Scheme Of Evaluation

Internal Assessment Test I – Nov 2024

Sub:	DIGITAL SIGNAL PROCESSING							**Code:**	BEC502
Date:	6/11/2024	Duration:	90mins	Max Marks:	50	**Sem:**	V	**Branch:**	ECE

Note: Answer Any Five Questions

Question #		Description	Marks Distribution		Max Marks
1	a)	**Sketch the even part and odd of the signal** $\boldsymbol{x(n) = \{1\ for\ 0 \leq n \leq 3\ \ -1\ for\ 4 \leq n \leq 7\ 0\ otherwise}$ ● Even Part ● Odd Part	3 M 3 M	06 M	10 M
	b)	Define signal and system. Give suitable examples for each term. ● Definition of signal ● Definition of system ● Examples	1 M 1 M 2 M	04 M	
2	a)	Compute the energy and power of the following signal. Determine whether the signal is an energy signal or power signal. $x(n) = cos\left(\frac{2\pi}{5}n\right)$ ● Energy=∞ ● Power =0.5	2 M 3 M	5 M	10 M
	b)	Check whether the following signal is periodic or not. If periodic, then find the period of the signal. $x(n) = cos\left(\frac{\pi}{7}n\right) + cos\left(\frac{\pi}{5}n\right)$ ● Stating property ● Computing X(z)	1 M 3 M	4 M	
3		Determine whether or not the following system is linear, time invariant, causal, memoryless and stable. Justify your answer. $y(n) = nx(n)$ where $x(n)$ is the input and $y(n)$ is the corresponding output.		10 M	10 M

		• Linear • Time Variant • Causal • Memoryless • Unstable	2 M 2 M 2 M 2 M 2 M		
4		Compute the convolution of the following signals. $x(n) = \alpha^n u(n)$ and $h(n) = \beta^n u(n)$ • Formula for convolution • Substitution and simplification	2 M 8 M	10 M	10 M
5		Find the step response of the LTI system whose impulse response is $h(n) = \left(\frac{1}{3}\right)^n u(n)$ • Formula for step response • Substitution and simplification	5 M 5 M	10 M	10 M
6		Find the z-transform of $x(n) = 2^n u(-n-1)$. Sketch the ROC. List out the properties of ROC. • Finding x(z) • Pole zero plot • Properties of ROC	4 M 2 M 4 M	10 M	10 M
7	a)	An LTI system is represented by the following difference equation. $y(n) - 3.5y(n-1) + 1.5y(n-2) = 3x(n) - 4x(n-1)$ i) Determine the transfer function of the system. ii) Determine the poles and zeroes of the system. iii) Determine the impulse response of the system assuming that the system is causal. • Determine the transfer function of the system. • Determine the poles and zeroes of the system. • Determine the impulse response of the system assuming that the system is causal.	3M 3 M 4 M	10 M	10 M

1 a.

$$\boldsymbol{x(n)} = \begin{cases} \mathbf{1} & \boldsymbol{for\ 0 \le n \le 3} \\ \mathbf{-1} & \boldsymbol{for\ 4 \le n \le 7} \\ \mathbf{0} & \boldsymbol{otherwise} \end{cases}$$

$$x(n) = \{\underset{\uparrow}{1},1,1,1,-1,-1,-1,-1\}$$

$$x(-n) = \{-1,-1,-1,-1,1,1,1,\underset{\uparrow}{1},\}$$

Even Component, $x_e(n) = \frac{1}{2}(x(n) + x(-n))$

$$x_e(n) = \frac{1}{2}\{-1,-1,-1,-1,1,1\ ,1\ ,2,1,1,1,-1,-1,-1,-1$$

Odd Component, $x_0(n) = \frac{1}{2}(x(n) - x(-n))$

$$x_o(n) = \frac{1}{2}\{1,1,1,1,-\ 1,-\ 1\ ,-1\ ,0,1,1,1,-1,-1,-1,-1\ \}$$

1 b. A ***signal*** is defined as any physical quantity that varies with time, space, or any other independent variable or variables. Mathematically, we describe a signal as a function of one or more independent variables. For example, the functions

$$s_1(t) = 5t$$
$$s_2(t) = 20t^2$$
$$s(x,y) = 3x + 2xy + 10y^2$$

For example, a speech signal (see Fig. 1.1) cannot be described functionally by expressions such as (1.1). In general, a segment of speech may be represented to a high degree of accuracy as a sum of several sinusoids of different amplitudes and frequencies, that is, as

$$\sum_{i=1}^{N} A_i(t)\sin[2\pi F_i(t)t + \theta_i(t)]$$

where $\{A_i(t)\}, \{F_i(t)\}, and\ \{\theta_i(t)\}$ are the sets of (possibly time-varying) amplitudes, frequencies, and phases, respectively, of the sinusoids.

