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Internal Assessment Test II - Nov 2024

Sub:	Theory Of Computation			Sub Code:	BCS503	Branch:	**AIDS & CSE(DS)**
Date:	**16/12/2024**	Duration:	**90 minutes**	Max Marks:	**50**	Sem/Sec:	**V -A, B & C**

		Answer any FIVE FULL Questions	**MARKS**	**OBE CO**	**RBT**
1	a	Check whether the given grammar G is ambiguous or not. $S \rightarrow aB \mid bA$ $A \rightarrow a \mid aS \mid bAA$ $B \rightarrow b \mid bS \mid aBB$	5	CO3	L2
	b	Define PDA and its Instantaneous Description (ID)	5	CO3	L3
2	a	Convert the following grammar to a PDA that accepts the same language. $S \rightarrow 0S1 \mid A$ $A \rightarrow 1A0 \mid S \mid \varepsilon$	5	CO3	L2
	b	Design PDA for the language $L=\{wcw^R\}$ where w belongs to {0,1}.	5	CO3	L2
3	a	Find a Greibach normal form grammar equivalent to the following CFG. $A1 \rightarrow A2A3$ $A2 \rightarrow A3A1/b$ $A3 \rightarrow A1A2/a$	5	CO3	L3
	b	If L1 and L2 are Context Free Languages then prove that L1UL2, L1.L2 & L1* are Context Free Language.	5	CO3	L3
4	a	Convert the following grammar into Chomsky Normal Form $S \rightarrow ASA \mid aB$, $S \rightarrow B \mid S$, $B \rightarrow b \mid \varepsilon$	5	CO3	L3
	b	State Pumping lemma for CFG and Show that the language $L = \{0^n1^n2^n\}$ where $n \geq 1$ is not CFG	5	CO3	L3
5	a	Construct a TM for multiplication of two unary numbers.	10	CO4	L3
6	a	Explain about Programming Techniques for Turing Machines.	5	CO4	L3
	b	Write in detail about universal Turing machine.	5	CO5	L3

CI **CCI** **HoD**

(1)

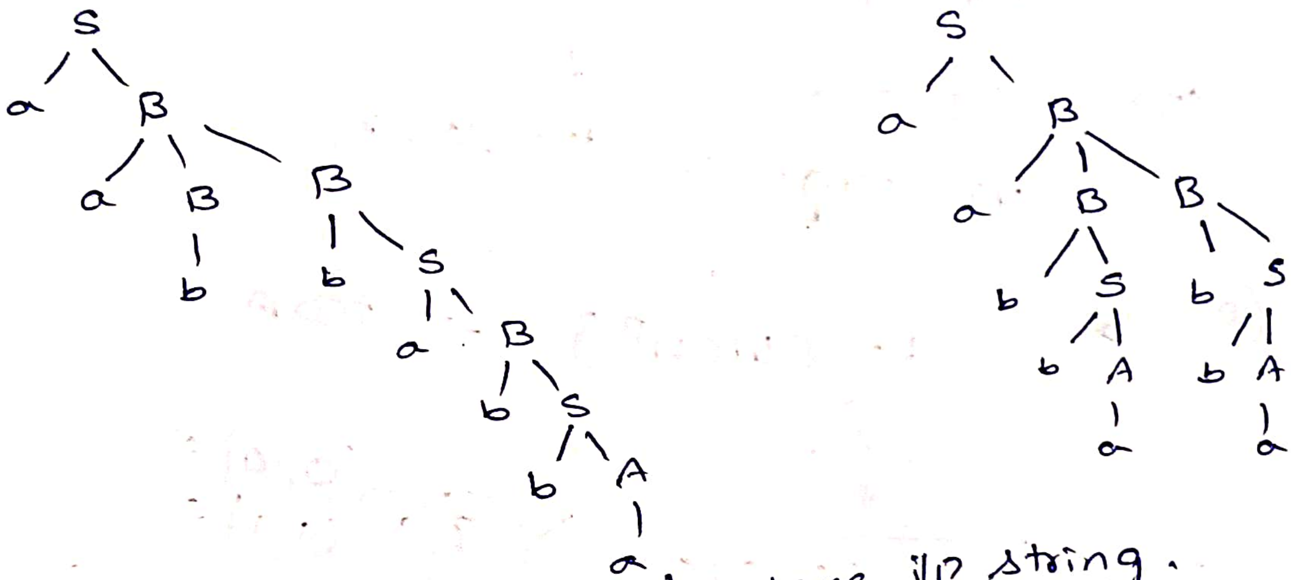
$S \rightarrow aB \mid bA$

$A \rightarrow a \mid aS \mid bAA$ check Ambiguity ?

