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Internal Assessment Test – I November 2024

Sub:	Mathematics for Computer Science							Code:	BCS301
Date:	06/11/2024	Duration:	90 mins	Max Marks:	50	Sem:	III	Branch:	AIML/AIDS/ CSE/ISE/ CS DS/CS ML

Question 1 is compulsory and Answer any 6 from the remaining questions.

<table>
<tr><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2">Marks</td><td colspan="2">OBE</td></tr>
<tr><td>CO</td><td>RBT</td></tr>
<tr><td>1</td><td>Every year a man trades his car for a new car in 3 brands of the popular company 'Tata motors'. If he has a 'Nexon' he trades it for a 'Punch'. If he has a 'Punch' he trades if for a 'Jaguar Land Rover'. If he has a 'Jaguar Land Rover' he is just as likely to trade it for a new 'Jaguar Land Rover' or for a 'Punch' or a 'Nexon' one. In 2020 he bought his first car which was 'Jaguar Land Rover'. Find the probability that he has
i) 2022 'Jaguar Land Rover' ii) 2022 'Nexon' iii) 2023 'Punch' iv) 2023 'Jaguar Land Rover'</td><td>[8]</td><td>CO3</td><td>L3</td></tr>
<tr><td>2</td><td>The probability distribution of a finite random variable X is given by the following data. i) Find the value of K, ii) $P(X \geq 5)$ and iii) $P(3 < X \leq 6)$.
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>P(X)</td><td>K</td><td>3K</td><td>5K</td><td>7K</td><td>9K</td><td>11K</td><td>13K</td></tr>
</table></td><td>[7]</td><td>CO1</td><td>L2</td></tr>
<tr><td>3</td><td>Find the mean and standard deviation of the Poisson distribution.</td><td>[7]</td><td>CO2</td><td>L2</td></tr>
<tr><td>4</td><td>Out of 800 families with 5 children each, how many families would you expect to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) at most 2girls, assuming equal probabilities for boys and girls.</td><td>[7]</td><td>CO2</td><td>L3</td></tr>
</table>

<table>
<tr><td>5</td><td>In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed. It is given that $\phi(1.2263) = P(0 \leq Z \leq 1.2263) = 0.39$ and $\phi(1.4757) = P(0 \leq Z \leq 1.4757) = 0.43$</td><td>[7]</td><td>CO2</td><td>L3</td></tr>
<tr><td>6</td><td>The joint distribution of two random variables X and Y is as follows.
<table>
<tr><td>Y
X</td><td>-4</td><td>2</td><td>7</td></tr>
<tr><td>1</td><td>1/8</td><td>1/4</td><td>1/8</td></tr>
<tr><td>5</td><td>1/4</td><td>1/8</td><td>1/8</td></tr>
</table>
Compute the following: (i) E(X) and E(Y) (ii) E(XY) (iii) $\rho(X, Y)$</td><td>[7]</td><td>CO2</td><td>L2</td></tr>
<tr><td>7</td><td>Define (i) Statistical Hypothesis (ii) Critical region of statistical test (iii) Type I and II error (iv) Test of significance</td><td>[7]</td><td>CO4</td><td>L1</td></tr>
<tr><td>8</td><td>It has been found from an experience that the mean breaking strength of a particular brand of thread is 275.6gms with standard deviation of 39.7gms. Recently a sample of 36 pieces of thread showed a mean breaking strength of 253.2gms. Can one conclude at a significance level of (a) 0.5 and (b) 0.01 that the thread has become inferior?
Tabel values $Z_{0.05} = \pm 1.645\ (One - tailed)$, $Z_{0.05/2} = \pm 1.96(Two - tailed)$,
$Z_{0.01} = \pm 2.33\ (One - tailed)$, $Z_{0.01/2} = \pm 2.58\ (Two - tailed)$,</td><td>[7]</td><td>CO4</td><td>L2</td></tr>
</table>

IAT-1

① Here, we will defines states for each car :-

a_1: Man is having nexon
a_2: Man is having Punch
a_3: Man is having Jaguar Land Rover.

+ This is a Markov chain problem

So, let our transition matrix can be :-

$$P = \begin{array}{ccc} (N) & (P) & (JLR) \\ \end{array}$$
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

given,
In 2020 he bought his first car ⇒ Jaguar Land Rover.

