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INTERNAL ASSESSMENT TEST – II

Sub:	DIGITAL COMMUNICATIONS							Code:	BEC503
Date:	16/ 12 / 2024	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE

Answer any 5 full questions

		Marks	CO	RBT
1	Derive an expression for the canonical representation of bandpass signals. Obtain a scheme for extracting in-phase and quadrature components of bandpass signals. Draw the corresponding block diagram.	[10]	CO1	L1
2	a. Define Hilbert transform. State and prove the properties of Hilbert transform. Plot the magnitude response and phase response of the ideal Hilbert transformer. b. Determine the Hilbert transform of the signal $x(t) = 2 \quad 0 \le t \le T \qquad 0$ otherwise	[5+5]	CO1	L3
3	a. Obtain the decision rule for ML decoding and explain correlation receiver b. With a neat block diagram, explain the generation and detection of QPSK.	[5+5]	CO1	L2

4	Derive the expression for error probability of binary BFSK using coherent detection.	[10]	CO2	L1
5	a) With a neat block diagram, explain the generation and detection of DPSK. b) In a FSK system the transmitted binary data is 2.5×10^6 bps. PSD of zero mean AWGN is 10^{-20} W/Hz. The amplitude of the received signal is 1µv. Find the probability of error using coherent detection. Given erf (2.5)=0.99959	[6+4]	CO2	L3
6	Obtain a set of orthonormal basis functions for the following set of signals. $x_1(t) = \begin{cases} 2 \text{ from } 0 \leq t \leq 1 \\ 0 \quad \text{otherwise} \end{cases}$ and $x_2(t) = \begin{cases} -4 \text{ from } 0 \leq t \leq 2 \\ 0 \quad \text{otherwise} \end{cases}$ and $x_3(t) = \begin{cases} 4 \text{ from } 0 \leq t \leq 3 \\ 0 \quad \text{otherwise} \end{cases}$ Express the signals as a linear combination of basis functions. Draw the signal space diagram (Constellation Diagram).	[10]	CO1	L3
7	Consider a (2,1,2) Convolutional encoder with g1=101 and g2=011. Decode the code sequence {11,01,01,10,00,01,11} using Viterbi algorithm.	[10]	CO5	L3

CI

CCI

HOD

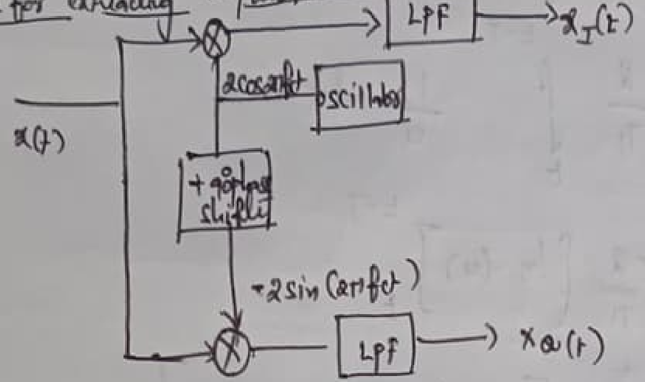
DC IAT 2 Solutions

Dr. Smitha Alamuru

1. $x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$

Derivation — 5M

scheme for extracting in phase quadrature



Mathematical Expressions — 3M

diagram — 2M

2. a) Hilbert Transform definition — 1M

properties — 3M

Magnitude & phase response — 1M.

b) $x(t) = 2 \quad 0 \le t \le T$

$= 0 \quad$ otherwise.