Thus signal generation is usually associated with a ***system*** that responds to a stimulus or force. In a speech signal, the system consists of the vocal cords and the vocal tract, also called the vocal cavity. The stimulus in combination with the system is called a ***signal source***.

A ***system*** may also be defined as a physical device that performs an operation on a signal. For example, a filter used to reduce the noise and interference corrupting a desired information-bearing signal is called a ***system***.

If the operation is linear, the system is called linear. If the operation on the signal is nonlinear, the system is said to be nonlinear, and so forth. Such operations are usually referred to as ***signal processing***.

In digital processing of signals on a digital computer, the operations performed on a signal consist of a number of mathematical operations as specified by a software program. In this case, the program represents an implementation of the system in ***software.***

2a.

$$x(n) = \cos\left(\frac{2\pi}{5}n\right)$$

Sol: Check for periodicity of $x(n)$. If it is periodic calculate the average power in one period.

$$\omega_0 = \frac{2\pi}{5}$$
$$2\pi f_0 = \frac{2\pi}{5}$$
$$f_0 = \frac{1}{5} = \frac{k}{N}$$

Hence $x(n)$ is periodic with period $N = 5$.

Average Power over one period $N = 5,\ \ 0 \le n \le 4$

$$P = \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2$$
$$= \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} \left|\cos\left(\frac{2\pi}{5}n\right)\right|^2$$

We know that $\cos 2\theta = 2\cos^2\theta - 1$

$$= \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} \frac{1}{2}\left(1 + \cos\left(\frac{4\pi}{5}n\right)\right)$$
$$= \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} \frac{1}{2} + \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} \frac{1}{2}\left(\cos\left(\frac{4\pi}{5}n\right)\right)$$
$$= \lim_{N\to\infty} \frac{1}{N}.\frac{1}{2}N + \lim_{N\to\infty} \frac{1}{N}.0$$
$$\boldsymbol{P = \frac{1}{2}W}$$

Hence the given sequence $x(n)$ is a power signal. Its energy is infinite $E = \infty$.

2b.

$$x(n) = cos\left(\frac{\pi}{7}n\right) + cos\left(\frac{\pi}{5}n\right)$$
$$x(n) = x_1(n) + x_2(n)$$

<u>To find periodicity of $x_1(n)$</u>

$$\omega_1 = \frac{\pi}{7}$$
$$2\pi f_1 = \frac{\pi}{7}$$

$$f_1 = \frac{1}{14} = \frac{k}{N_1}$$

Hence $x_1(n)$ is periodic with period $N_1 = 14$.

To find periodicity of $x_2(n)$

$$\omega_2 = \frac{\pi}{5}$$

$$2\pi f_2 = \frac{\pi}{5}$$

$$f_2 = \frac{1}{10} = \frac{k}{N_2}$$

Hence $x_2(n)$ is periodic with period $N_2 = 10$.

$$\frac{N_2}{N_1} = \frac{10}{14} = \frac{5}{7} \text{ (Rational Number)}$$

$$N = 7N_2 = 5N_1$$

$$N = 7(10) = 5(14) = 70$$

Therefore, $x(n)$ is periodic with period $N = 70$.

3.

$$y(n) = nx(n)$$

i. The given system is **static**.

ii. For two input sequences $x_1(n)$ and $x_2(n)$, the corresponding outputs are

$$y_1(n) = nx_1(n) \qquad (1)$$

$$y_2(n) = nx_2(n) \qquad (2)$$

A linear combination of the two input sequences results in the output

$$y_3(n) = \mathcal{T}[a_1x_1(n) + a_2x_2(n)] = n[a_1x_1(n) + a_2x_2(n)]$$

$$= a_1nx_1(n) + a_2nx_2(n) \qquad (3)$$

On the other hand, a linear combination of the two outputs in (1) and (2) results in the output

$$a_1y_1(n) + a_2y_2(n) = a_1nx_1(n) + a_2nx_2(n) \qquad (4)$$

Since the right-hand sides of (3) and (4) are identical, **the system is linear**.