$B \rightarrow b \mid bS \mid aBB$

Sol:

I/p : aabbabba



Two different parse trees for same i/p string.

$\therefore$ Grammar is Ambiguous.

(2)

PDA :

$M = (Q, \Sigma, \delta, q_0, Z_0, \Gamma, F)$

Q : finite set of states

Σ : i/p alphabet

δ : Mapping function.

$Q \times (\Sigma \cup \varepsilon) \times \Gamma \rightarrow (Q \times \Gamma^*)$

Z_0 : bottom of the stack

Γ : stack alphabet

F : Finite set of final states $F \subseteq Q$.

ID:

(i) $(q, a, x) = (q, ax)$

push into the stack

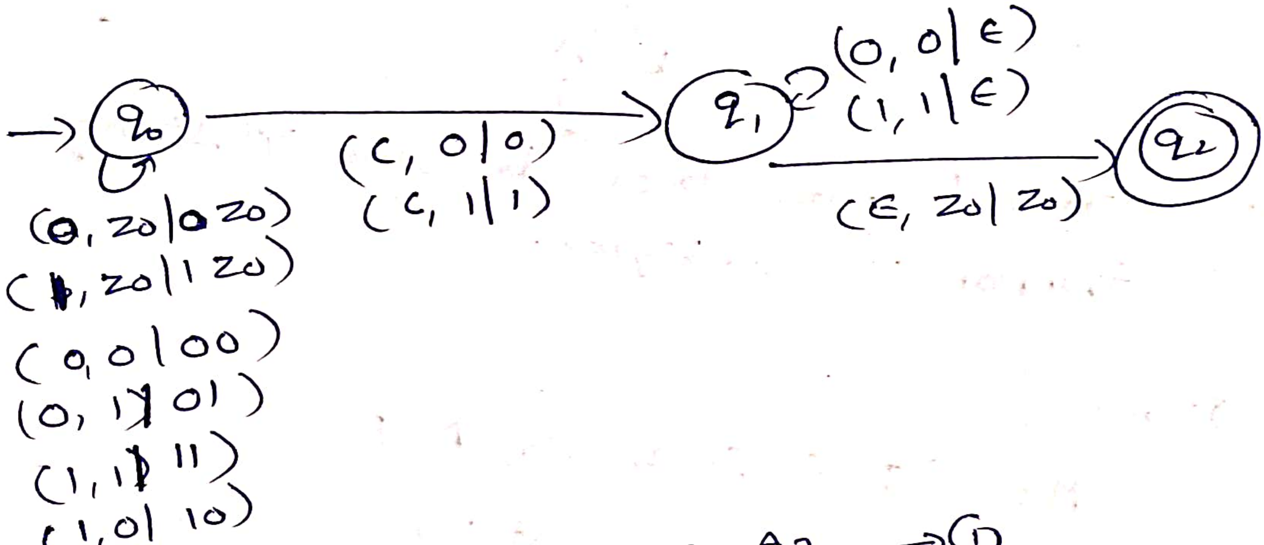
(ii) $(q, a, x) = (q, \epsilon)$

pop from the stack

(iii) $(q, a, x) = (q, x)$

No change in the stack.

