CMR INSTITUTE OF TECHNOLOGY	

Internal Assessment Test – 1 November 2024 Solutions

Sub:	Mathematics-I for Electrical & Electronics Engineering Stream							Code:	BMATE101
Date:	19/11/2024	Duration:	90 mins	Max Marks:	50	Sem:	I	Section:	**M – P**

Question 1 is compulsory and Answer any 6 from the remaining questions.

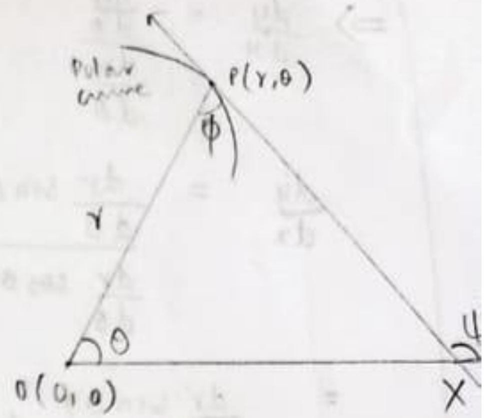
		Marks	OBE CO	OBE RBT
1.	With usual notations prove that $\tan\phi = r\frac{d\theta}{dr}$	[8]	CO1	L2
2.	Find the rank of the matrix $\begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & 2 & 4 & 3 \\ 3 & 6 & 12 & 9 \\ 3 & 3 & 3 & 6 \end{bmatrix}$	[7]	CO5	L3
3.	Solve the system of equations by Gauss elimination method $3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4$	[7]	CO5	L2
4.	Solve the system of equations using Gauss – Seidel method by taking $(0,0,0)$ as an initial approximate root $2x - 3y + 20z = 25, 20x + y - 2z = 17, 3x + 20y - z = -18$	[7]	CO5	L2

5.	Using Rayleigh's power method find the dominant eigenvalue and the corresponding eigenvector of by taking $[1 \quad 0 \quad 0]^T$ as the initial eigenvector. $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$	[7]	CO5	L2
6.	Find the angle between the curves $r^2 \sin 2\theta = 4$ and $r^2 = 16\sin 2\theta$	[7]	CO1	L2
7.	Find the pedal equation of the curve $r = a\left(e^{\theta\cot\alpha}\right)$	[7]	CO1	L3
8.	Expand $e^{\sin x}$ by Maclaurin's series up to the term containing x^4	[7]	CO2	L2

1. <u>To prove</u> : $\tan\phi = r\frac{d\theta}{dr}$

<u>Given</u> :

$P(r,\theta)$ is any point on the plane. $OP = r$ is <u>radius of vector</u> making angle with polar curve 'ϕ' and 'ψ' is angle between <u>tangent of the curve PX</u> with initial line $\overline{OX}$. 'θ' is the angle between radius vector and initial line OX.



We know that,

$$\boxed{\psi = \phi + \theta}$$

taking tan both sides

$$\tan\psi = \tan(\phi + \theta)$$

$$\boxed{\tan\psi = \frac{\tan\phi + \tan\theta}{1 - \tan\phi.\tan\theta}}$$

We also know that,

$$\boxed{m = \frac{dy}{dx} = \tan\psi} \quad \text{———— (1)}$$

$\Rightarrow$ $x = r\cos\theta$ $\Rightarrow$ $y = r\sin\theta$

diff wrt to θ

$$\frac{dx}{d\theta} = -\sin\theta\, r\frac{d\theta}{d\theta} + \frac{dr}{d\theta}\cos\theta \qquad \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + \cos\theta . r$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + \cos\theta . r}{\frac{dr}{d\theta}\cos\theta - \sin\theta . r}$$

$$= \frac{\frac{\frac{dr}{d\theta}\sin\theta . r}{\frac{dr}{d\theta}\cos\theta} + \frac{\cancel{\cos}\theta . r}{\frac{dr}{d\theta}\cancel{\cos}\theta}}{\frac{\frac{dr}{d\theta}\cos\theta}{\frac{dr}{d\theta}\cancel{\cos}\theta} - \frac{\sin\theta . r}{\frac{dr}{d\theta}\cos\theta}}$$

$$\frac{dy}{dx} = \frac{\tan\theta + r.\frac{d\theta}{dr}}{1 - \tan\theta r\frac{dr}{d\theta}} \quad \text{---} ②$$

comparing eqn ① and ②.

$$\tan\psi = \frac{\tan\theta + r.\frac{d\theta}{dr}}{1 - \tan\theta r.\frac{dr}{d\theta}} = \left[\frac{\tan\theta + \tan\phi}{1 - \tan\theta . \tan\phi}\right]$$

$$\therefore \boxed{\tan\phi = r\frac{d\theta}{dr}} \;//$$

Hence proved..

