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FOR 3A AND 3C ONLY

Internal Test 2 –December 2024

Sub:	Network Analysis							Code:	BEC304
Date:	17/12/2024	Duration:	90 mins	Max Marks:	50	Sem:	**3 A&C**	Branch:	ECE

Note: Answer any FIVE full questions with neat diagram wherever necessary.

		Marks	OBE CO	OBE RBT
1	**Find Y parameters of the network shown in Figure Q1.**	[5+5]	CO5	L3

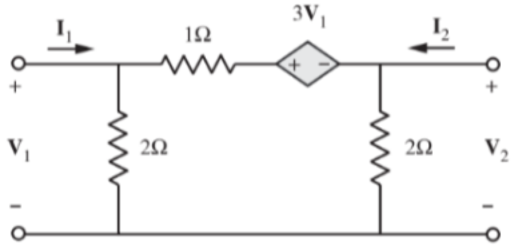


Figure Q 1

Figure 7.13

SOLUTION

Converting the voltage source into an equivalent current source, we get the circuit diagram shown in Fig. 7.14(a).

To find $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, the output terminals of Fig. 7.14(a) are shorted and connect a current source $\mathbf{I}_1$ to the input terminals. This results in a circuit diagram as shown in Fig. 7.14(b).

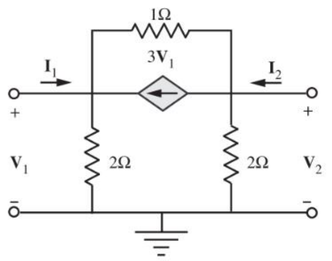


Figure 7.14(a)

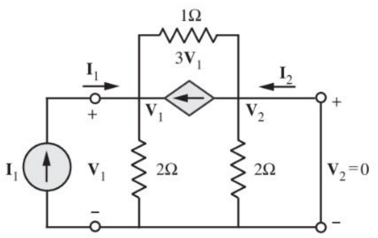


Figure 7.14(b)

KCL at node $\mathbf{V}_1$:

$$\frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1} = \mathbf{I}_1 + 3\mathbf{V}_1$$

Since $\mathbf{V}_2 = 0$, we get

$$\frac{\mathbf{V}_1}{2} + \mathbf{V}_1 = \mathbf{I}_1 + 3\mathbf{V}_1$$

$$\Rightarrow \qquad \mathbf{I}_1 = \frac{-3}{2}\mathbf{V}_1$$

Hence, $$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = \frac{-3}{2}\text{S}$$

KCL at node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2}{2} + 3\mathbf{V}_1 + \frac{\mathbf{V}_2 - \mathbf{V}_1}{1} = \mathbf{I}_2$$

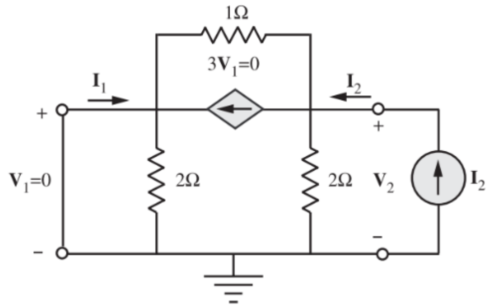
Since $\mathbf{V}_2 = 0$, we get

$$0 + 3\mathbf{V}_1 - \mathbf{V}_1 = \mathbf{I}_2$$

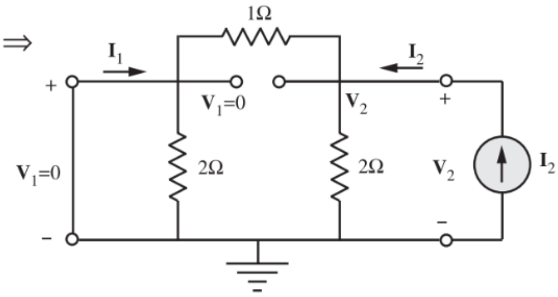
$$\Rightarrow \qquad \mathbf{I}_2 = 2\mathbf{V}_1$$

Hence $$\mathbf{y}_{21} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 2\text{S}$$

To find $\mathbf{y}_{21}$ and $\mathbf{y}_{22}$, the input terminals of Fig. 7.14(a) are shorted and connect a current source $\mathbf{I}_2$ to the output terminals. This results in a circuit diagram as shown in Fig. 7.14(c).