(2) (b) $L = \{WCW^R\} \rightarrow$ PDA



(3) (a) GNF:

$A_1 \rightarrow A_2A_3 \rightarrow (1)$

$A_2 \rightarrow A_3A_1 \mid b \rightarrow (2)$

$A_3 \rightarrow A_1A_2 \mid a \rightarrow (3)$

eq (3)

$A_3 \rightarrow A_1A_2 \mid a$

$\rightarrow A_2A_3A_2 \mid a$

$A_3 \rightarrow A_3A_1A_3A_2 \mid bA_3A_2 \mid a$

$A \rightarrow A\alpha \mid \beta_1 \mid \beta_2$

$A_3 \rightarrow bA_3A_2 \mid a$

$A_3 \rightarrow bA_3A_2Z \mid aZ$

$Z \rightarrow A_1A_3A_2 \mid A_1A_3A_2Z \rightarrow (4)$

eq (2) $A_2 \rightarrow A_3A_1 \mid b$

$\rightarrow bA_3A_2A_1 \mid aA_1 \mid b \mid bA_3A_2ZA_1 \mid aZA_1$

eq (1) $A_1 \rightarrow A_2A_3$

$A_1 \rightarrow bA_3A_2A_1A_3 \mid aA_1A_3 \mid bA_3 \mid bA_3A_2ZA_1A_3 \mid aZA_1A_3$

eq (4) $Z \rightarrow A_1A_3A_2 \mid A_1A_3A_2Z$

$Z \rightarrow bA_3A_2A_1A_3A_3A_2 \mid aA_1A_3A_3A_2 \mid bA_3A_3A_2 \mid bA_3A_2ZA_1A_3A_3A_2 \mid aZA_1A_3A_3A_2$

$Z \rightarrow bA_3A_2A_1A_3A_3A_2Z \mid aA_1A_3A_3A_2Z \mid bA_3A_3A_2Z \mid bA_3A_2ZA_1A_3A_3A_2Z \mid aZA_1A_3A_3A_2Z$

(2) (a) CFG to PDA

$S \rightarrow 0S1 \mid A$

$A \rightarrow 1A0 \mid S \mid \epsilon$

$(q_0, \epsilon, S) = (q_0, 0S1)$

$(q_0, \epsilon, A) = (q_0, 1A0)$

$(q_0, \epsilon, S) = (q_0, A)$

$(q_0, \epsilon, A) = (q_0, S)$

$(q_0, \epsilon, A) = (q_0, \epsilon)$

$(q_0, 0, 0) = (q_0, \epsilon)$

$(q_0, 1, 1) = (q_0, \epsilon)$

3 (b) Union:

If L_1 & L_2 are two CFL, then $L_1 \cup L_2$ is also CFL

Ex: $L_1 = \{a^n b^n c^m \mid m \geq 0 \ \& \ n \geq 0\}$

$L_2 = \{a^n b^m c^m \mid n \geq 0 \ \& \ m \geq 0\}$

then $L_3 = L_1 \cup L_2$

$= \{a^n b^n c^m \cup a^n b^m c^m \mid n \geq 0, m \geq 0\}$

is also CFL.

So CFL is closed under union

Concatenation

If L_1 & L_2 are CFL, then $L_1 \cdot L_2$ is also CFL

$L_1 = \{a^n b^n \mid n \geq 0\}$, $L_2 = \{c^m d^m \mid m \geq 0\}$

$L_1 \cdot L_2 = \{a^n b^n c^m d^m \mid m, n \geq 0\}$ is also CFL.

Kleen closure:

If L_1 is CFL, then L_1^* is also a CFL.

$L_1 = \{a\}$

then $L_1^* = a^*$ is also a CFL.

(4) (a) CFG to CNF

$S \rightarrow ASA \mid aB$

$S \rightarrow B \mid S$

$B \rightarrow b \mid \epsilon$

Eliminate Null production :

$S \to ASA \mid a \mid aB$
$S \to \epsilon \mid S$
$B \to b$

Eliminate unit productions

$S \to ASA \mid a \mid aB \mid \epsilon$
$B \to b$

$S \to ASA$ is useless

$\therefore S \to a \mid aB \mid \epsilon$
$B \to b$

$S \to aB$ is not in CNF

$S \to AB \mid \epsilon$
$A \to a$
$S \to a$
$B \to b$

CNF

(4) (b) pumping Lemma for CFL :

Let L be a CFL, Then there exist a constant 'n' such that if z is any string in L such that |z| is atleast n, then we write z = uvwxy satisfy :

(1) $|vwx| \leq n$
(2) $|vx| \neq \epsilon$
(3) $uv^iwx^iy \in L$ for all $i \geq 0$

$L = \{0^n 1^n 2^n / n \geq 1\}$

$\omega = \underset{u}{0}\underset{v}{0}\underset{w}{11}\underset{x}{2}\underset{y}{2}$