⇒ So, our initial probability vector = $(0 \quad 0 \quad 1) = p^0$

To find
(i) Probability of having 2022 'Jaguar Land Rover'
(ii) Probability of having 2022 'Nexon'
(iii) ,, ,, ,, 2023 'Punch'
(iv) ,, ,, ,, 2023 Jaguar Land Rover

So, for finding probabilities for year 2022, we have to find P^2

$$P \times P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{bmatrix}$$

Now, for finding probabilite for differed car,

$$P^2 \times P^0 : \begin{bmatrix} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{bmatrix} \left[\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \right]$$

$$P^* = \left[\frac{1}{9} \quad \frac{4}{9} \quad \frac{4}{9} \right] \Rightarrow$$ Required transition probability for 2022

∴ (i) Probability that he has Jaguar Land Rover in 2022 $= 4/9$

(ii) Probability that he has Nexon in 2022 : $1/9$

Now, for calculating probability for 2023 :-

$$P^* \times P = \begin{bmatrix} 1/9 & 4/9 & 4/9 \end{bmatrix} \begin{bmatrix} 0 & \overset{1}{\cancel{0}} & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$= \left[\frac{4}{27} \quad \frac{7}{27} \quad \frac{16}{27} \right] \Rightarrow$$ Required transition probability for 2023.

∴ (iii) Probability of having Punch in 2023 $= \frac{7}{27}$

(iv) Probability of having Jaguar Land Rover in 2023 $= \frac{16}{27}$

② Given,
Probability distribution for a finite random variable X :-

X	0	1	2	3	4	5	6
P(x)	K	3K	5K	7K	9K	11K	13K

So, because its probability distribution for finite X,

$$P(X) \geq 0 \quad \& \quad \sum_i P(x_i) = 1$$

So, for fulfilling these mandatory conditions,

$\Rightarrow K \geq 0$, Also, $\sum_i P(x_i) = 1$

$$\therefore \sum_i P(x_i) = 1$$

$$\Rightarrow K + 3K + 5K + 7K + 9K + 11K + 13K = 1$$

$$\Rightarrow 49K = 1$$

$$\Rightarrow \boxed{K = \frac{1}{49}}$$

Now, our table :-

X	0	1	2	3	4	5	6
P(x)	$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49} = \frac{1}{7}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$

Now, (ii) $P(x \geq 5) = P(6)$

$$= \boxed{13/49}$$

(iii) $P(3 < x \leq 6) = P(4) + P(5) + P(6)$

$$= \frac{9}{49} + \frac{11}{49} + \frac{13}{49} = \frac{33}{49}$$

$$= \boxed{\frac{33}{49}}$$

③ $P(x)$ for Poisson distribution = $\frac{m^x e^{-m}}{x!}$

Mean for Poisson distribution =

$$\sum_{x=0}^{\infty} x \cdot P(x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{m^x e^{-m}}{x!}$$

$$= \sum_{x=0}^{\infty} \cancel{x} \cdot \frac{m^x e^{-m}}{\cancel{x}(x-1)!}$$

$$= \sum_{x=1}^{\infty} \frac{m^x e^{-m}}{(x-1)!}$$

$$= \sum_{x=1}^{\infty} \frac{m . m^{x-1} e^{-m}}{(x-1)!}$$

$$= me^{-m} \sum_{x=1}^{\infty} \frac{m^{x-1}}{(x-1)!}$$

$$= me^{-m} \left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \cdots \right]$$

⇓

[This is expansion of e^m]

$$= me^{-m} . (e^m)$$

$$= me^{-m+m}$$

$$= \boxed{me^0}$$

$$= m$$

∴ Mean of Poisson distribution = m

$$\text{Variance} = \left[\sum_{x=1}^{\infty} x^2 . P(x)\right] - (\mu)^2 \quad \left[\mu = \text{Mean}\right] \rightarrow ①$$