$\Rightarrow$



KCL at node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2 - 0}{1} = \mathbf{I}_2$$

$$\Rightarrow \qquad \frac{3}{2}\mathbf{V}_2 = \mathbf{I}_2$$

Hence, $$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{3}{2}\text{S}$$

KCL at node $\mathbf{V}_1$:

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1} = 0$$

Since $\mathbf{V}_1 = 0$, we get

$$\mathbf{I}_1 = -\mathbf{V}_2$$

Hence, $$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = -1\text{S}$$

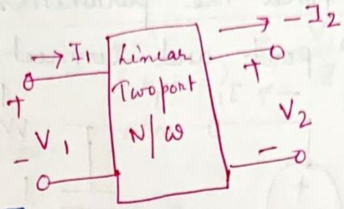
2a	**Define ABCD parameter.**	[4]	CO5	L1

(39)

Transmission Parameters:

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$



In matrix form:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}.$$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ ⇒ open circuit voltage ratio

$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$ ⇒ negative short circuit transfer imp.

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ ⇒ open circuit transfer admittance

$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$ ⇒ negative short circuit current ra

2b	**Z parameters of a network are obtained from an experiment. Explain how h parameters can be obtained from the data.**	[6]	CO5	L1

(45)

b) Relation between Z parameters and h parameters

The Z parameter equations are $V_1 = Z_{11} I_1 + Z_{12} I_2$ — (1)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — (2).

h-parameter equations are

$V_1 = h_{11} I_1 + h_{12} V_2$ — (3)

$I_2 = h_{21} I_1 + h_{22} V_2$ — (4)

from eqn (4) $h_{22} V_2 = I_2 - h_{21} I_1$.

$V_2 = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1$ — (5).

Substituting (5) in eqn (3).

$$V_1 = h_{11} I_1 + h_{12}\left[\frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1\right]$$

$$\Rightarrow V_1 = I_1\left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right] + \frac{h_{12}}{h_{22}} I_2$$

$$\Rightarrow \boxed{V_1 = I_1\left[\frac{\Delta h}{h_{22}}\right] + \frac{h_{12}}{h_{22}} I_2} \rightarrow (6)$$

	from eqn (5) $V_2 = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1$ $\boxed{\text{Or } V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \quad -(7)}$ Comparing (1) with (6) and (2) with (7) $Z = \begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$			
3a	**Prove Initial and Final value Theorem with respect to Laplace Transform.** Initial Value Theorem: Initial value theorem allows us to find the initial value $x(0)$ directly from its L.T $X(s)$ $x(0) = \lim_{s \to \infty} X(s)$. Proof: We know that, $\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = sX(s) - x(0) = \int_0^\infty \frac{dx}{dt} e^{-st} dt = 0$ $\because e^{-\infty} = 0$. If $s \to \infty$, then $\lim_{s \to \infty} [sX(s) - x(0)] = 0$; $\lim_{s \to \infty} [sX(s)] - x(0) = 0$. $\boxed{x(0) = \lim_{s \to \infty} [sX(s)]}$ FINAL VALUE THEOREM: The final value theorem allow us to find the final value $x(\infty)$ directly from its LT $X(s)$. If $x(t)$ casual signal. $\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$ Proof: $\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = sX(s) - x(0) = \int_0^\infty \frac{dx}{dt} e^{-st} dt$	[5]	CO4	L1

if $s \to 0$

$$\lim_{s\to 0}\left[sX(s) - x(0)\right] = \lim_{s\to 0}\int_0^\infty \frac{dx}{dt}e^{-st}\,dt$$

$$= \int_0^\infty \frac{dx(t)}{dt}\left[\lim_{s\to 0} e^{-st}\right]dt$$

$$= \int_0^\infty \frac{dx(t)}{dt}\,dt = x(t)\Big|_0^\infty$$

$$= x(\infty) - x(0).$$

$$\lim_{s\to 0}\left[sX(s) - x(0)\right] = \lim_{s\to 0} sX(s) - x(0).$$

$$x(\infty) - x(0) = \lim_{s\to 0} sX(s) - x(0).$$

$$\boxed{x(\infty) = \lim_{s\to 0} sX(s).}$$

3b	**Determine Laplace Transform of a) Unit Step Function, b) Ramp Function.**	[5]	CO4	L1