(i) $|vwx| \leq n$

(ii) $|vx| \neq \epsilon$

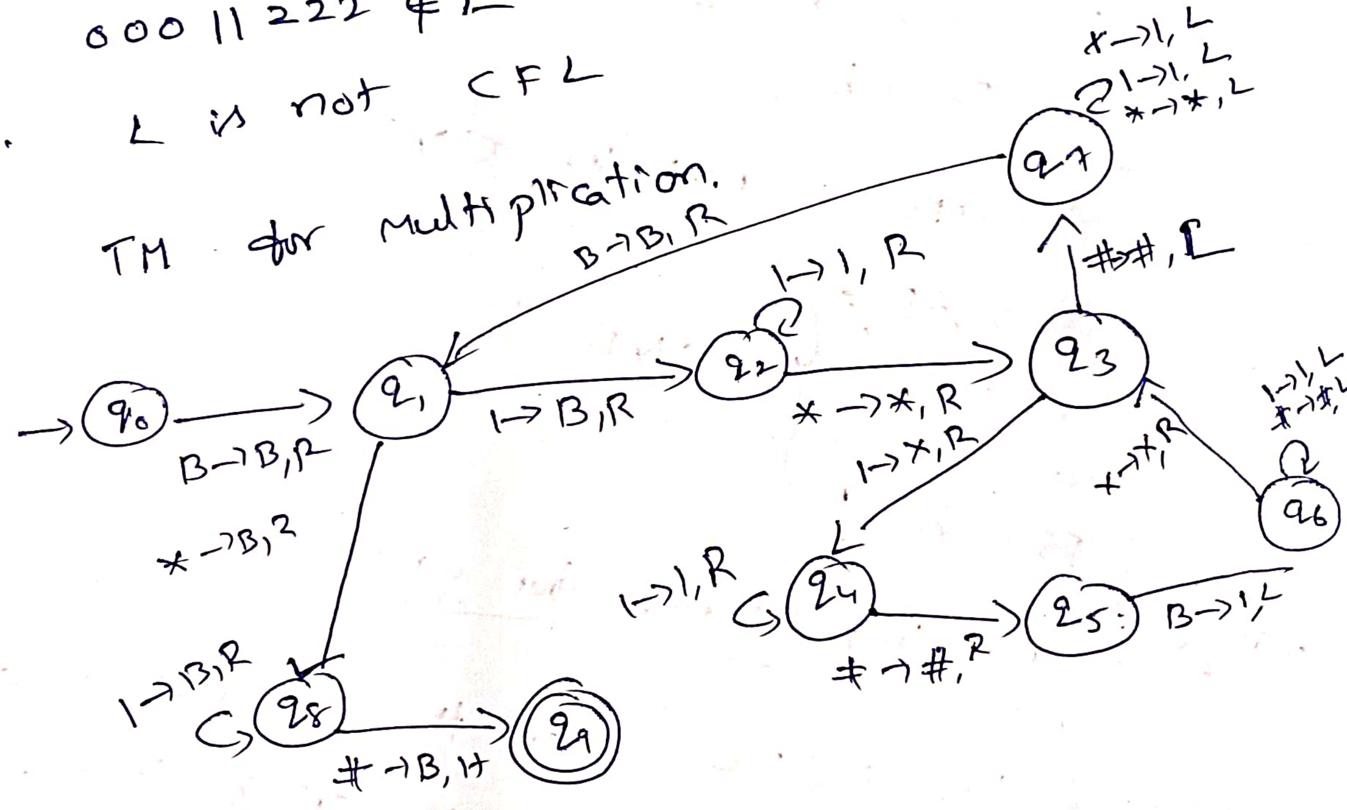
(iii) $u v^i w x^i y \quad \forall i \geq 0$

$i = 0 \quad 0112 \notin L$

$i = 2 \quad 0001122 2 \notin L$

$\therefore$ L is not CFL

(5) (a) TM for Multiplication.



(6) (a) Halting problem:

Halting problem is a recursive enumerable problem that is also undecidable.