$\boxed{h(t) = \frac{1}{\pi t}}$

$\hat{x}(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_0^T 2 \cdot \frac{1}{\pi(t-\tau)}\, d\tau$$

$$= \frac{2}{\pi}\int_0^T \frac{1}{t-\tau}\, d\tau$$

$t-\tau = u$ when $\tau = 0 \quad u = t$

$-d\tau = du$ $\quad \tau = T \quad u = t-T$

$d\tau = -du$

$$\hat{x}(t) = \frac{2}{\pi}\int_{t}^{t-T} \frac{1}{u}\,(-du)$$

$$= \frac{-2}{\pi}\int_{t}^{t-T} \frac{1}{u}\,du$$

$$= \frac{-2}{\pi}\Big[\ln(u)\Big]_{t}^{t-T}$$

$$= \frac{-2}{\pi}\Big[\ln(t-T) - \ln(t)\Big]$$

$$\boxed{\hat{x}(t) = \frac{-2}{\pi}\ln\left(\frac{t-T}{t}\right)}$$

3. a) decision rule for ML decoding

set $\hat{m} = m_i$ if

$P(m_i \text{ sent} / x) \geq P(m_k \text{ sent} / x)$ for all $k \neq i$

— 2M

Correlation receiver

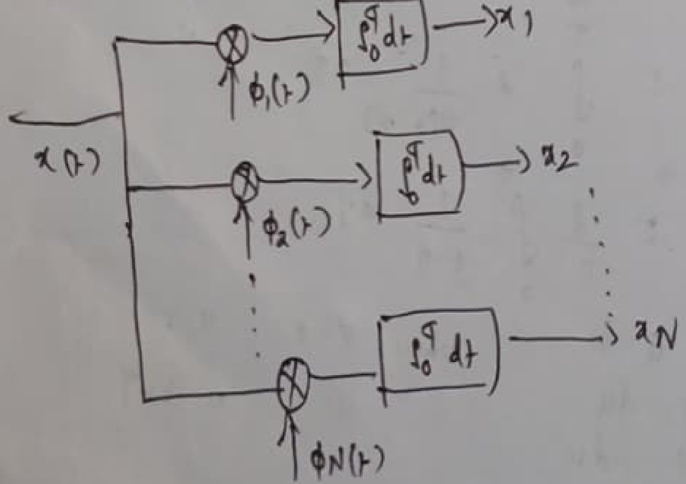
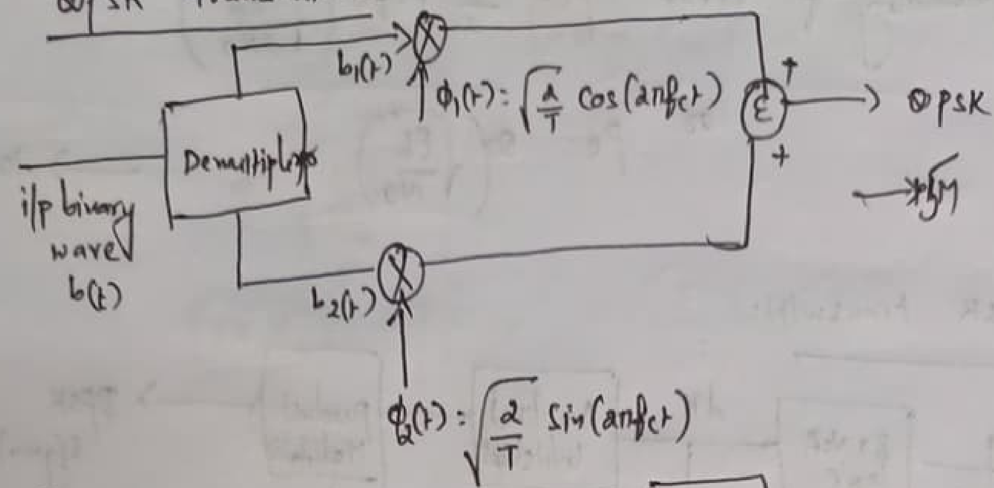