Laplace Transform of unit step function:

$$u(t) = \begin{cases} 1 & t \geq 0^+ \\ 0 & t \leq 0^- \end{cases}$$

$$\mathcal{L}\{u(t)\} = F(s) = \int_0^\infty 1\cdot e^{-st}\,dt = \left[-\frac{1}{s}e^{-st}\right]_0^\infty$$

$$= \frac{1}{s}.$$

<table>
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<td>Laplace transform of ramp function.
Ramp function is defined as $x(t) = tu(t)$
$\mathcal{L}\{x(t)\} = \mathcal{L}\{tu(t)\} = \int_0^\infty tu(t)e^{-st}\,dt$
$= \int_0^\infty te^{-st}\,dt$
$= \left[\frac{te^{-st}}{s}\right]_0^\infty - \int_0^\infty (1)\frac{e^{-st}}{-s}\,dt$
$= 0 - \left[\frac{e^{-st}}{s^2}\right]_0^\infty = \left(\cancel{0 - \frac{1}{s^2}}\right) = \frac{1}{s^2}$</td>
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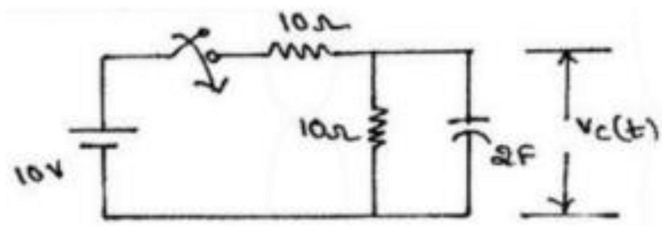
<td>4.</td>
<td>The switch in the network shown in Figure Q4 is closed at t=0. Find the voltage across Capacitor using Laplace transform.


Figure Q 4</td>
<td>[10]</td>
<td>CO4</td>
<td>L3</td>
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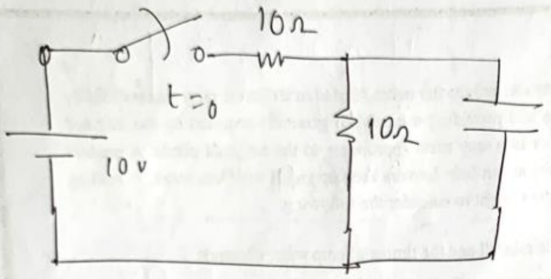
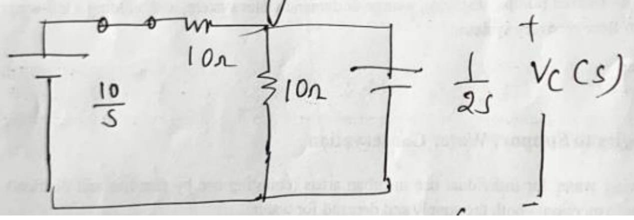


as circuit not activated at $t=0^-$,

so $v_c(0^-) = 0$

Hence $v_c(0^+) = 0$

Circuit in Frequency domain



Applying KCL at node $v_c(s)$

$$V_c(s)\left[\frac{1}{10}+\frac{1}{10}+2s\right] = \frac{\cancel{10}}{s\times\cancel{10}}$$

or $V_c(s)\ \dfrac{2+20s}{10} = \dfrac{1}{s}$

or $V_c(s) = \dfrac{5}{s(10s+1)}$

$$= \frac{A}{s} + \frac{B}{10s+1}$$

$$= \frac{10As + Bs + A}{s(10s+1)}$$

Comparing $10 \cdot A + B = 0$

$A = 5$

$\therefore B = -50.$

$$\therefore V_c(s) = \frac{5}{s} - \frac{5\cancel{0}}{\cancel{10}(s+0.1)}$$

$$v_c(t) = 5u(t) - 5e^{-0.1t}u(t)$$

$$v_c(t) = 5u(t)\left[1 - e^{-0.1t}\right] \text{ Ans}$$

5 **Given the signal** [10] CO4 L3

$$x(t) = \begin{cases} 3, & t < 0 \\ -2 & 0 < t < 1 \\ 2t - 4 & t > 1 \end{cases}$$

Find Laplace Transform in terms of singularity function.