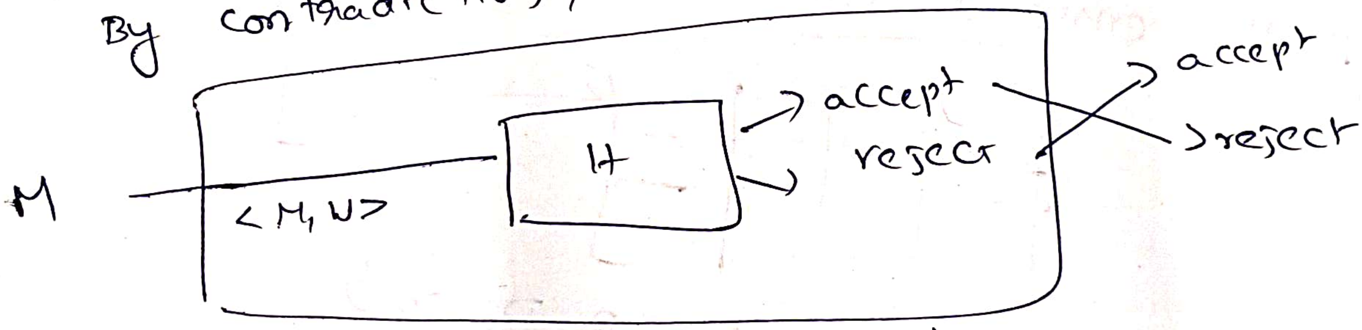
Does a TM halt on a given i/p w?

Build TM called 'H' that will output

$$\begin{cases} \text{accept} & \text{if } M \text{ accepts } W \text{ \&} \\ \text{reject} & \text{otherwise} \end{cases}$$

$$H\langle M, W\rangle = \begin{cases} \text{accept} & \text{if } M \text{ accepts } W \\ \text{reject} & \text{if } M \text{ not accept } W \end{cases}$$

By contradiction, let us assume 'H' exists.



A program can be i/p to itself.

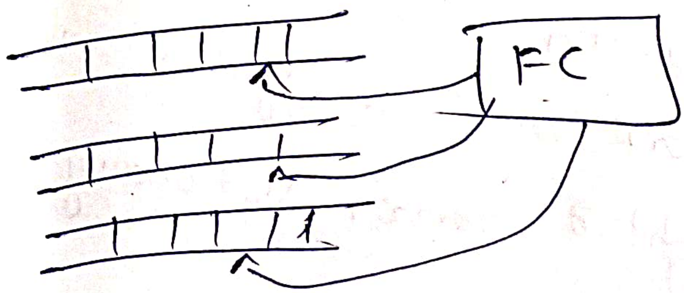
$$D\langle M\rangle = \begin{cases} \text{accept} & \text{if } M \text{ doesn't accept } \langle M\rangle \\ \text{reject} & \text{if } M \text{ accepts } \langle M\rangle \end{cases}$$

<u>Contradiction</u>

Neither D nor H can exist

<u>Extended TM:</u>

<u>Multitape TM:</u>



TM with several tapes, each with its own independently controlled read/write head.

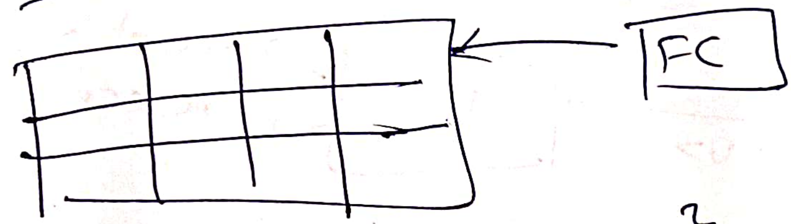
$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

$\delta : Q \times \Gamma^n \to Q \times \Gamma^n \times \{L, R\}^n$

(2) Non-deterministic TM:

$\delta(q, x) = \{(q_1, Y_1, D)\ (q_2, Y_2, D_2) \ldots (q_n, Y_k, D_k)\}$

Multi dimensional TM:



$Q \times \Gamma \to Q \times \Gamma\ (L, R, D_1, D_2\}$

D_1 - up
$D_2 \to$ down

TM with more than one dimension.

Multihead TM:

TM has some fixed no. of heads called Multihead TM.

Based on state & symbol, the head can move left, right or remain stationary.