	USN	CMRIT

Internal Assessment Test – I November - 2024

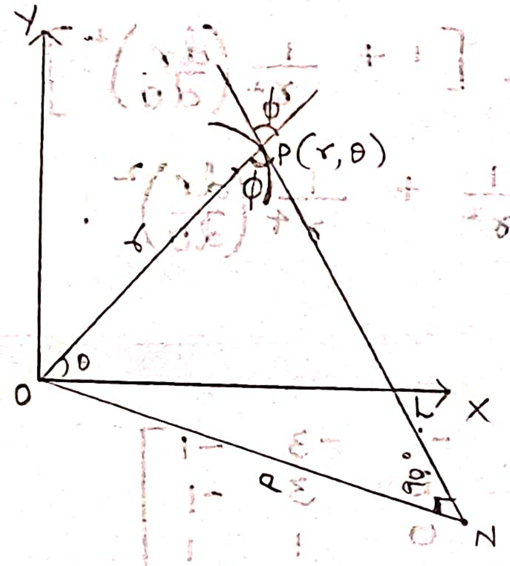
Sub:	Mathematics-1 for CSE Stream							Code:	BMATS101
Date:	19-11-2024	Duration:	90 mins	Max Marks:	50	Sem:	I	SEC	I, J, K, L (CHE CYCLE)

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE CO	OBE RBT
1	With usual notations prove that. $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$	[08]	CO1	L3
2	Find the rank of the matrix $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$	[07]	CO5	L3
3	Solve the following system using Seidel method **(perform only 3 iterations)**: $20x + y - 2z = 17, \quad 3x + 20y - z = -18, \quad 2x - 3y + 20z = 25.$	[07]	CO5	L3

4	Investigate the values of $\boldsymbol{a}$ and $\boldsymbol{b}$ such that the following system may have (i) infinitely many solutions (ii no solution (iii) unique solutions. $x + 2y + 3z = 6, \quad x + 3y + 5z = 9, \quad 2x + 5y + az = b.$	[07]	CO5	L3
5	Find the pedal Equation for the curve $\frac{2a}{r} = (1 + cos\theta)$.	[07]	CO1	L3
6	Find the angle between the following pairs of curves r=a(1+cosθ) and r=a(1-cosθ)	[07]	CO1	L3
7	Find the radius of curvature of the curve $x^4 + y^4 = 1$ at the point (1, 1).	[07]	CO1	L3
8	Expand $y(x) = tanx$ by Maclaurin's series up to the term containing x^3 .	[07]	CO2	L3

1.



Let 'O' be the pole, and $P(r,\theta)$ be any point on the curve. Let PL be a perpendicular line to X-axis.

$\angle OPN = \phi$, $\angle PON = \theta$, $\angle ONP = 90^\circ$, $ON = p$, $OP = r$.

from the right angle triangle $\angle NOP$,

$$\sin\phi = \frac{opp}{hyp}$$

$$\sin\phi = \frac{ON}{OP}$$

$$\sin\phi = \frac{p}{r}$$

$$\boxed{p = r\sin\phi} \longrightarrow ①$$

inverse and square eq ① on both side.

$$\frac{1}{p^2} = \frac{1}{r^2}\frac{1}{\sin^2\phi}$$

w.k.t, $\dfrac{1}{\sin^2\phi} = \operatorname{Cosec}^2\phi$

$$\frac{1}{p^2} = \frac{1}{r^2}\operatorname{Cosec}^2\phi$$

$$\frac{1}{p^2} = \frac{1}{r^2}\left[1 + \operatorname{Cot}^2\phi\right]$$

also w.k.t, $\operatorname{Cot}\phi = \dfrac{1}{r}\dfrac{dr}{d\theta}$.

$$\frac{1}{p^2} = \frac{1}{r^2}\left[1 + \frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2\right]$$

$$\boxed{\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2}$$

---

2. Given,

$$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$ $\quad -3-6 \quad -1+2$

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -5 & -9 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -5 & -9 & 1 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_3 \rightarrow \frac{R_3}{-2}$

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -5 & -9 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -5 & -9 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to 5R_3 + R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -5 & -9 & 1 \\ 0 & 0 & -4 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to \frac{R_2}{-5}$; $R_3 \to \frac{R_3}{-4}$

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 9/5 & -1/5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\therefore \ \rho(A) = 3.}$

