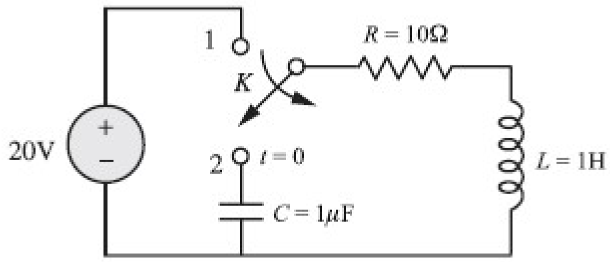
CMR INSTITUTE OF TECHNOLOGY	USN	

Internal Test 2–December 2024

Scheme and solution

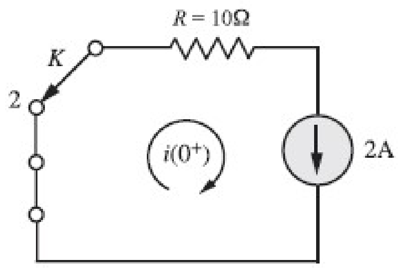
Sub:	Network Analysis	Code:	BEC304

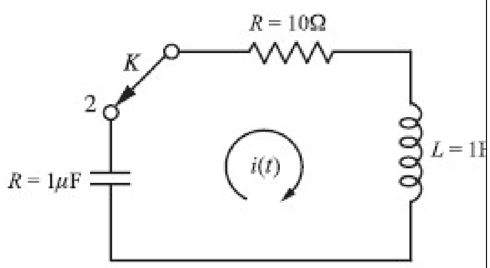
Note: Answer any FIVE full questions with neat diagram wherever necessary.

		Marks	OBE CO	OBE RBT
1.	 Refer to the circuit shown. The switch K is changed from position 1 to position 2 at t=0. The steady-state condition has been reached at position 1, find the values of i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at t=0^+	[10]	CO3	L3

$$i\left(0^-\right) = \frac{20}{10} = 2\text{A}$$

………………2M





……………2M

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{0^+} i(\tau)d\tau = 0$$

$$\Rightarrow \qquad Ri(t) + L\frac{di(t)}{dt} + v_C(t) = 0$$

At $t = 0^+$ equation (4.3a) becomes

$$Ri\left(0^+\right) + L\frac{di\left(0^+\right)}{dt} + v_C\left(0^+\right) = 0$$

$$\Rightarrow \qquad R \times 2 + L\frac{di\left(0^+\right)}{dt} + 0 = 0$$

$$\Rightarrow \qquad 20 + \frac{di\left(0^+\right)}{dt} = 0$$

$$\Rightarrow \qquad \frac{di\left(0^+\right)}{dt} = -\mathbf{20\ A/sec}$$

Differentiating equation (4.3) with respect to t, we get

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

At $t = 0^+$, we get

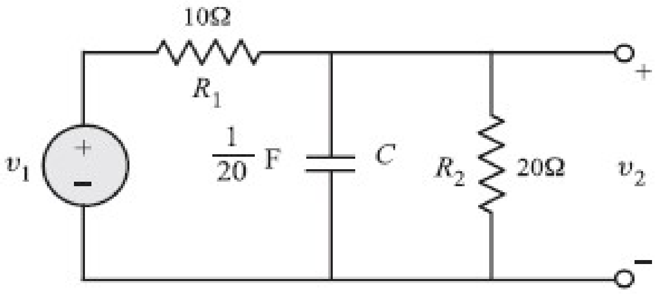
$$R\frac{di\left(0^+\right)}{dt} + L\frac{d^2i\left(0^+\right)}{dt^2} + \frac{i\left(0^+\right)}{C} = 0$$

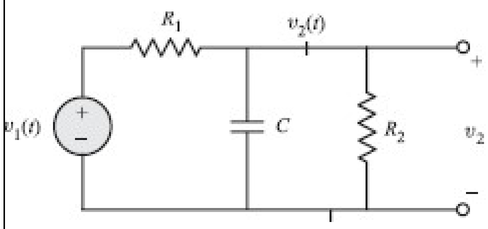
$$\Rightarrow \qquad R \times (-20) + L\frac{d^2i\left(0^+\right)}{dt^2} + \frac{2}{C} = 0$$

