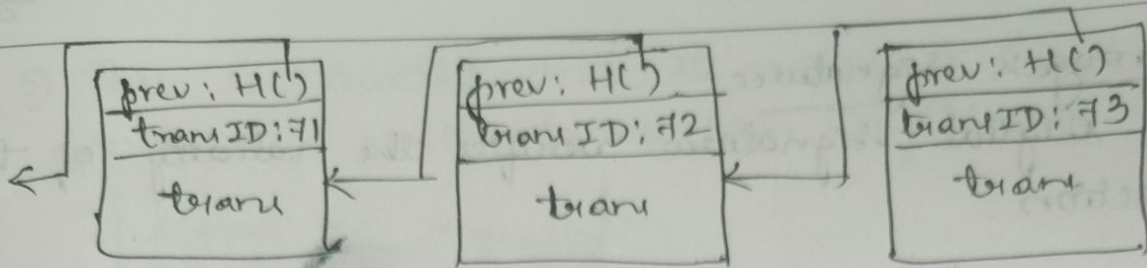
1) Append-only ledger:

→ There is a public ledger which has all transaction records.

→ It is append-only i.e., no modifications are allowed which makes the system transparent & secure.

2) Blockchain structure:

→ It has the blockchain structure where each block points to the previous block with a hash pointer.



* ScroogeCoin structure

3) Transaction types:

→ Create coins: It can create coin and assign initial owner.

transID: 73			type: CreateCoins
Coin created			
num	value	recipient	
0	3.1	0x...	→ 73[0]
1	3.6	0x...	→ 73[1]
2	7.1	0x...	→ 73[2]

→ pay coins: It pays the amount by creating new coin to the receiver and destroying spent coins.

transID: 74			type: payCoins
consumed coinIDs: 69[0], 80[4], 90[1]			
Coin created			
num	value	recipient	
0	3.1	0x...	→ 73[0]
1	3.6	0x...	→ 73[1]
2	7.1	0x...	→ 73[2]
Digital signatures.			

4) Validation rules:

- coins are created before being consumed.
- coins are not spent in previous transactions
- the value of coin created is equal to those coins spent.
- All participants (owners of the coin) sign the

5) Scrooge's signature:
The digital signature verifies the validity of the transaction.

→ Some drawbacks of Goofy coin are:

- 1) Distrust: Since coins can be easily duplicated, it is not a secure currency.
- 2) Devalues the currency: Since double spending is possible, the currency loses its value.

→ All the features mentioned above overcome the drawbacks of goodycoin.

④ Double Spending attack,

→ This attack is visible in goodycoin.

→ It occurs when the same coin is transferred to multiple recipients.

→ For example:

- 1) Alice has created a coin.
- 2) Transfers the coin to Bob by signing the transaction.
- 3) Now, without notifying anyone, Alice signs another transaction transferring the coin to Chuck.
- 4) This enables both Bob and Chuck to claim that coin.

5) This is invalid and it makes it difficult to find the valid transaction / legitimate transaction.

Signed by sk Alice sk Alice
pay to pk Bob: H(G)

↓

Signed by sk Goofy
pay to pk Alice: H(G)

↓

Signed by sk Goofy
create coin [unique coin ID]

* Double spending attack in
Goofy coin

- These coins are created by Goofy.
- The coin creation requires the signature of Goofy using secret key.
- This is paid to Alice with a valid digital signature of Goofy.
- Now Alice pays this coin to Bob by signing the statement "Pay this to Bob".
- Afterwards, Alice also pays it to Chuck.
- Now both have valid claims to this coin.
- This is called Double-spending attack.

* Double spending occurs due to:

- 1) No centralized ledger like in ScroogeCoin.
- 2) No prevention mechanism.

* Double spending creates:

- 1) Distrust among users.
- 2) Devalues the authenticity of the currency.

⑤ * Digital Signatures:

→ Digital Signature scheme shows authenticity of the transactions or messages and helps in verifying the validity of the message.

→ There are 3 key algorithms:

1) Key generation:

→ It is used to generate a pair of keys:
 $(sk, pk) := \text{generate}(key\ size)$

→ Input: key size is given as input.

→ Output: Secret key (sk): private & used for signing.

Public key (pk): publicly available &

→ It is necessary to process further steps.

2) Signing:

→ It is used to authenticate or sign the message.

$$\text{sig} := \text{sign}(\text{sk}, \text{message})$$

→ Input: Secret key (sk)

A message

→ Output: Digital signature created by the owner.

