SOLUTION TO VTU PHYSICS QUESTION PAPER ECE STREAM 2024-25 ODD SEM

1A

Debroglie's theory:

<u>Statement</u>: By the law of symmetry of nature, a particle exhibits wave properties in addition to its particle properties.

The wavelength of the group of waves associated with particle of mass m moving with a velocity v is given by the expression

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \frac{h}{\sqrt{2mE}}$$

where his the Planck's constant

<u>Time independent Schrödinger equation</u>

A matter wave can be represented in complex form as

$$\Psi = A \sin k\, x(\cos w\, t + i \sin w\, t)$$

$$\Psi = A \sin k\, x e^{iwt}$$

Differentiating wrt x

$$\frac{d\Psi}{dx} = kA \cos k\, x e^{iwt}$$

$$\frac{d^2\Psi}{dx^2} = -k^2 A \sin k\, x e^{iwt} = -k^2\Psi \quad \text{……………………..}$$

(1)

From debroglie's relation

$$\frac{1}{\lambda} = \frac{h}{mv} = \frac{h}{p}$$

$$\text{k} = \frac{2\pi}{\lambda} = \frac{2\Pi p}{h}$$

$k^2 = 4\Pi^2 \frac{p^2}{h^2}$……………………… (2)

Total energy of a particle E = Kinetic energy + Potential Energy

$$E = \frac{1}{2}m\, v^2 + V$$

$$E = \frac{p^2}{2m} + V$$

$$p^2 = (E - V)2m$$

Substituting in (2)

$$k^2 = \frac{4\Pi^2(E - V)2m}{h^2}$$

∴From (1)

$$\boxed{\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 m(E - V)\Psi}{h^2} = 0}$$

For three dimensions

$$\frac{d^2\Psi}{dx^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\frac{8\Pi^2 m(E - V)\Psi}{h^2} = 00$$

1B

HEISENBERG'S UNCERTAINTY PRINCIPLE:

The position and momentum of a particle cannot be determined accurately and simultaneously.The product of uncertainty in the measurement of position (Δx) and momentum (Δp) is always greater than or equal to$\frac{h}{2\Pi}$.

$$\boxed{(\Delta x).(\Delta p) \geq \frac{h}{4\Pi}}$$

This uncertainty is not due to discrepancy with the apparatus or with the method of measurement, but because of the very wave nature of the object. This uncertainty persists as long as matter possesses wave nature.

Different forms of Heisenberg’s Principle:

$$(\mathit{\Delta}x).(\mathit{\Delta}p) \geq \frac{h}{4\mathit{\Pi}}$$

$$(\mathit{\Delta}L).(\mathit{\Delta}\theta) \geq \frac{h}{4\pi}$$

$$(\mathit{\Delta}E).(\mathit{\Delta}t) \geq \frac{h}{4\pi}$$

Here ΔL is the uncertainty in angular momentum

Δθ is the uncertainty in the measurement of angular displacement

ΔE is the uncertainty in the measurement of energy

Δt is the uncertainty in the measurement of time interval during which the particle exists in the state E

Physical Significance:

1.It introduces the concept of probability.

2.It can be used to find life time of electrons in an excited state.

3.It can be used to show that electrons do not exist inside the nucleus.

TO SHOW THAT ELECTRON DOES NOT EXIST INSIDE THE NUCLEUS:

We know that the diameter of the nucleus is of the order of 10^{-14}m.If the electron is to exist inside the nucleus, then the uncertainty in its position Δx cannot exceed the size of the nucleus

$$\mathit{\Delta}x = 5x10^{-15}m$$

Now the uncertainty in momentum is

$$\mathit{\Delta}x = 5x10^{-15}m$$

$$\Delta P = \frac{h}{4\pi x \Delta x} = 0.1x10^{-19}kg.\frac{m}{s}$$

Then the momentum of the electron can atleast be equal to the uncertainty in momentum.