Hence. $$\frac{d^2i\left(0^+\right)}{dt^2} \approx -\mathbf{2 \times 10^6\ A/sec^2}$$

………..6M

2.	In the network shown , the input voltage V1(t) $=e^{-t}$ for t ≥ 0 and is zero for all t<0. If the capacitor in the network is initially uncharged, determine $\frac{dv}{dt}$, $\frac{d^2v}{dt^2}$ and $\frac{d^3v}{dt^3}$at t=0^+ at t=0^+.	[10]	CO3	L3





……….2M

$$\frac{v_2(t) - v_1(t)}{R_1} + C\frac{dv_2(t)}{dt} + \frac{v_2(t)}{R_2} = 0$$

$$\Rightarrow \quad \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_2(t) + C\frac{dv_2(t)}{dt} = \frac{v_1(t)}{R_1}$$

………..2M

Putting $t = 0^+$, we get

$$0.15v_2(0^+) + 0.05\frac{dv_2(0^+)}{dt} = 0.1$$

$$\Rightarrow \quad 0.15 \times 0 + 0.05\frac{dv_2(0^+)}{dt} = 0.1$$

$$\Rightarrow \quad \frac{dv_2(0^+)}{dt} = \frac{0.1}{0.05} = 2 \text{ Volts/ sec}$$

……………….3M

Differentiating equation (4.5) with respect to t, we get

$$0.15\frac{dv_2}{dt} + 0.05\frac{d^2v_2}{dt^2} = -0.1e^{-t}$$

Putting $t = 0^+$ in equation (4.6), we find that

$$\frac{d^2v_2(0^+)}{dt^2} = \frac{-0.1 - 0.3}{0.05} = \mathbf{-8 \text{ Volts/ sec}^2}$$

Again differentiating equation (4.6) with respect to t, we get

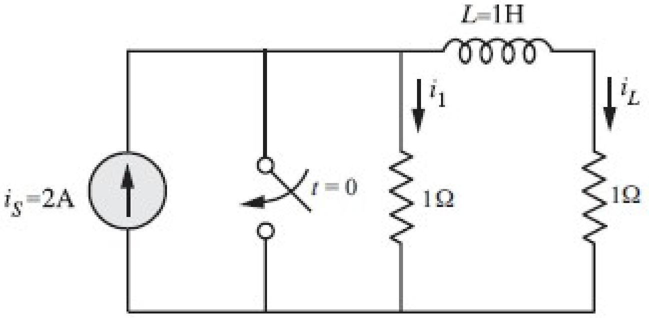
$$0.15\frac{d^2v_2}{dt^2} + 0.05\frac{d^3v_2}{dt^3} = 0.1e^{-t}$$

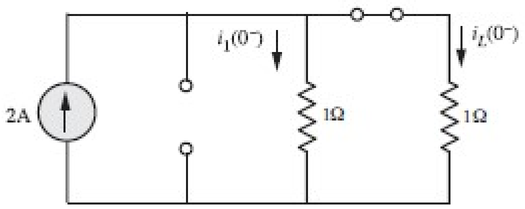
Putting $t = 0^+$ in equation (4.7) and solving for $\frac{d^3v_2}{dt^3}(0^+)$, we find that

$$\frac{d^3v_2(0^+)}{dt^3} = \frac{0.1 + 1.2}{0.05} = \mathbf{26 \text{ Volts/ sec}^3}$$

……….3M

3. Refer the circuit shown . Find $i_1(0_+)$ and $i_L(0_+)$. The circuit is in steady state for $t < 0$. (Use any method) [10] CO3 L3





………………..3M

$$i_L(0^-) = \frac{2 \times 1}{1+1} = 1\text{A}$$

……………..2M

Since the current in an inductor cannot change instantaneously, we have

$$i_L(0^+) = i_L(0^-) = 1\text{A}$$

At $t = 0^-$, $i_1(0^-) = 2 - 1 = 1\text{A}$. Please note that the current in a resistor can change instantaneously. Since at $t = 0^+$, the switch is just closed, the voltage across R_1 will be equal to zero because of the switch being short circuited and hence,

$$i_1(0^+) = 0\text{A}$$

Thus, the current in the resistor changes abruptly form 1A to 0A.