Given the signal

$$x(t) = \begin{cases} 3, & t < 0 \\ -2, & 0 < t < 1 \\ 2t - 4, & t > 1 \end{cases}$$

Express $x(t)$ in terms of singularity functions. Also find $\mathscr{L}\{x(t)\}$.

SOLUTION

The signal $x(t)$ may be viewed as follows:

(i) in the interval, $t < 0$, $x(t)$ may be regarded as $3u(-t)$

(ii) in the interval, $0 < t < 1$, $x(t)$ may be viewed as $-2[u(t) - u(t-1)]$ and

(iii) for $t > 1$, $x(t)$ may be viewed as $(2t - 4)u(t-1)$

Thus, $\quad x(t) = 3u(-t) - 2[u(t) - u(t-1)] + (2t-4)u(t-1)$

$$\Rightarrow \quad x(t) = 3[1 - u(t)] - 2u(t) + 2u(t-1) + 2tu(t-1) - 4u(t-1)$$
$$= 3 - 5u(t) - 2u(t-1) + 2(t-1+1)u(t-1)$$
$$= 3 - 5u(t) - 2u(t-1) + 2(t-1)u(t-1) + 2u(t-1)$$
$$= \mathbf{3 - 5u(t) + 2r(t-1)}$$

$\mathscr{L}\{x(t)\}$ cannot be found because $x(t)$ contains a constant 3 for $-\infty < t < 0$ (a noncausal signal).

6. **Determine the Laplace transform of the following stair case wave form using gate function.** [10] CO4 L4

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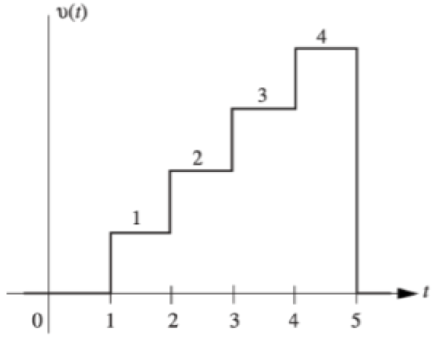

Figure Q6

Solution: $v(t) = \begin{cases} 1 & 1 < t < 2 \\ 2 & 2 < t < 3 \\ 3 & 3 < t < 4 \\ 4 & 4 < t < 5 \\ 0 & \text{else where.} \end{cases}$

$\therefore v(t) = [u(t-1) - u(t-2)] + 2[u(t-2) - u(t-3)] + 3[u(t-3) - u(t-4)] + 4[u(t-4) - u(t-5)]$