Now,

$$\sum_{x=0}^{\infty} x^2 . P(x) = \sum_{x=1}^{\infty} x^2 . \frac{m^x e^{-m}}{x!}$$

$$= \sum_{x=0}^{\infty} \left[x(x-1)+x\right] . \frac{m^x e^{-m}}{x!}$$

$$= \left[\sum_{x=0}^{\infty} x(x-1) \frac{m^x e^{-m}}{x!}\right] + \left[\sum_{x=0}^{\infty} x \frac{m^x e^{-m}}{x!}\right]$$

$$= \sum_{x=0}^{\infty} \frac{\cancel{x(x-1)} m^x e^{-m}}{\cancel{x(x-1)}(x-2)!} + \sum_{x=0}^{\infty} \cancel{x} \frac{m^x e^{-m}}{\cancel{x}(x-1)!}$$

$$= \sum_{x=0}^{\infty} \frac{m^x e^{-m}}{(x-2)!} + \sum_{x=1}^{\infty} \frac{m . m^x e^{-m}}{(x-1)!}$$

$\Downarrow$

= m (by first mean derivation)

$$= \sum_{x=2}^{\infty} \frac{m^2 . m^{x-2} e^{-m}}{(x-2)!} + m$$

$$= m^2 e^{-m} \sum_{x=2}^{\infty} \frac{m^{x-2}}{(x-2)!} + m$$

$$= m^2 e^{-m} \left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \ldots\right] + m$$

$$= m^2 e^{-m} \times e^{m} + m$$

$$= \boxed{m^2 + m}$$

Putting this value in ①, we get

$$\text{Variance } (\sigma^2) = m^2 + m - (u)^2 \qquad [\because \text{Mean} = m]$$

$$= \cancel{m^2} + m - \cancel{m^2}$$

$$\sigma^2 = m$$

$\therefore$ We can say that in Poisson distribution, Mean = Variance.

Now, Standard deviation, $\sigma = \sqrt{m}$ //

$$\therefore \boxed{\begin{array}{l} \text{Mean} = m \\ S.D = \sqrt{m} \end{array}}$$

④ let ~~no.~~ probability of family having boy be x

$n = 5$, $P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

So, for 800 families, lets say $f(x) = 800\, P(x)$ //

$p = 1/2$, $q = 1/2$ [Probability for boy & girls are equal]

(i) $f(3)$

$$\Rightarrow {}^{5}C_{3} \left(\tfrac{1}{2}\right)^{3} \left(\tfrac{1}{2}\right)^{2} \times 800$$

$$= 10 \times \frac{1}{8} \times \frac{1}{4} \times 800$$

$$= \frac{5}{16} \times 800 = 250$$ families will have 3 boys

(ii) ~~f(~~

(ii) 5 girls

⇒ 0 boys

$\Rightarrow \ {}^5C_0(1/2)^0 \ (1/2)^5 \times 800$

$= 25$

∴ 25 families will have 5 girls

(iii) either 2 or 3 boys

$= (f(2) + f(3)) \times 800$

$= \left[{}^5C_2 \ (1/2)^2 (1/2)^3 + {}^5C_3 \ (1/2)^3 (1/2)^2\right] \times 800$

$= \left[\overset{5}{\cancel{10}} \times \frac{1}{\cancel{4}_2} \times \frac{1}{8} + 10 \times \frac{1}{8} \times \frac{1}{4}\right] \times 800$

$= \frac{5}{\cancel{8}} \ \cancel{\times 80} \times \cancel{800}^{100}$

$= \boxed{500}$ families will have either 2 or 3 boys

~~∴ 5~~

(iv) atmost 2 girls

⇒ 5 boys + 4 boys 1 girl + 3 boys 2 girls

$= f(5) + f(4) + f(3)$

$= {}^5C_5 (1/2)^5 \times (1/2)^0 + {}^5C_4 (1/2)^4 \left(\frac{1}{2}\right)^1 + {}^5C_3 (1/2)^3 (1/2)^2$

$= 1 \times \frac{1}{32} \times 1 + 5 \times \frac{1}{16} + \frac{1}{2} + \overset{5}{\cancel{10}} \times \frac{1}{\cancel{8}_4} \times 1/4$

$= 1/32 + \frac{5}{32} + \frac{5}{16} = \frac{1+5+10}{32} = \frac{\cancel{16}}{\cancel{32}} \times \cancel{800}^{400} =$ 400 families