……………5M

4.	State and prove initial and final value theorems.	[10]	CO4	L3

The initial-value theorem allows us to find the initial value $x(0)$ directly from its Laplace transform $X(s)$.

If $x(t)$ is a causal signal,

then, $$x(0) = \lim_{s \to \infty} sX(s)$$

***Proof*:**

To prove this theorem, we use the time differentiation property.

$$\mathscr{L}\left\{\frac{dx(t)}{dt}\right\} = sX(s) - x(0) = \int_0^\infty \frac{dx}{dt} e^{-st} dt$$

If we let $s \to \infty$, then the integral on the right side of equation (5.10) vanishes due to damping factor, e^{-st}.

Thus, $$\lim_{s \to \infty} [sX(s) - x(0)] = 0$$

$$\Rightarrow \quad x(0) = \lim_{s \to \infty} sX(s)$$

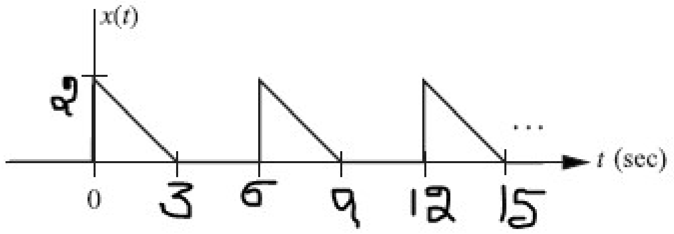
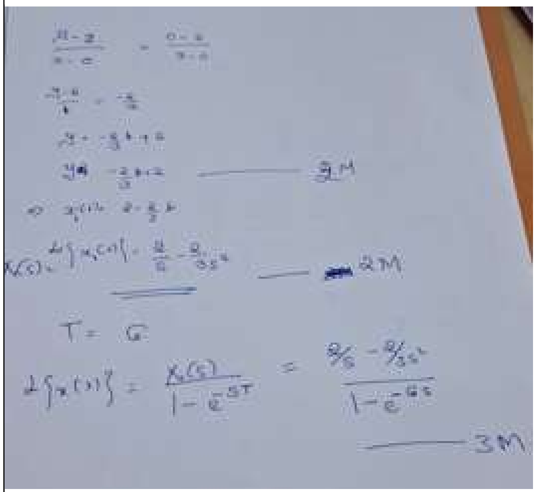
Statement:2M
Proof: 3M

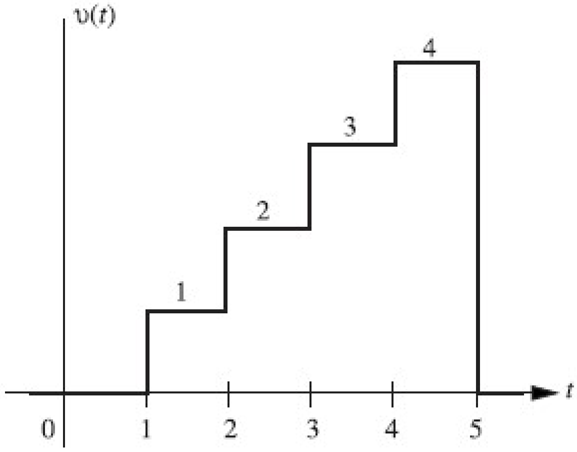
The final-value theorem allows us to find the final value $x(\infty)$ directly from its Laplace transform $X(s)$.