2. $A = \begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & 2 & 4 & 3 \\ 3 & 6 & 12 & 9 \\ 3 & 3 & 3 & 6 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 2 & 1 & -1 & 3 \\ 3 & 6 & 12 & 9 \\ 3 & 3 & 3 & 6 \end{bmatrix}$$

$R_2 \rightarrow 2R_1 - R_2$, $R_3 \rightarrow 3R_1 - R_3$, $R_4 \rightarrow 3R_1 - R_4$

$$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 3 & 9 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 9 & 3 \end{bmatrix}$$

$R_4 \rightarrow R_2 - R_4$

$$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 3 & 9 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

Rank

3. $3x + y + 2z = 3$
$2x - 3y - z = -3$
$x + 2y + z = 4$

$$\left[\begin{array}{ccc:c} 3 & 1 & 2 & 3 \\ 2 & -3 & -1 & -3 \\ 1 & 2 & 1 & 4 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 4 \\ 2 & -3 & -1 & -3 \\ 3 & 1 & 2 & 3 \end{array}\right]$$

$R_2 \rightarrow 2R_1 - R_2$, $R_3 \rightarrow 3R_1 - R_3$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 4 \\ 0 & 7 & 3 & 11 \\ 0 & 5 & 1 & 9 \end{array}\right]$$

$R_3 \rightarrow 5R_2 - 7R_3$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 4 \\ 0 & 7 & 3 & 11 \\ 0 & 0 & 8 & -8 \end{array}\right]$$

$x + 2y + z = 4$ $\quad x + 4 - 1 = 4$

$7y + 3z = 11$ $\quad 7y - 3 = 11$

$\quad y = 2$

$8z = -8$

$\therefore \boxed{z = -1}$, $\boxed{y = 2}$, $\boxed{x = 1}$

4. $$\begin{aligned} 2x - 3y + 20z &= 25 \\ 20x + y - 2z &= 17 \\ 3x + 20y - z &= -18 \end{aligned} \longrightarrow \begin{aligned} \overset{a_1}{20}x + \overset{a_2}{}y - \overset{a_3}{2}z &= \overset{d_1}{17} \\ \overset{b_1}{3}x + \overset{b_2}{20}y \overset{b_3}{-}z &= \overset{d_2}{-18} \\ \overset{c_1}{2}x - \overset{c_2}{3}y + \overset{c_3}{20}z &= \overset{d_3}{25} \end{aligned}$$

$$x_1 = \frac{1}{a_1}(d_1 - a_2 y - a_3 z)$$

$$y_1 = \frac{1}{b_2}(d_2 - b_1 x - b_3 z)$$

$$z_1 = \frac{1}{c_3}(d_3 - c_1 x - c_2 y)$$

$$\Rightarrow \quad x_1 = \frac{1}{20}(17 - y + 2z) = \frac{1}{20}(17 - 0 + 0) = \frac{17}{20} = 0.85$$

$$y_1 = \frac{1}{20}(-18 - 3x + z) = \frac{1}{20}(-18 - \overset{2.55}{3(0.85)} + 0) = -1.0275$$

$$z_1 = \frac{1}{20}(25 - 2x + 3y) = \frac{1}{20}(25 - \overset{-1.7}{2(0.85)} + \overset{-3.0825}{3(-1.0275)}) = 1.0$$

taking $(0,0,0)$ as initial approx.

$$\Rightarrow \quad x_2 = \frac{1}{20}(17 + 1.0275 + 2(1.01)) = 1.002$$

$$y_2 = \frac{1}{20}(-18 - 3(1.002) + 1.01) = -0.999$$

$$z_2 = \frac{1}{20}(25 - 2(1.002) + \overset{2.997}{3(-0.999)}) = 0.999$$

$$x_2 = 1.002 \approx 1$$
$$y_2 = -0.999 \approx -1$$
$$z_2 = 0.999 \approx 1$$

$$\therefore \boxed{\begin{aligned} x &= 1 \\ y &= -1 \\ z &= 1 \end{aligned}}$$

5. $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ $\quad x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$AX = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix} = \underset{\lambda_1}{4} \begin{bmatrix} 1 \\ 0.5 \\ -0.5 \end{bmatrix} \rightarrow X_1$$