iii. This system is described by the input–output equations

$$y(n) = \mathcal{T}[x(n)] = nx(n) \qquad (1)$$

If the input is delayed by k units in time and applied to the system, then the output is given by

$$y(n, k) = \mathcal{T}[x(n-k)] = nx(n-k) \qquad (2)$$

If we delay $y(n)$ by k units in time. Substitute n by $n-k$, we get in (1)

$$y(n-k) = (n-k)x(n-k) \qquad (3)$$

From (1) and (3), $\boldsymbol{y(n,k) \neq y(n-k)}$

Therefore, the system is NOT time invariant

iv. The given system is **Causal** since present output depends only on the present input.

v. If the input $x(n)$ is bounded

$$|x(n)| \leq M_x < \infty,$$

Then magnitude of the output $|y(n)| = |nx(n)|$

As $n \to \infty, |y(n)| \to \infty$ which makes the output $y(n)$ unbounded

Hence the given system is **UNSTABLE**

4. Compute the convolution of the following signals $x(n) = \alpha^n u(n)$ and $h(n) = \beta^n u(n)$

Sol:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$x(k) = \alpha^k u(k) = \begin{cases} \alpha^k, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

$$h(n-k) = \beta^{n-k}u(n-k) = \begin{cases} \beta^{n-k}, & n-k \geq 0 \\ & k \leq n \\ 0, & k < n \end{cases}$$

$$y(n) = \sum_{k=-\infty}^{\infty} \alpha^k u(k) \beta^{n-k} u(n-k)$$

i. $n < 0$

There is no overlap between $x(k)$ and $h(n-k)$

$$y(n) = 0$$

ii. $n \geq 0$

Common overlap interval between $x(k)$ and $h(n-k)$ for $k = 0$ to $k = n$.

$$y(n) = \sum_{k=0}^{n} \alpha^k \beta^{n-k}$$

$$= \beta^n \sum_{k=0}^{n} \alpha^k \beta^{-k} = \beta^n \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k$$

Recall:

$$\sum_{k=}^{N-1} a^k = \begin{cases} \dfrac{1-a^N}{1-a} & \text{if } a \neq 1 \\ N & \text{if } a = 1 \end{cases}$$

If $\alpha = \beta$, $y(n) = \beta^n(n+1)$

If $\alpha \neq \beta$,

$$y(n) = \beta^n \left\{ \frac{1 - \left(\frac{\alpha}{\beta}\right)^{n+1}}{1 - \left(\frac{\alpha}{\beta}\right)} \right\} = \left\{ \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} \right\}$$

$$y(n) = \begin{cases} 0, & n < 0 \\ \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha}, & n \geq 0,\ \alpha \neq \beta \\ \beta^n (n+1), & n \geq 0, \alpha = \beta \end{cases}$$

5. Find the step response of the LTI system whose impulse response is $h(n) = \left(\frac{1}{3}\right)^n u(n)$

Sol: The step response is defined as the output due to a unit step input signal.

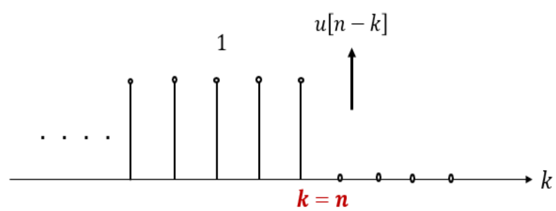
Let $h[n]$ be the impulse response of a discrete-time LTI system, and denote the step response $s[n]$.

$$s[n] = h[n] * u[n]$$

$$= \sum_{k=-\infty}^{\infty} h[k] u[n-k]$$

Now, since

$$u[n-k] = \begin{cases} 1, & n-k \geq 0 \\ 0, & otherwise \end{cases} = \begin{cases} 1, & k \leq n \\ 0, & otherwise \end{cases}$$



Step response is the running sum of the impulse response.