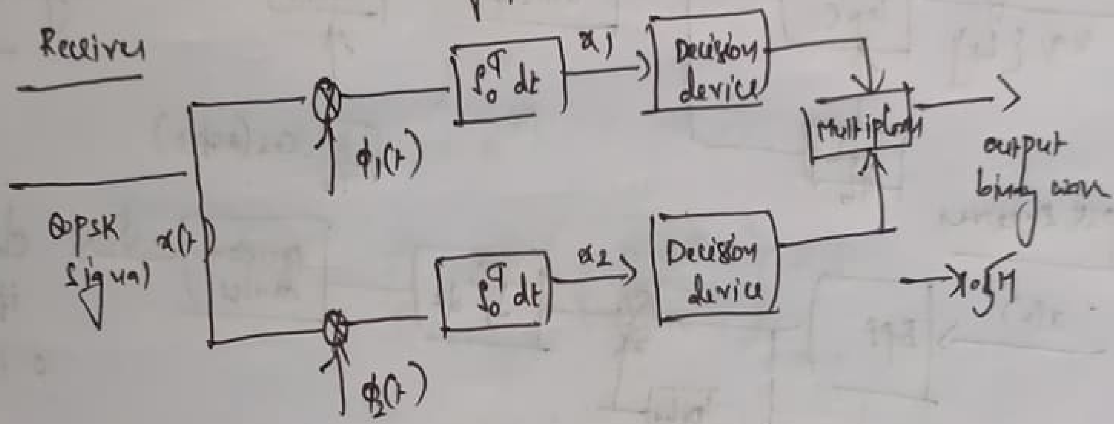
diagram — 2M

Explanation - 2M

3. b) QPSK Transmitter



Receiver



Explanation — 2M.

4. BFSK

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \quad 0 \le t \le T_b$$

$= 0$ elsewhere

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t) \quad 0 \le t \le T_b$$

$= 0$ elsewhere

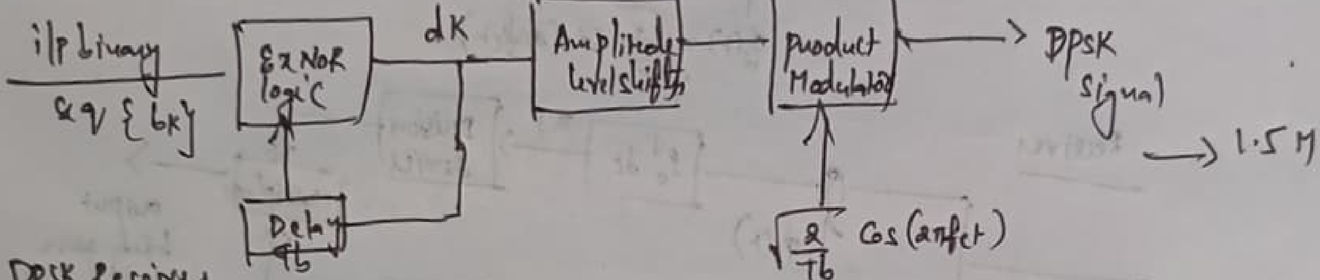
$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t) \quad 0 \le t \le T_b$$

$= 0$ elsewhere

probability of error $P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$

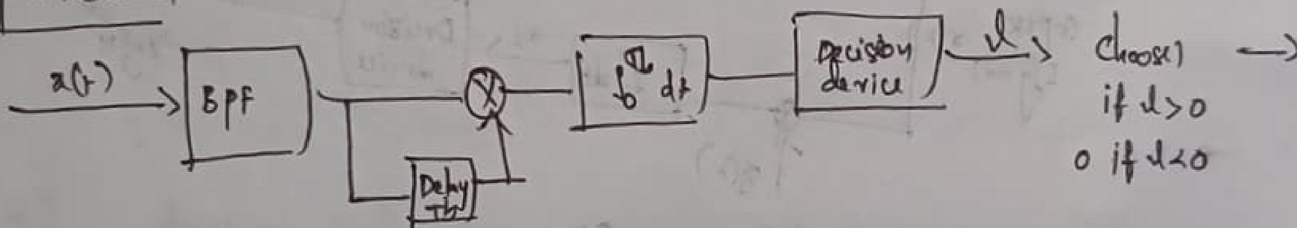
or $P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$ $\longrightarrow$ 10M

4. a) <u>DPSK transmitter</u>



$\longrightarrow$ 1.5M

<u>DPSK Receiver</u>



Explanation — 2M. with Example — 1M.