$P \approx \Delta P = 0.1x10^{-19}kg.\frac{m}{s}$

Now the energy of the electron with this momentum supposed to be present in the nucleus is given by (for small velocities -non-relativistic-case)

$$E = \frac{p^2}{2m} = 5.5x10^{-11}J = 343MeV$$

The beta decay experiments have shown that the kinetic energy of the beta particles (electrons) is only a fraction of this energy. This indicates that electrons do not exist within the nucleus. They are produced at the instant of decay of nucleus ($n \rightarrow p + e + \bar{\nu}$ / $p \rightarrow n + \overset{+}{e} + \nu$).

1C

$$E = \frac{n^2h^2}{8mL^2}$$
$$L = 10\ nm$$
$$m_e = 9.1x10^{-31}kg$$

n = 1

$$E_1 = 6.038x10^{-23}\mathrm{J}$$

2A

MATTER WAVES:

1. Matter waves represent the probability density variation in a region.

2. A matter wave in complex form is written as $\Psi = A\, sin\, k\, x(cos\, w\, t + i\, sin\, w\, t)$. It is obtained as general solution to Schrodinger’s equation.

3. Matter waves are neither transverse nor longitudinal and their velocity is equal to that of Particle.

4. They propagate as group of waves.

5. Group velocity of matter waves is equal to particle velocity

Group Velocity(V_g): It is the velocity with which the resultant envelope formed by the superposition of two or more waves propagates .

$$V_g = \frac{\Delta\omega}{\Delta k} = \underset{dk\to 0}{Lt}\frac{d\omega}{dk}$$

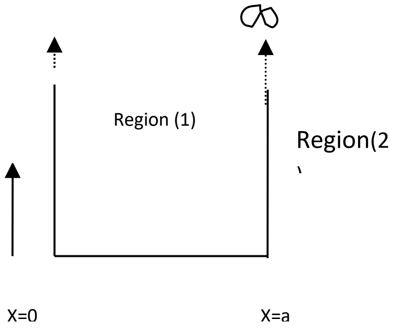
Wave function: In general, it represents matter wave for a particle. It is obtained as solution to Schrodinger equation representing a quantum particle in a specific state. Ex: For a particle in an infinite potential well, the wave function is

$$\Psi_n = \sqrt{\frac{2}{a}}\, sin\left(n\frac{\Pi}{a}\right)x$$

2B

Particle in an infinite potential well problem:

Consider a particle of mass m moving along X-axis in the region from X=0 to X=a in a one dimensional potential well as shown in the diagram. The potential energy is assumed to be zero inside the region and infinite outside the region.



Applying, Schrodingers equation for region (1) as particle is supposed to be present in region (1)

$$\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 mE\psi}{h^2} = 0 \because V = 0$$

But $k^2 = \frac{8\Pi^2 mE}{h^2}$

$$\therefore \frac{d^2\Psi}{dx^2} + k^2\Psi = 0$$

Auxiliary equation is $(D^2 + k^2)x = 0$

Roots are D = +ik and D = -ik

The general solution is

$$\begin{aligned} x &= Ae^{ikx} + Be^{-ikx} \\ &= A(\cos k\,x + i\sin k\,x) + B(\cos k\,x - i\sin k\,x) \\ &= (A+B)\cos k\,x + i(A-B)\sin k\,x \\ &= C\cos k\,x + D\sin k\,x \end{aligned}$$

The boundary conditions are

1. At x=0, $\Psi = 0 \therefore C = 0$

2. At x=a, $\Psi = 0$

D sin ka = 0 ⇒ka = nΠ ………(2)

where n = 1, 2 3…

$$\therefore \Psi = D\sin\left(n\frac{\Pi}{a}\right)x$$

From (1) and (2) Eigen value E = $\frac{n^2h^2}{8ma^2}$

To evaluate the constant D:

Normalisation: For one dimension

$$\int_0^a \Psi^2\,dx = 1$$

$$\int_0^a D^2\sin^2\left(\frac{n\Pi}{a}\right)x dx = 1$$

But $\cos 2\theta = 1 - 2\sin^2\theta$

$$\int_0^a D^2 \frac{1}{2}\left(1 - \cos 2\left(\frac{n\Pi}{a}\right)x\right)dx = 1$$

$$\int_0^a \frac{D^2}{2} dx - \int_0^a \frac{1}{2} \cos 2\,(\frac{n\Pi}{a})x)dx = 1$$

$$\frac{D^2 a}{2} - [\sin 2\left(\frac{n\Pi}{a}\right)\frac{x}{2}]_0^a = 1$$

$$D^2 \frac{a}{2} - 0 = 1$$

$$\mathrm{D} = \sqrt{\frac{2}{a}}$$

Eigen function $\therefore \Psi_n = \sqrt{\frac{2}{a}} \sin\left(n\frac{\Pi}{a}\right)x$

2C

.

$$\Delta x \times \Delta p = \frac{h}{4\pi}$$

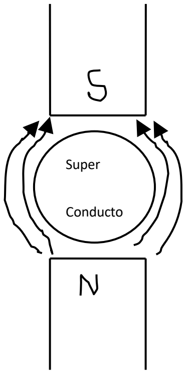
$$\Delta p = m.\Delta v$$

Minimum uncertainty in velocity = $\Delta v = \frac{h}{4\pi\Delta x.m} = 7.2x10^3 m/s$

3A

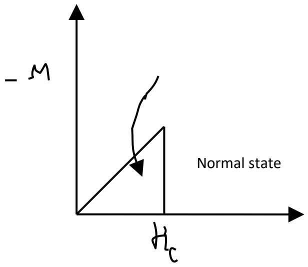
When a superconductor is placed in a magnetic field, it expels the magnetic flux out of its body and behaves like a diamagnet.This effect is known as **Meisseners effect.**

Using the experiments on superconducting cylinders in presence of small magnetic fields it was demonstrated that as temperature is lowered to Tc, the magnetic flux inside the superconductor is suddenly and completely expelled as the specimen becomes superconductor as shown in the diagram. This effect is reversible.



Types of superconductors

Type 1 Superconductors: Soft Superconductors



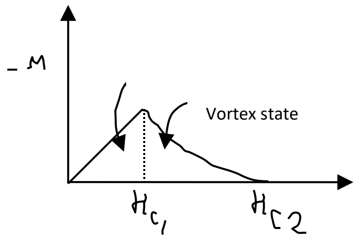
These are pure superconductors.

When kept in magnetic field, initially they continue to exhibit superconductivity and the negative magnetic moment increases. At critical magnetic field there is a sharp transition to normal state due to the penetration of magnetic flux lines. The transition is sharp.

These possess low critical magnetic fields. Their critical temperatures also low.They are generally pure metals.

Ex: Al, Pb

Type 2 superconductor:Hard Superconductors



These are generally alloys.

When kept in magnetic field, initially they continue to exhibit superconductivity and the negative magnetic moment increases. At lower critical magnetic field H_{C1}, the flux lines start penetrating .As the magnetic field is increased, the super conductivity coexists with magnetic field and this phase is known as mixed state(vortex state). At higher critical magnetic field H_{C2}, the penetration is complete and the material transforms to normal state. They possess higher critical magnetic fields. Their critical temperatures are high.

Ex: Nb_3Ge, $YBa_2Cu_3O_7$

3B

Polar dielectrics: The atoms of these materials are permanently polarized in nature and possess dipole moment. They show orientation polarization.

Ex: Water, Kcl, NH_3

Different polarization mechanisms:

There are 4 mechanisms.

1. Electronic polarization
2. Ionic polarization
3. Orientation polarization
4. Space charge polarization

Electronic polarization: These are generally seen in the case of covalent compounds.

When a covalent compound is placed in electric field, displacement of electron cloud takes place relative to the nucleus. This displacement creates a dipole which develops dipole moment.