400 families

⑤ Given,

Probability of students scoring less than 35% marks

$= 7\% = 0.07$

$P(x < 35) = 0.07$

Probability of students scoring less than 60% marks $= 89\% = 0.89$

$P(x < 60) = 0.89$

$Z_1 = \frac{x-\mu}{\sigma} = \frac{35-\mu}{\sigma}$ ~~= 0~~. $\rightarrow$ (*)

$Z_2 = \frac{x-\mu}{\sigma} = \frac{60-\mu}{\sigma} \rightarrow$ (#)

$P(Z < Z_1) = 0.07$ ~~⇒ 0~~ $0.5 + \phi(z_1) = 0.07 \rightarrow$ ①

$P(Z < Z_2) = 0.89 \Rightarrow$ ~~-~~ $0.5 + \phi(z_2) = 0.89 \rightarrow$ ⑪

In ①

$\phi(z_1) = 0.07 - 0.5$

$\phi(z_1) = -0.43$ (By given data)

$\boxed{Z_1 = -1.4757}$

In ⑪

$0.5 + \phi(z_2) = 0.89$

$\phi(z_2) = 0.89 - 0.5$

$\phi(z_2) = 0.39$

$\boxed{Z_2 = 1.2263}$ (by given data)

⇒ Substituting these values of Z_1 & Z_2 in (I) & (II)

$$\frac{35-\mu}{\sigma} = -1.4757$$

$$\frac{60-\mu}{\sigma} = 1.2263 \longrightarrow \text{(III)}$$

$$\Rightarrow \frac{35-\mu}{\sigma} = -1.4757$$

$$\Rightarrow 35-\mu = -1.4757\sigma$$

$$-\mu = -1.4757\sigma - 35$$

$$\Rightarrow \mu = 35 + 1.4757\sigma$$

Putting this value of μ in eqⁿ (III)

$$\frac{60-(35+1.4757\sigma)}{\sigma} = 1.2263$$

$$\Rightarrow 60-35-1.4757\sigma = 1.2263\sigma$$

$$\Rightarrow 25 = 1.2263\sigma + 1.4757\sigma$$

$$\Rightarrow 2.702\sigma = 25$$

$$\Rightarrow \sigma = \frac{25}{2.702}$$

$$\Rightarrow \sigma = 9.252$$

Now, $\mu = 35 + 1.4757(9.252)$

$= 48.65$

∴ Mean = 48.65

S.D = 9.252

⑥

$x \backslash y$	-4	2	7	$f(x_i)$
1	$1/8$	$1/4$	$1/8$	$1/2$
5	$1/4$	$1/8$	$1/8$	$1/2$
$g(y_j)$	$3/8$	$3/8$	$1/4$	$1 \rightarrow$ sum

So, our marginal distribution table for $X \Rightarrow$

X	1	5
$f(X)$	$1/2$	$1/2$

and marginal distribution table for $Y \Rightarrow$

y	-4	2	7
$g(y)$	$3/8$	$3/8$	$1/4$

Now, $E(X) = \sum_i x_i f(x_i)$

$= 1 \times \frac{1}{2} + 5 \times \frac{1}{2}$

$= \frac{1}{2} + \frac{5}{2}$

$= \frac{6}{2} = 3$

$\therefore \boxed{E(X) = 3}$

Now, $E(Y) = \sum_j y_j g(y_j)$

$= -4 \times \frac{3}{8} + 2 \times \frac{3}{8} + 7 \times \frac{1}{4}$

$= \frac{-12 + 6 + 14}{8} = \frac{8}{8} = 1$

$\therefore E(Y) = 1$

Now, $E(XY) = \Sigma x_i y_j J_{ij}$

$$= \left(1\times -\cancel{4}\times\frac{1}{\underset{2}{\cancel{8}}}\right) + \left(1\times\cancel{2}\times\frac{1}{\underset{2}{\cancel{4}}}\right) + \left(1\times 7\times\frac{1}{8}\right)$$

$$+ \left(5\times -\cancel{4}\times\frac{1}{\cancel{4}}\right) + \left(5\times\cancel{2}\times\frac{1}{\underset{4}{\cancel{8}}}\right) + \left(5\times 7\times 1/8\right)$$