---

3. Given, $20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25.$

$$x = \frac{1}{20}\left[17 - y + 2z\right]$$

$$y = \frac{1}{20}\left[-18 - 3x + z\right]$$

$$z = \frac{1}{20}\left[25 - 2x + 3y\right]$$

I Iteration :

$x = 0$; $y = 0$; $z = 0$

$$x^{(1)} = \frac{1}{20}[17 - 0 - 2(0)]$$

$\rightarrow x^{(1)} = 0.85$

$$y^{(1)} = \frac{1}{20}[-18 - 3(0.85) + 0]$$

$\rightarrow y^{(1)} = -1.0275$

$$z^{(1)} = \frac{1}{20}[25 - 2(0.85) + 3(-1.0275)]$$

$\rightarrow z^{(1)} = 1.0109$

II iteration :

$x = 0.85$ $y = -1.0275$ $z = 1.0109$

$$x^{(2)} = \frac{1}{20}[17 - (-1.0275) + 2(1.0109)]$$

$\rightarrow x^{(2)} = 1.0025$

$$y^{(2)} = \frac{1}{20}[-18 - 3(1.0025) + 1.0109)]$$

$\rightarrow y^{(2)} = -0.9998$

$$z^{(2)} = \frac{1}{20}[25 - 2(1.0025) + 3(-0.9998)]$$

$\rightarrow z^{(2)} = 0.9998$

III iteration :

$x = 1.0025$ $y = -0.9998$ $z = 0.9998$

$$x^{(3)} = \frac{1}{20}[17 - (-0.9998) + 2(0.9998)]$$

$\rightarrow x^{(3)} = 1.0000$

$$y^{(3)} = \frac{1}{20}[-18 - 3(1) + 0.9998]$$

$\rightarrow y^{(3)} = -1.0000$

$z^{(3)} = \frac{1}{20}[25 - 2(1) + 3(-1)]$

$\rightarrow z^{(3)} = 1.000$.

$\therefore \boxed{x = 1}$, $\boxed{y = -1}$, $\boxed{z = 1}$.

4. Given, $x + 2y + 3z = 6$

$x + 3y + 5z = 9$

$2x + 5y + az = b$.

$$\therefore [A ; B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 1 & 3 & 5 & 9 \\ 2 & 5 & a & b \end{array}\right]$$

$R_2 \rightarrow R_2 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 2 & 5 & a & b \end{array}\right]$$

$R_3 \rightarrow R_3 - 2R_1$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & a-6 & b-12 \end{array}\right]$$

$R_3 \rightarrow R_3 - R_2$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & a-8 & b-15 \end{array}\right]$$

i) infinitely many solutions.

$a = 8$ and $b = 15$.

ii) no solution

~~$a = 6$ and $b \neq 12$~~

$a = 8$ and $b \neq 15$.

iii) unique solutions.

$a \neq 8$ and $b \neq 15$

5. Given, $\frac{2a}{r} = (1 + \cos\theta)$

$$r = \frac{2a}{1+\cos\theta}$$

apply log on both sides.

$$\log r = \log\left(\frac{2a}{1+\cos\theta}\right)$$

$$\log r = \log 2a - \log(1+\cos\theta).$$

$$\frac{1}{r}\frac{dr}{d\theta} = 0 - \frac{1}{1+\cos\theta}(-\sin\theta).$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{\sin\theta}{1+\cos\theta}$$

$$\cot\phi = \frac{\cancel{2}\sin\theta/2\,\cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$$

$$\cot\phi = \tan\theta/2$$

$$\cot\phi = \cot\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$\boxed{\phi = \frac{\pi}{2} - \frac{\theta}{2}}$$

CMR

w.k.t. $p = r \sin\phi$

$$p = r \sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$p = r \cos\frac{\theta}{2} \rightarrow (1)$$

from the given equation.

$$\frac{2a}{r} = 1 + \cos\theta$$

$$\frac{\cancel{2}a}{r} = \cancel{2}\cos^2\theta/2$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{a}{r}}$$

Sub in (1)

$$p = r\sqrt{\frac{a}{r}}$$

Square on both sides.