Electronic polarisability $\alpha_e = \frac{\varepsilon_o(\varepsilon_r - 1)}{N}$

N is number of dipoles per unit volume

It is independent of temperature.

Ionic polarization:

This is exhibited by ionic compounds.

When ionic compounds are kept in an electric field, displacement of positive and negative ions occurs developing a dipole moment.

Ionic polarisability $\alpha_i = \frac{\varepsilon_o(\varepsilon_r - 1)}{N_i} \approx 0.1\alpha_e$

Orientation polarization:

Polar molecules exhibit this mechanism.

When polar molecules are kept in an electric field, already existing dipoles tend to align in the direction of applied electric field .This increases the dipole moment.

Orientation polarization $\alpha_o = \frac{\mu^2}{kT}$

Space charge polarization:

This polarization exists in materials possessing different phases due to difference in temperatures. In such materials charge carriers drift and accommodate in certain regions of higher conductivity (electrodes) causing dipole moment. It occurs in ferrites and semiconductors.Its magnitude is very small compared to other mechanisms.

Ex: Depletion regions in semiconductors, discharge tube,Vaccuum tubes

3C

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$
$$E - E_F = -0.2eV = -0.2x1.6x10^{-19}J$$
$$f(E) = \frac{1}{e^{\frac{0.2x1.6x10^{-19}}{1.38x10^{-23}x300}} + 1} = 0.99$$

4A

Fermi probability factor: It represents the probability of occupation of an energy level.

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)}+1}$$

To show that energy levels below Fermi energy are completely occupied:

For E **<** E_F, at T = 0,

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)}+1} = 1$$

To show that energy levels above Fermi energy are empty:

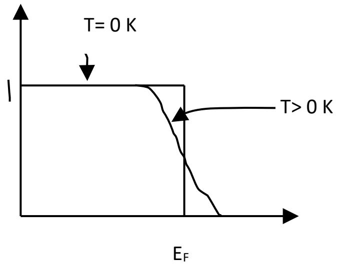
For E >E_F, at T=0

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)}+1} = 0$$

At ordinary temperatures, for E = E_F,

$$f(E) = \frac{1}{2}$$

Fermi energy for T > 0k, $E_f = E_{f0} \left[1 - \frac{\Pi^2}{12}\left(\frac{kT}{E_F}\right)^2\right]$



4B

INTERNAL FIELDS IN A DIELECTRIC:

It is the resultant of the applied field and the field produced due to all the dipoles.

CLAUSIUS – MOSOTTI RELATION:

This expression relates dielectric constant of an insulator (ε) to the polarization of individual atoms(α) comprising it.

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$$

where N is the number of atoms per unit volume

α is the polrisability of the atom

ε_r is the relative permittivity of the medium

ε_o is the permittivity of free space.

Proof:

If there are N atoms per unit volume,the electric dipole moment per unit volume –known as polarization is given by

$$P = N\alpha E_i$$

$$E_i = \frac{P}{N\alpha}$$

By the definition of polarization P, it can be shown that

$$P = \varepsilon_0 E_a(\varepsilon_r - 1)$$

$$E_a = \frac{P}{\varepsilon_o(\varepsilon_r - 1)} \text{……………………(1)}$$

The internal field at an atom in a cubic structure (γ =1/3) is of the form

$$E_i = E_a + \frac{\gamma p}{\varepsilon_0}$$

$$\frac{P}{N\alpha} = \frac{P}{\varepsilon_o(\varepsilon_r - 1)} + \frac{P}{3\varepsilon_o}$$

$$\frac{1}{N\alpha} = \frac{1}{\varepsilon_o(\varepsilon_r - 1)} + \frac{1}{3\varepsilon_o}$$

$$\frac{1}{N\alpha} = \frac{3 + \varepsilon_r - 1}{3\varepsilon_o(\varepsilon_r - 1)}$$

$$\frac{N\alpha}{3\varepsilon_o} = \frac{1}{\left[\frac{\varepsilon_r + 2}{\varepsilon_r - 1}\right]} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2}$$