b) $R_b = 2.5 \times 10^{6}$ bps

$\frac{N_0}{2} = 10^{-20}$ w/Hz

$A = 1\mu V$

$P_e = ?$

for BFSK using coherent detection

$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$

$A = \sqrt{\frac{2E_b}{T_b}}$ $T_b = \frac{1}{R_b} = \frac{1}{2.5 \times 10^{6}} = 0.4 \times 10^{-6} = 4 \times 10^{-7}$

$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$A = \sqrt{\frac{2E_b}{T_b}}$$

$$(1\times 10^{-6})^2 = \left(\sqrt{\frac{2E_b}{T_b}}\right)^2$$

$$10^{-12} = \frac{\cancel{2}\times E_b}{\underset{2}{\cancel{4}}\times 10^{-7}}$$

$$E_b = 2\times 10^{-19}$$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{2\times 10^{-19}}{2\times2\times 10^{-20}}}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{0.5\times 10}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(2.23606\right)$$

Given $\mathrm{erf}(2.5) = 0.99959$

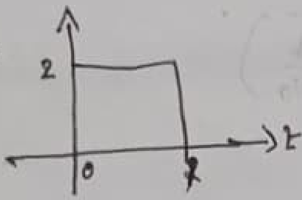
$$= \frac{1}{2}\,\mathrm{erfc}(2.5)$$

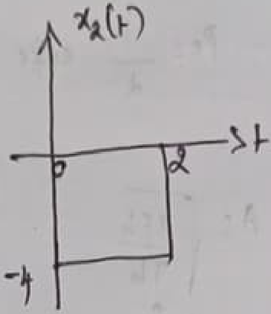
$\mathrm{erfc}(2.5) = 1-\mathrm{erf}(2.5)$

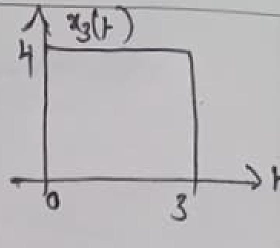
$$= \frac{1}{2}\left[1-0.99959\right]$$

$$\boxed{P_e = 2.05\times 10^{-4}}$$

5. $x_1(t)$:







$s_1(t) = x_1(t)$

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

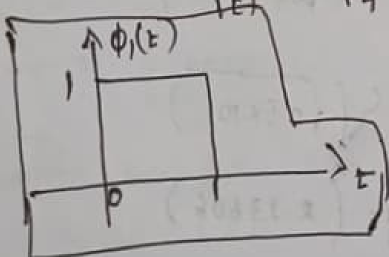
$s_2(t) = x_2(t)$

$s_3(t) = x_3(t)$

$$E_1 = \int_0^{q} s_1^2(t)\, dt$$

$$= \int_0^1 (2)^2\, dt = 4.$$

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} = \frac{2}{\sqrt{4}} = 1$$

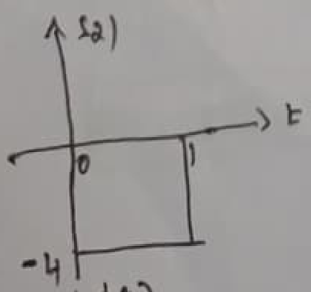


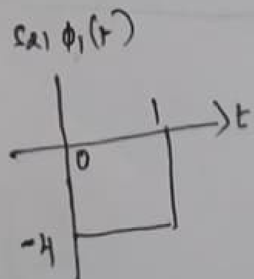
$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t)$$

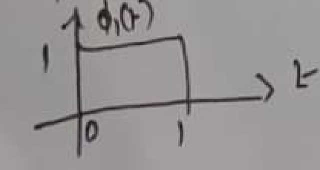
$$s_{21} = \int_0^{q} s_2(t)\,\phi_1(t)\, dt$$

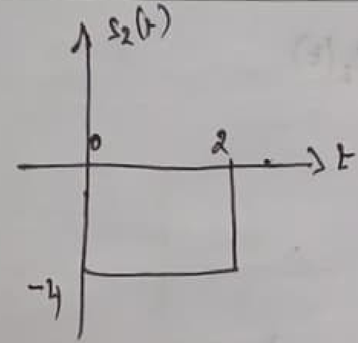
$$= \int_0^1 (-4)\cdot 1\, dt + \int_1^2 (-4)\cdot 0$$