If $x(t)$ is a causal signal,

then $$\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$$

	***Proof*:** The Laplace transform of $\frac{dx(t)}{dt}$ is given by $$sX(s) - x(0) = \int_0^\infty \frac{dx(t)}{dt} e^{-st} dt$$ Taking the limit $s \to 0$ on both the sides, we get $$\lim_{s\to 0}[sX(s) - x(0)] = \lim_{s\to 0}\int_0^\infty \frac{dx(t)}{dt} e^{-st} dt$$ $$= \int_0^\infty \frac{dx(t)}{dt}\left[\lim_{s\to 0} e^{-st}\right] dt$$ $$= \int_0^\infty \frac{dx(t)}{dt} dt$$ $$= x(t)\Big\vert_0^\infty$$ $$= x(\infty) - x(0)$$ Since, $\lim_{s\to 0}[sX(s) - x(0)] = \lim_{s\to 0}[sX(s)] - x(0)$ we get, $x(\infty) - x(0) = \lim_{s\to 0}[sX(s) - x(0)]$ Hence, $x(\infty) = \lim_{s\to 0}[sX(s)]$ This proves the final value theorem. Statement:2M Proof: 3M			
5.	Obtain the Laplace transform of i) Step function ii) Ramp function iii) Impulse function **Unit step function** $$f(t) = u(t)$$ $$\mathcal{L}\{u(t)\} = F(s) = \int_0^\infty e^{-st} dt = \frac{1}{s}$$ ………………..3M **Impulse function** $$f(t) = \delta(t)$$ $$\mathcal{L}\{\delta(t)\} = F(s) = \int_0^\infty \delta(t) e^{-st} dt = e^{-st}\Big\vert_{t=0} = 1$$ ……………………….3M	[10]	CO4	L3

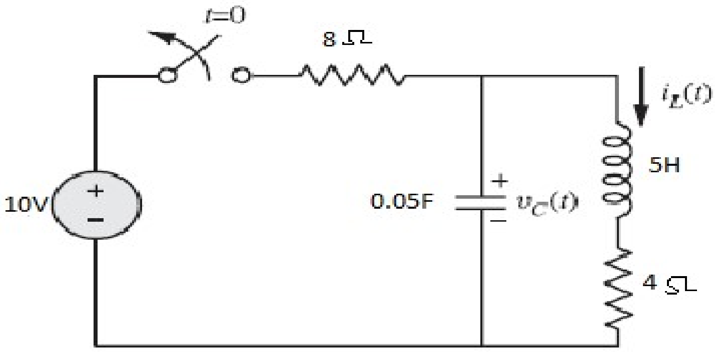
	Ramp function F(t)=t u(t) $\mathscr{L}\{u(t)\} = F(s) = \int_0^{\infty} t\, e^{-st} dt$ F(s)=1/s^2 ………4M			
6.	a. Find the Laplace transform of the periodic signal x(t).   b. List any three properties of Laplace transform and write the relevant equations. $\mathcal{L}\{f_1(t)+f_2(t)\} = F_1(s) + F_2(s)$. $\mathcal{L}\{x(t-t_0)\,u(t-t_0)\} = e^{-t_0 s} X(s)$ $\mathcal{L}\{e^{at}\,u(t)\} = \frac{1}{s-a}$ 3*1=3M	[7] [3]	CO4 CO4	L3 L2
7.	a.Develop the time domain equation v(t) of the staircase waveform shown. (in terms of unit step functions).	[2]	CO4	L3



$$v(t) = [u(t-1) - u(t-2)] + 2[u(t-2) - u(t-3)]$$
$$+3[u(t-3) - u(t-4)] + 4[u(t-4) - u(t-5)]$$
$$= u(t-1) + u(t-2) + u(t-3) + u(t-4) - 4u(t-5)$$