→ The purpose is to show that the message is legitimate.

3) Verification:

→ It is used to verify the validity of the message

$$\text{isValid} := \text{verify}(\text{pk}, \text{msg}, \text{sig})$$

→ Input: public key (pk)

A message (msg)

Digital signature (sig)

→ Output: isValid is a boolean value. If it is true, it shows that the message is valid otherwise it is invalid.

→ The purpose is to verify the security and data integrity & reliability.

* Properties of Digital Signatures:

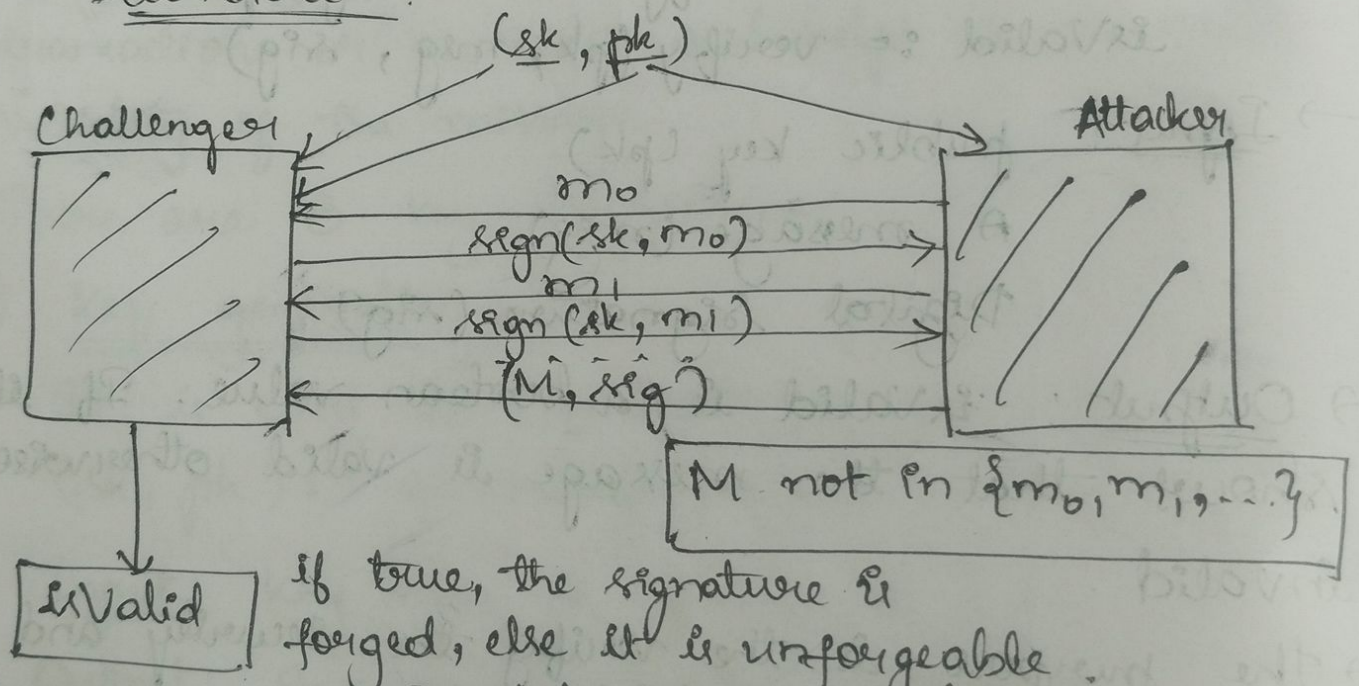
1) Validity: It is used to determine if the message is valid and from a legitimate sender.

$$\text{verify}(\text{pk}, \text{msg}, \text{sign}(\text{sk}, \text{msg})) = \text{true}$$

d) Unforgeability: If the verification returns true, the message is unforgeable or it is forged. It says, ^{that} it is impossible to determine the message without the digital signature & secret key. A message can be signed only if the secret key is known.

It is not possible by knowing either:
 → The public key
 → Or the digital signatures of other messages.

* Illustration:



Setup:

A challenger creates 2 keys: secret and public (sk, pk). The attacker only has access to public key (pk).

Challenge:

The attacker sends a new message M along with its digital signature created by the attacker.

Outcome:

- If the verification of this signature returns true, it implies that the signature can be forged otherwise the signature is unforgeable.
- It is intended that the Challenger signs the message sent by the Attacker to validate it.

Example: ECDSA (Elliptic Curve Digital Signature Algorithm).