$$= -\frac{\cancel{1}}{\cancel{2}} + \frac{\cancel{1}}{\cancel{2}} + \frac{7}{8} - 5 + \frac{5}{4} + \frac{35}{8}$$

$$= \frac{7-40+10+35}{8}$$

$$= \frac{\cancel{12}\ \cancel{6}\ 3}{\cancel{8}\ \cancel{4}\ 2}$$

$$= \frac{3}{2}$$

$\sigma_x^2 = \Sigma x_i^2 \times f(x) - E(x)^2$

$= 1^2\times\frac{1}{2} + 5^2\times\frac{1}{2} - 9$

$= \frac{1}{2} + \frac{25}{2} - 9$

$= \frac{26}{2} - 9$

$\sigma_x^2 = 13-9$, ~~$\sigma_x = \sqrt{13} = 3.605$~~

$\Rightarrow \sigma_x^2 = 4$, $\boxed{\sigma_x = 2}$

$\sigma_y^2 = \Sigma y_j^2 f(y_j) - E(Y)^2$

$= \overset{2}{\cancel{16}}\times\frac{3}{\cancel{8}} + 4\times\frac{3}{\underset{2}{\cancel{8}}} + 49\times\frac{1}{4} - E(Y)^2$

$= 6 + \frac{3}{2} + \frac{49}{4} = \frac{24+6+49}{4} = \frac{79}{4} = 19.75$

~~$\sigma^2 y = 19.75$~~ $\sigma^2 y = 19.75 - (1)^2$

~~$\sigma y = \sqrt{19.75}$~~ $\sigma^2 y = 18.75$

~~$= 4.44$~~ $\sigma y = 4.33$

$Cov(XY) = E(XY) - E(X)E(Y)$.

~~$= \frac{3}{2} - 3 \times 1$~~

~~$= \frac{\quad}{2}$~~

$= \frac{3}{2} - (3)(1)$

$= \frac{3 - 6}{2} = -\frac{3}{2}$

(iii) $\rho(X,Y) = \frac{Cov(X,Y)}{\sigma_x \cdot \sigma_y}$

$= \frac{-\frac{3}{2}}{2 \times 4.33}$

$= -0.173$

$\therefore$ $E(X) = 3$

$E(Y) = 1$

$E(XY) = 3/2$

$\rho(X,Y) = -0.173$

⑦ (i) Statistical hypothesis ⇒ ~~Stat~~ ~~In~~ ~~D~~

We are checking the observed value with the expected value and operating statistics and data using different statistical methods

Two types : (i) Null Hypothesis (H_0) ⇒ Proportion population = Hypothesized populati

(ii) H_a

⇓

Proportion population ≠ hypothesized population

(ii) Critical Region of statistical test ⇒ Also known as rejection region

⇒ Contains range of values for which our null hypothesis can be rejected.

(iii) Type I error ⇒ When null hypothesis is rejected even when its true. P= 'α'

Type II error ⇒ Failing to reject null hypothesis ~~if~~ even if its false.

P= 'β'.

(iv) Test of significance ⇒ This is implemented to determine ~~how much~~ how much significant deviation of observed value as compared to expected value is there under our null hypothesis.

⑧ Given,

$\mu = 275.6$ gms

$\sigma = 39.7$ gms

$n = 36$

$\bar{x} = 253.2$ gms

Hypothesis: $H_0 : \mu = 275.6$

$H_1 : \mu \neq 275.6$

⇒ It is a two tailed Test

$$Z_c = \left| \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \right| = \left| \frac{253.2 - 275.6}{\frac{39.7}{\sqrt{36}}} \right|$$

$$Z_c = \left| -3.385 \right|$$

∴ $Z_c = 3.385$

~~∴ $Z_c >$~~

(i) For significance level of 0.05

$Z_c > Z_{\alpha = 0.05}$ $\left(Z_{\alpha=0.05} = \pm 1.96 \right)$

∴ H_0 is rejected

⇒ It is significant

(ii) For significance level of 0.01

$Z_c > Z_{\alpha = 0.01}$ $\left(Z_{\alpha=0.01} = \pm 2.58 \right)$

⇒ Ho is rejected

⇒ It is significant

∴ It is significant at both 0.05 & 0.01 significance level.