$= u(t-1) + u(t-2) + u(t-3) + u(t-4) \cancel{+} -4u(t$

$\therefore V(s) = \frac{1}{s}e^{-s} + \frac{e^{-2s}}{s} + \frac{e^{-3s}}{s} + \frac{e^{-4s}}{s} - \frac{4e^{-5s}}{s}$
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<td>7</td>
<td>A series RLC circuit has a resistance of 10Ω, an inductance of 0.3H and a capacitance of 100μF. The applied voltage is 220 V. Find i) Resonant Frequency, ii) Quality Factor, iii) Lower and upper cut off frequencies, iv) Band width, v) current at resonance, and vi) voltage across inductance at resonance.</td>
<td>[10]</td>
<td>CO6</td>
<td>L1</td>
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	$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.3\times100\times10^{-6}}}$ $f_0 = 29.05\ Hz$ $B.W = \frac{R}{2\pi L} = \frac{10}{2\pi\times0.3} = 5.30\ Hz.$ $Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = 5.47$ $B.W = f_2 - f_1 = 5.30$ $f_0 = \sqrt{f_1 f_2}$ $f_0^2 = f_1 f_2 = 843.90$ $f_1(530+f_1) = 843.90$ $f_1^2 + 530 f_1 - 843.9 = 0$ $f_1 = 1.58\ Hz$ $f_2 = 1.58 + 5.30 = 6.88\ Hz$ Current at resonance $= \frac{V}{R} = \frac{220}{10} = 22A$ Voltage across inductance $= \cancel{jX_L V}\ Q_0 V$ $= j\,5.47\times220$ $= j\,1203V$			
8	**Derive the expressions of half power frequencies in terms of resonant frequency, also derive the expression for bandwidth in terms circuit parameters for a series R, L , C circuit.** Current in the series RLC circuit is given by, $I = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}}$ —(1) Also at ~~resonance~~ cut off frequency. $I = \frac{I_0}{\sqrt{2}}$, where I_0 is maximum current	[10]	CO6	L2

$$I = \frac{V}{R\sqrt{2}} \quad \text{—(2)}$$

Equating (1) and (2)

$$\frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{R\sqrt{2}}$$

$$\Rightarrow 2R^2 = R^2 + (X_L - X_C)^2$$

$$\Rightarrow R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\text{Or } \omega L - \frac{1}{\omega C} = \pm R. \quad \text{—(3)}$$

At upper cut off frequency, f_2

$$R = \omega_2 L - \frac{1}{\omega_2 C} \quad \text{—(4)}$$

At lower cut off frequency

$$-R = \omega_1 L - \frac{1}{\omega_1 C} \quad \text{—(5)}$$

Adding (4) and (5)

$$\omega_2 L - \frac{1}{\omega_2 C} + \omega_1 L - \frac{1}{\omega_1 C} = R - R$$

$$\Rightarrow L(\omega_2 + \omega_1) - \frac{1}{C}\left(\frac{1}{\omega_2} + \frac{1}{\omega_1}\right) = 0$$

$$\Rightarrow L(\omega_2 + \omega_1) = \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)$$

$$L = \frac{1}{C} \times \frac{1}{\omega_1 \omega_2}$$

9

$$\Rightarrow \omega_1 \omega_2 = \frac{1}{LC}$$

$$\Rightarrow \omega_1 \omega_2 = \omega_0^2 \quad \left[\text{as } \omega_0^2 = \frac{1}{LC}\right].$$

$$\Rightarrow \boxed{f_1 f_2 = f_0^2} \Rightarrow$$ In series resonance circuit resonant frequency f_0 is geometrical mean of f_1 & f_2.

Subtracting eqn (4) and (5)

$$\omega_2 L - \frac{1}{\omega_2 C} - \omega_1 L + \frac{1}{\omega_1 C} = R - (-R)$$

$$\Rightarrow L(\omega_2 - \omega_1) - \frac{1}{C}\left(\frac{\omega_1 - \omega_2}{\omega_1 \omega_2}\right) = 2R$$

$$\Rightarrow L(\omega_2 - \omega_1) + \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) = 2R$$

$$\Rightarrow (\omega_2 - \omega_1)\left[L + \frac{1}{C\omega_0^2}\right] = 2R$$

$$\Rightarrow (\omega_2 - \omega_1)\left[\frac{LC\omega_0^2 + 1}{C\omega_0^2}\right] = 2R$$

$$\Rightarrow (\omega_2 - \omega_1)\left[\frac{\cancel{LC} \times \frac{1}{\cancel{LC}} + 1}{C\omega_0^2}\right] = 2R$$

$$\Rightarrow \frac{(\omega_2 - \omega_1)}{C\omega_0^2} = R$$

Or $$R = \frac{\omega_2 - \omega_1}{\cancel{C} \times \frac{1}{L\cancel{C}}} \Rightarrow \omega_2 - \omega_1 = \frac{R}{L}$$

$$\boxed{\text{Or } f_2 - f_1 = \frac{R}{2\pi L}}$$