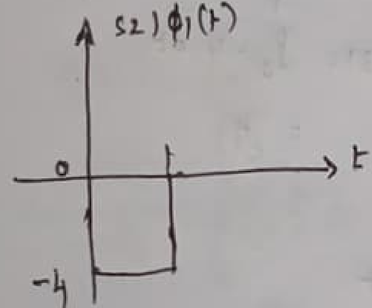
$$= -4.$$



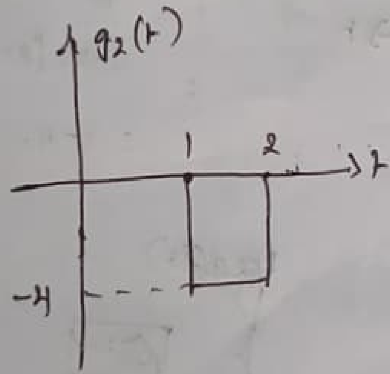








$$g_2(t) = s_2(t) - s_{21}\phi_1(t)$$

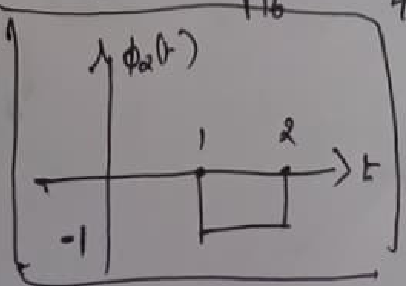


$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_2}}$$

$$E_2 = \int_1^2 (-4)^2 \, dt$$

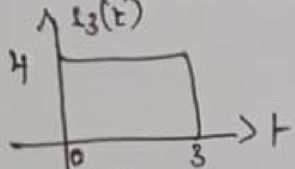
$$= 16.$$

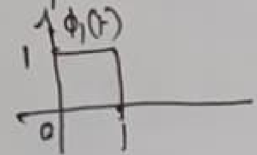
$$\phi_2(t) = \frac{-4}{\sqrt{16}} = \frac{-4}{4} = -1.$$



$$g_3(t) = s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)$$

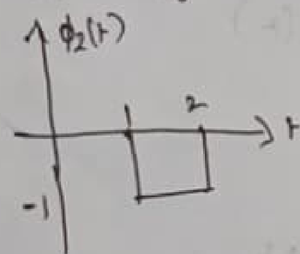
$$s_{31} = \int_0^T s_3(t)\,\phi_1(t)\,dt$$



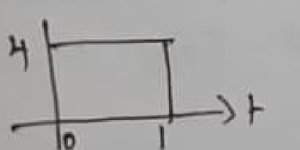


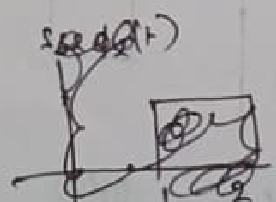
$$s_{31} = \int_0^1 4\,dt$$
$$= 4.$$

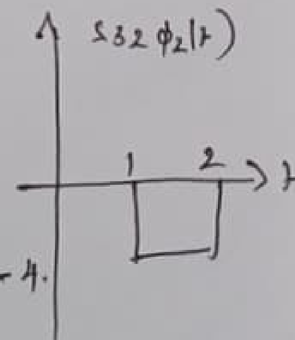
$$s_{32} = \int_0^T s_3(t)\,\phi_2(t)\,dt$$

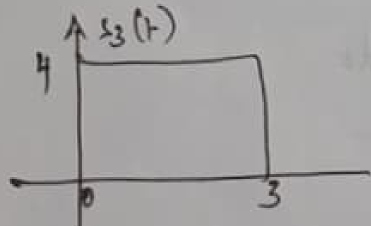


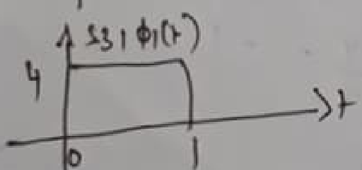
$$s_{32} = \int_1^2 4\cdot(-1)\,dt$$
$$= -4\cdot[2-1]$$
$$= -4.$$