$$AX_1 = \begin{bmatrix} 4+0.5+0.5 \\ 2+1.5+0.5 \\ -2+0.5-2.5 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ -4 \end{bmatrix} = \underset{\lambda_2}{5} \begin{bmatrix} 1 \\ 0.8 \\ -0.8 \end{bmatrix} \rightarrow X_2$$

$$AX_2 = \begin{bmatrix} 4+0.8+0.8 \\ 2+2.4+0.8 \\ -2+0.8-4 \end{bmatrix} = \begin{bmatrix} 5.6 \\ 5.2 \\ -5.2 \end{bmatrix} = \underset{\lambda_3}{5.6} \begin{bmatrix} 1 \\ 0.92 \\ -0.92 \end{bmatrix} \rightarrow X_3$$

$$AX_3 = \begin{bmatrix} 4+0.92+0.92 \\ 2+2.76+0.92 \\ -2+0.92-4.6 \end{bmatrix} = \begin{bmatrix} 5.84 \\ 5.68 \\ -5.68 \end{bmatrix} = \underset{\lambda_4}{5.84} \begin{bmatrix} 1 \\ 0.97 \\ -0.97 \end{bmatrix} \rightarrow X_4$$

$$AX_4 \begin{bmatrix} 4+0.97+0.97 \\ 2+2.91+0.97 \\ -2+0.97-4.85 \end{bmatrix} = \begin{bmatrix} 5.94 \\ 5.88 \\ -5.88 \end{bmatrix} = \underset{\lambda_5}{5.94} \begin{bmatrix} 1 \\ 0.98 \\ -0.98 \end{bmatrix} \rightarrow X_5$$

$$AX_5 \begin{bmatrix} 4+0.98+0.98 \\ 2+2.94+0.98 \\ -2+0.98-4.9 \end{bmatrix} = \begin{bmatrix} 5.96 \\ 5.92 \\ -5.92 \end{bmatrix} = \underset{\lambda_6}{5.96} \begin{bmatrix} 1 \\ 0.99 \\ -0.99 \end{bmatrix} \rightarrow X_6$$

$$AX_6 = \begin{bmatrix} 4+0.99+0.99 \\ 2+2.97+0.99 \\ -2+0.99-4.95 \end{bmatrix} = \begin{bmatrix} 5.98 \\ 5.96 \\ -5.96 \end{bmatrix} = \underset{\lambda_7}{5.98} \begin{bmatrix} 1 \\ 0.99 \\ -0.99 \end{bmatrix} \rightarrow X_7$$

using rayleigh's power method,

dominant eigen value(λ) = $\underline{\underline{5.98}}$

eigen vector(X) = $\begin{bmatrix} 1 \\ 0.99 \\ -0.99 \end{bmatrix}$ //.

6. Angle between curves;

$r^2 \sin 2\theta = 4$ —①

$2r\frac{dr}{d\theta}\cdot\sin 2\theta + 2\cos 2\theta \cdot r^2 = 0$

$\cancel{2}\cancel{r}\frac{dr}{d\theta}\cdot\sin 2\theta = -\cancel{2}\cos 2\theta \cdot r^{\cancel{2}}$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{-\cos 2\theta}{\sin 2\theta}$

$\underbrace{\frac{1}{r}\frac{dr}{d\theta}}_{\downarrow} = -\cot 2\theta$

$\cot\phi = -\cot 2\theta$

$\phi_1 = -2\theta$

$r^2 = 16\sin 2\theta$ —②

$\cancel{2}r\frac{dr}{d\theta} = \frac{3\cancel{2}\cos 2\theta}{16}$

$\frac{r}{r^2}\frac{dr}{d\theta} = \frac{16\cos 2\theta}{r^2}$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{\cancel{16}\cos 2\theta}{\cancel{16}\sin 2\theta}$ $(r^2 = 16\sin 2\theta)$

$\underbrace{\frac{1}{r}\frac{dr}{d\theta}}_{\downarrow} = \cot 2\theta$ $\left(\frac{1}{r}\frac{dr}{d\theta} = \right.$