$$p^2 = r^2\left(\frac{a}{r}\right)$$

$$\boxed{p^2 = ar} \text{ (or) } \boxed{r = \frac{p^2}{a}}$$

6. Given, $r = a(1+\cos\theta)$ and $r = a(1-\cos\theta)$.

- $r = a(1+\cos\theta)$

  apply log on both side

$$\log r = \log a + \log(1+\cos\theta)$$

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta}(-\sin\theta)$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{-\sin\theta}{1+\cos\theta}.$$

$$\cot \phi_1 = \frac{-\cancel{2}\sin \theta/2 \cos \theta/2}{\cancel{2}\cos^2 \theta/2}$$

$$\cot \phi_1 = -\tan \theta/2$$

$$\cot \phi_1 = \cot\left(\frac{\pi}{2} + \frac{\theta}{2}\right)$$

$$\phi_1 = \frac{\pi}{2} + \frac{\theta}{2}.$$

- $r = a(1 - \cos\theta)$

apply log on both side.

$$\log r = \log a + \log(1 - \cos\theta)$$

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{\sin\theta}{1 - \cos\theta}$$

$$\cot \phi_2 = \frac{\cancel{2}\sin \theta/2 \cos \theta/2}{\cancel{2}\sin^2 \theta/2}$$

$$\cot \phi_2 = \cot \theta/2.$$

$$\phi_2 = \theta/2.$$

the angle equals:

$$|\phi_1 - \phi_2| = \left|\frac{\pi}{2} + \frac{\theta}{2} - \frac{\theta}{2}\right|$$

$$= \left|\frac{\pi}{2}\right|$$

$$\boxed{|\phi_1 - \phi_2| = \frac{\pi}{2}}$$

7. Given $x^4 + y^4 = 1$

differentiate w.r.t to x.

$$4x^3 + 4y^3 y_1 = 0$$

$$4y^3 y_1 = -4x^3$$

$$y_1 = -\frac{x^3}{y^3}$$

again diff/: w.r.t to x.

$$y_2 = -\left[\frac{y^3 3x^2 - x^3 3y^2 y_1}{y^6}\right]$$

$$y_2 = \frac{3x^3y^2y_1 - 3y^3x^2}{y^6}$$

Let $y_1 = -\frac{x^3}{y^3}$ at points $(1, 1)$

$$y_1 = -\frac{1^3}{1^3}$$

$$y_1 = -1.$$

$$y_2 = \frac{3x^3y^2y_1 - 3y^3x^2}{y^6} \text{ at } (1, 1).$$

$$y_2 = \frac{3(1)^3(1)^2(-1) - 3(1)^3(1)^2}{1^6}$$

$$y_2 = -3 - 3$$

$$y_2 = -6.$$

w.r.t radius of curvature.

$$\rho = \frac{(1 + y_1^2)^{3/2}}{y_2}$$

$$\rho = \frac{(1+(-1)^2)^{3/2}}{-6}$$

$$\rho = \frac{(1+1)^{3/2}}{-6}$$

$$\rho = \frac{-\cancel{2}\sqrt{2}}{-\cancel{2}\times 3}$$

$$\boxed{\rho = -\frac{\sqrt{2}}{3}}$$

8. Given, $y(x) = \tan x \rightarrow (a)$.

w.k.t

$$y(x) = y(0) + x y_1(0) + \frac{x^2}{2!} y_2(0) + \frac{x^3}{3!} y_3(0) \rightarrow (1)$$

from eq (a)

$y(x) = \tan x \longrightarrow y(0) = \tan 0 = 0$

$y_1(x) = \sec^2 x \longrightarrow y_1(0) = \sec^2(0) = 1$

$y_2(x) = 2\sec x \sec x \tan x$

$= 2\sec^2 x \tan x \longrightarrow y_2(0) = 2\sec^2(0)\tan(0)$

$= 0$

$y_3(x) = 2[\sec^2 x \sec^2 x + \tan x \cdot 2\sec x \sec x \tan x]$

$= 2[\sec^4 x + 2 \cdot \sec^2 x \tan^2 x]$

$= 2[\sec^4(0) + 2(\sec^2(0))\tan^2(0)]$

$= 2[1 + 2(1)(0)]$

2

eq ① becomes :

$$y(x) = 0 + x(1) + \frac{x^2}{2!}(0) + \frac{x^3}{3!}(2).$$

$$\tan x = x + \frac{2x^3}{3!}$$

$$\tan x = x + \frac{\cancel{2}x^3}{3 \times \cancel{2}}$$

$$\boxed{\tan x = x + \frac{x^3}{3}.}$$