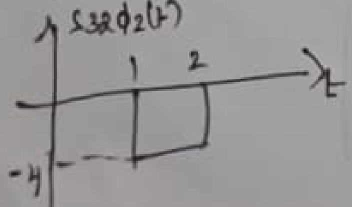












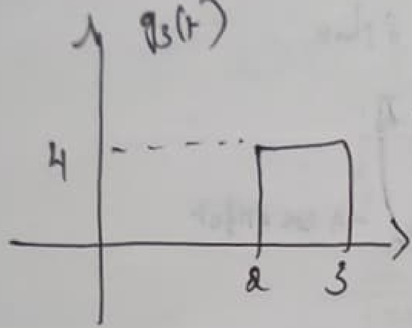
$$g_3(t) = s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)$$

$$\int_0^1 (4-4-0) + \int_1^2 (4-0-(-4)) + \int_2^3 (4-0-0)\,dt$$

$$= \int_2^3 4\,dt$$

$$= 4\cdot(3-2)$$

$$= 4. \quad 2 \le t \le 3$$

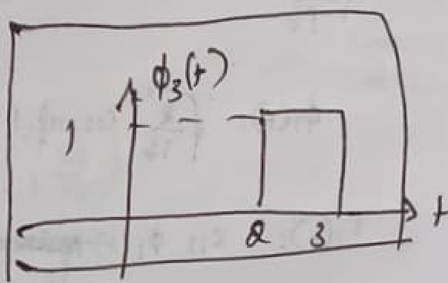


$$\phi_3(t) = \frac{g_3(t)}{\sqrt{E_3}}$$

$$E_3 = \int_2^3 (4)^2\,dt$$

$$= 16.$$

$$\phi_3(t) = \frac{4}{\sqrt{16}} = 1$$



$$s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t)$$

$$s_1(t) = s_{11}\,\phi_1(t) + s_{12}\,\phi_2(t) + s_{13}\,\phi_3(t)$$

$$s_{11} = \int_0^1 s_1(t)\,\phi_1(t)$$

$$= 2.$$

$$\boxed{s_1(t) = 2\,\phi_1(t)}$$