$\cot\phi = \cot 2\theta$

$\phi_2 = 2\theta$

Angle between both curves = $|\phi_1 - \phi_2|$

$= |-2\theta - 2\theta| = |-4\theta| = \underline{\underline{4\theta}}$

equating eq^n ① and ②

$r^2 = \frac{4}{\sin 2\theta}$, $r^2 = 16 \sin 2\theta$

$$\frac{4}{\sin 2\theta} = 16 \sin 2\theta$$

$$\frac{\cancel{4}}{16}\ \frac{1}{4} = \sin^2 2\theta$$

$$\sqrt{\frac{1}{4}} = \sqrt{\sin^2 2\theta}$$

$$\frac{1}{2} = \sin 2\theta$$

$$\therefore \sin \frac{\pi}{6} = \frac{1}{2}$$

$$\frac{\pi}{6} = 2\theta$$

$$\boxed{\theta = \frac{\pi}{12}} //.$$

$\therefore$ Angle between both curves

$$= 4\theta = \cancel{4} \times \frac{\pi}{\cancel{12}\ 3} = \frac{\pi}{3} //.$$

7) $r = a\left(e^{\theta \cot \alpha}\right)$

$r = a e^{\theta \cot \alpha}$ ——— (1)

diff w. r to 'θ'

$$\frac{dr}{d\theta} = a \cdot e^{\theta \cot \alpha} \cdot \cot \alpha$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{(a \cot \alpha)\, e^{\theta \cot \alpha}}{r}$$

$$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{(\cancel{a} \cot \alpha)\, \cancel{e^{\theta \cot \alpha}}}{\cancel{a} \cancel{e^{\theta \cot \alpha}}}$$

$$\Rightarrow \cot \phi = \cot \alpha$$

$$\Rightarrow \phi = \alpha$$

WKT,

$$p = r \sin \phi$$

$$\boxed{p = r \sin \alpha}$$

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8. Maclaurin's series

$$\Rightarrow f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{iv}(0) \quad \text{—} \ (1)$$

$$f(x) = e^{\sin x}$$

$$f(0) = e^{\sin 0} = e^0 = 1$$

$$f'(x) = e^{\sin x} \cdot \cos x \qquad f'(0) = e^{\sin 0} \cdot \cos 0 = 1$$

$$f''(x) = e^{\sin x} \cdot \cos^2 x + -\sin x \cdot e^{\sin x} \qquad f''(0) = e^{\sin 0} \cdot \cos^2 0 - \sin 0 \cdot e^{\sin 0}$$

$$= 1 - 0 = 1$$

$$f'''(x) = e^{\sin x} \cdot \cos x \cdot \cos^2 x + 2\cos x(-\sin x) \cdot e^{\sin x} - (\cos x \cdot e^{\sin x} + e^{\sin x} \cdot \cos x \cdot \sin x)$$

$$= e^{\sin x} \cdot \cos x \cdot \cos^2 x - 2\sin x \cos x \cdot e^{\sin x} - e^{\sin x} \cdot \cos x - e^{\sin x} \cos x \cdot \sin x$$

$$f'''(0) = e^{\sin 0} \cdot \cos 0 \cdot \cos^2 0 - \cancel{2\sin 0} - e^{\sin 0} \cos 0 - \cancel{e^{\sin 0} \cos 0 \sin 0}$$

$$= 1 - 1 = 0$$

$$f'''(x) = e^{\sin x} \cos^3 x - \sin 2x \cdot e^{\sin x} - e^{\sin x} \cos x - \frac{e^{\sin x} \sin 2x}{2}$$

$$f^{iv}(x) = e^{\sin x} \cdot \cos^4 x + 3\cos^2 x(-\sin x)(e^{\sin x}) - (2\cos 2x \cdot e^{\sin x} + e^{\sin x} \cos x \cdot \sin 2x) - (e^{\sin x} \cdot \cos^2 x + \sin x \cdot e^{\sin x}) - \frac{(e^{\sin x} \cos x \cdot \sin 2x + 2\cos 2x)}{2}$$

$$f^{iv}(0) = e^{\sin 0} \cos^4 0 + 0 - (2\cos 0 \cdot e^{\sin 0} + 0) - (e^{\sin 0} \cdot \cos^2 0 - 0) - \frac{(0 + 2\cos 0)}{2}$$

$$= 1 + 0 - 2 - 1 - 1 = -3$$

Substituting in eqn (1)

$$e^{\sin x} = \overset{f(0)}{1} + \frac{x}{1}\overset{f'(0)}{(1)} + \frac{x^2}{2}\overset{f''(0)}{(1)} + \frac{x^3}{6}\overset{f'''(0)}{(0)} + \frac{x^4}{24_{8}}\overset{f^{iv}(0)}{(-3)}$$

$$e^{\sin x} = 1 + x + \frac{x^2}{2} + 0 - \frac{x^4}{8}$$

$$e^{\sin x} = 1 + x + \frac{x^2}{2} - \frac{x^4}{8}$$