……..2M

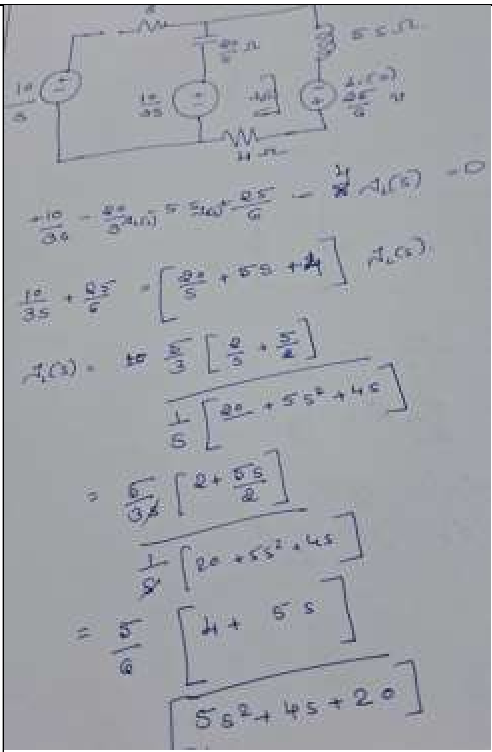
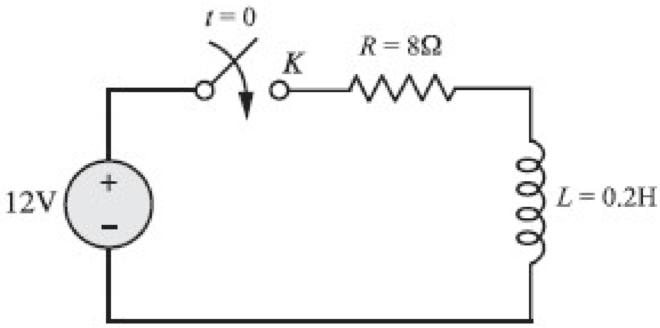
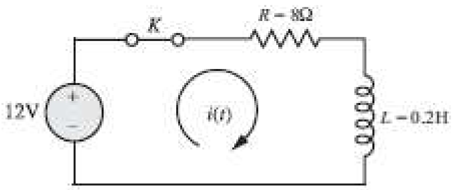
b. Determine the current $i_L(t)$ for t> 0 for the circuit shown using Laplace transform. Write the equivalent circuit in complex frequency domain for t>0.



[8]

CO4

L3

	 $i_L(t) = \left(1 - e^{-2t}\right)\ A$			
8.	a. In the given network, K is closed at t= 0 with zero current in the inductor. Find the values of i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at t=0⁺   ………………2M	[8] [2]	CO3 CO3	L3 L2

Applying KVL clockwise to the circuit shown in Fig. 4.12(b), we get

$$Ri + L\frac{di}{dt} = 12$$

$$\Rightarrow \quad 8i + 0.2\frac{di}{dt} = 12$$

At $t = 0^+$, the equation (4.1) becomes

$$8i(0^+) + 0.2\frac{di(0^+)}{dt} = 12$$

$$\Rightarrow \quad 8 \times 0 + 0.2\frac{di(0^+)}{dt} = 12$$

$$\Rightarrow \quad \frac{di(0^+)}{dt} = \frac{12}{0.2}$$

$$= \mathbf{60\ A/sec}$$

………3M

Differentiating equation (4.1) with respect to t, we get

$$8\frac{di}{dt} + 0.2\frac{d^2i}{dt^2} = 0$$

At $t = 0^+$, the above equation becomes

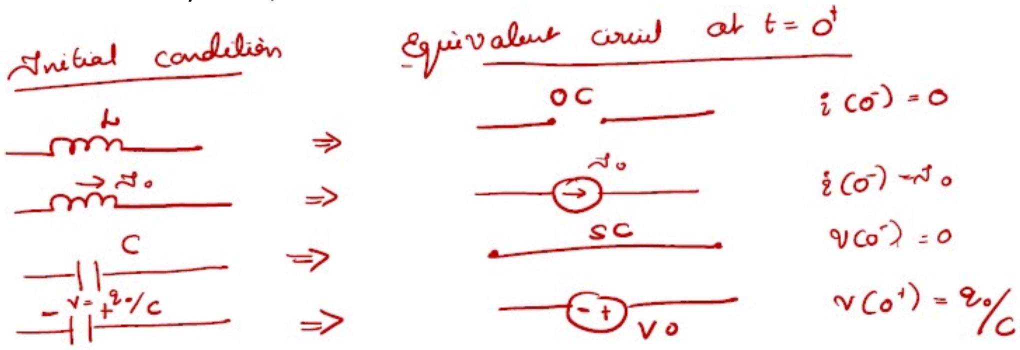
$$8\frac{di(0^+)}{dt} + 0.2\frac{d^2i(0^+)}{dt^2} = 0$$

$$\Rightarrow \quad 8 \times 60 + 0.2\frac{d^2i(0^+)}{dt^2} = 0$$

Hence $$\frac{d^2i(0^+)}{dt^2} = \mathbf{-\,2400\ A/sec^2}$$

…………………..3M

b. List the initial conditions of inductor and capacitor with relevant equivalent circuit elements at steady state / t=0^- and t=0^+



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