$$s_2(t) = s_{21}\,\phi_1(t) + s_{22}\,\phi_2(t) + s_{23}\,\phi_3(t)$$

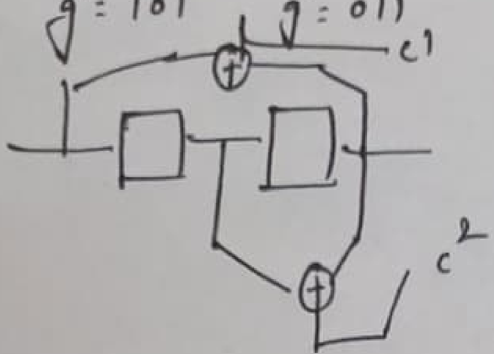
$$= -4\,\phi_1(t) + 4\,\phi_2(t)$$

$$s_3(t) = s_{31}\,\phi_1(t) + s_{32}\,\phi_2(t) + s_{33}\,\phi_3(t)$$

$$= 4\,\phi_1(t) - 4\,\phi_2(t) + 4\,\phi_3(t)$$

(4) (2,1,2) Convolutional Encoder

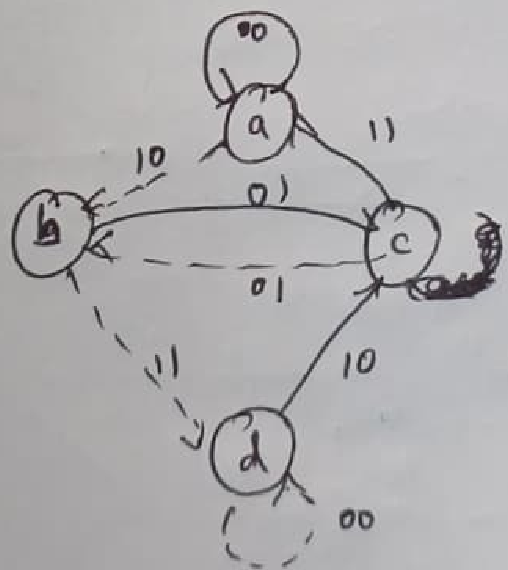
$g^1 = 101$ $g^2 = 011$

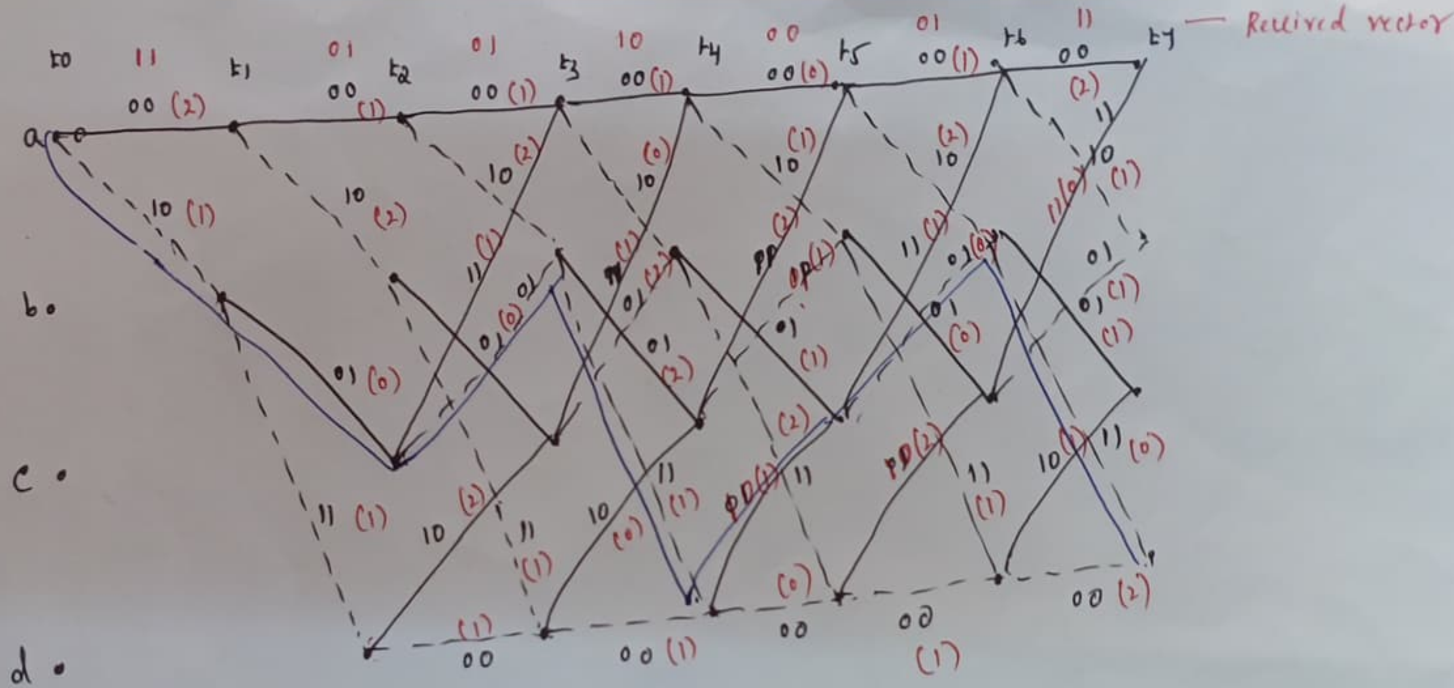


$n=2$ $k=1$ $m=2$

2ff's — 4 states

PS		NS $x=0$	NS $x=1$	o/p $x=0$	o/p $x=1$
00	(a)	00 (a)	10 (b)	00	10
10	(b)	01 (c)	11 (d)	01	11
01	(c)	00 (a)	10 (b)	11	01
11	(d)	01 (c)	11 (d)	10	00





The shortest path with minimum Hamming Metric is shown

according to that the transmitted codeword is

10 01 01 11 10 01 11

the msg seq is 1 0 1 1 0 1 0