$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

$$h(k) = \left(\frac{1}{3}\right)^k u(k) = \begin{cases} \left(\frac{1}{3}\right)^k, & k \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$i)$ For $n < 0$, the step response

$$s[n] = 0$$

$ii)$ For $n \geq 0$, the step response

$$s[n] = \sum_{k=0}^{n} h[k] = \sum_{k=0}^{n} \left(\frac{1}{3}\right)^k$$

$$= \frac{1 - \left(\frac{1}{3}\right)^{n+1}}{1 - \left(\frac{1}{3}\right)} = \frac{3}{2}\left\{1 - \left(\frac{1}{3}\right)^{n+1}\right\} = \left[\frac{3}{2} - \frac{1}{2}\left(\frac{1}{3}\right)^n\right]$$

Hence the step response,

$$s[n] = \left[\frac{3}{2} - \frac{1}{2}\left(\frac{1}{3}\right)^n\right]$$

6. $x(n) = 2^n u(-n-1)$.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

Substituting for $x[n]$, we get

$$X(z) = \sum_{n=-\infty}^{\infty} 2^n u[-n-1]\ z^{-n}$$

$$= \sum_{n=-\infty}^{-1} (2z^{-1})^n$$

Substitute $k = -n$

when $n = -\infty,\ k = \infty$

$n = -1,\ k = 1$

$$X(z) = \sum_{k=1}^{\infty} (2z^{-1})^{-k} = \sum_{k=1}^{\infty} \left(\frac{z}{2}\right)^k$$

$$= 1 - \sum_{k=0}^{\infty} \left(\frac{z}{2}\right)^k$$

The sum converges, provided that $\left|\frac{z}{2}\right| < 1$, or $|z| < |2|$

$$X(z) = 1 - \frac{1}{1 - 2^{-1}z}, |z| < |2|$$

$$= 1 - \frac{1}{1 - 2^{-1}z},$$

$$= \frac{2^{-1}z}{1 - 2^{-1}z} \left(\frac{-2}{-2}\right)$$

$$= \frac{-z}{z-2}, |z| < |2|$$

$$X(z) = \frac{-1}{1 - 2z^{-1}}, \qquad |z| < |2|$$

$$x[n] = 2^n u[-n-1] \xleftrightarrow{z} X(z) = \frac{-1}{1-2z^{-1}} \quad \text{ROC } |z| < |2|$$

The ROC is now the interior of a circle having radius 2.

7. $y(n) - 3.5y(n-1) + 1.5y(n-2) = 3x(n) - 4x(n-1)$

Apply Z transform on both sides of this difference equation using time shift property

$$Y(z) - 3.5z^{-1}Y(z) + 1.5z^{-2}Y(z) = 3X(z) - 4z^{-1}X(z)$$

$$Y(z)\{1 - 3.5z^{-1} + 1.5z^{-2}\} = \{3 - 4z^{-1}\}X(z)$$

System Function or Transfer Function is given by

$$H(z) = \frac{Y(z)}{X(z)}$$

$$\boldsymbol{H(z) = \frac{3-4z^{-1}}{1-3.5z^{-1}+1.5z^{-2}}}$$

<u>Zeros:</u> $3-4z^{-1}=0,$

$$\boldsymbol{z=\frac{4}{3}}$$

<u>Poles:</u> $1-3.5z^{-1}+1.5z^{-2}=0,$

$$(1-3z^{-1})\left(1-\frac{1}{2}z^{-1}\right)=0$$

$$\boldsymbol{z=\frac{1}{2},\qquad z=3}$$

To determine the impulse response $h(n)$ of the system assuming that the system is causal.

ROC must be exterior

Expanding $\boldsymbol{H(z)}$ using Partial Fractions

$$H(z)=\frac{3-4z^{-1}}{(1-3z^{-1})\left(1-\frac{1}{2}z^{-1}\right)}=\frac{A}{(1-3z^{-1})}+\frac{B}{\left(1-\frac{1}{2}z^{-1}\right)}$$

$$3-4z^{-1}=A\left(1-\frac{1}{2}z^{-1}\right)+B(1-3z^{-1})$$

Solving, we get $A=2,\quad B=1$

$$H(z)=\frac{3-4z^{-1}}{(1-3z^{-1})\left(1-\frac{1}{2}z^{-1}\right)}=\frac{2}{(1-3z^{-1})}+\frac{1}{\left(1-\frac{1}{2}z^{-1}\right)}$$

Recall:

$$a^n u(n)\xleftrightarrow{z}\frac{1}{1-az^{-1}},\qquad \text{ROC: }|z|>a$$

For an infinite duration causal sequence, the Z transform has exterior ROC.

Therefore the impulse response $h(n)$ of the system assuming that the system is causal.

$$\boldsymbol{h(n)=2(3)^n u(n)+\left(\frac{1}{2}\right)^n u(n)}$$