Simplify to get

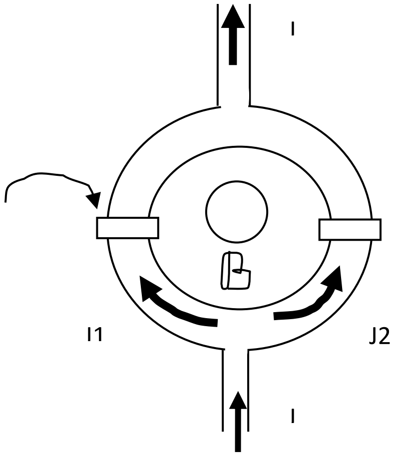
$$\left[\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right] = \frac{N\alpha}{3\varepsilon_0}$$

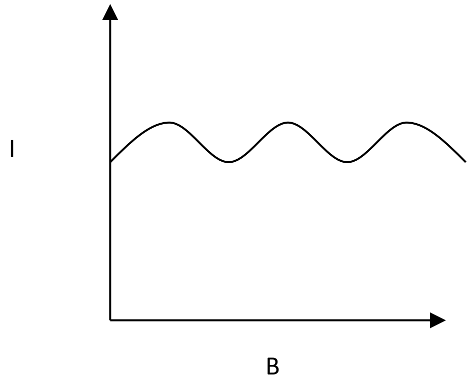
4C

DC SQUID:

Principle: DC Josephsons junction

Imagine a pair of Josephsons junctions connected in a network . When magnetic field is applied, current is induced in the SQUID which opposes the external magnetic field. Magnetic Flux through the SQUID is quantized .The current through the SQUID varies periodically with the external magnetic field.





$$\varphi = \frac{nhc}{2e}$$

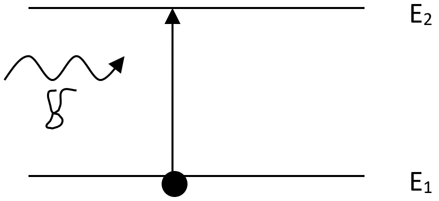
5A

INTERCTION OF RADIATION WITH MATTER

Induced absorption:

It is a process in which an atom at a lower level absorbs a photon to get excited to the higher level.

Let E_1 and E_2 be the energy levels in an atom and N_1 and N_2 be the number density in these levels respectively. Let U_γ be the energy density of the radiation incident.



$$\gamma = \frac{E_2 - E_1}{h}$$

Rate of absorption is proportional to the number of atoms in lower state and also on the energy density $U_{\gamma.}$

$$\text{Rate of absorption} = B_{12}\ N_1 U_\gamma$$

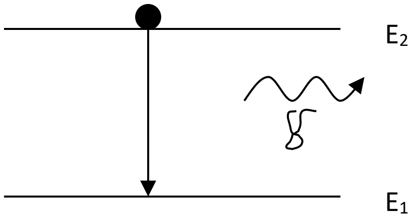
Here B_{12} is a constant known as Einsteins coefficient of spontaneous absorption.

Spontaneous emission:

It is a process in which ,atoms at the higher level voluntarily get excited emitting a photon. The rate of spontaneous emission representing the number of such deexcitations is proportional to number of atoms in the excited state.

$$\text{Rate of spontaneous emission} = A_{21}\ N_2$$

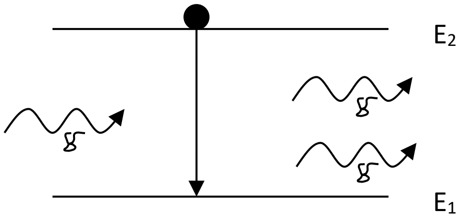
Here B_{12} is a constant known as Einsteins coefficient of spontaneous emission.



$$\gamma = \frac{E_2 - E_1}{h}$$

Stimulated emission:

In this process, an atom at the excited state gets deexcited in the presence of a photon of same energy as that of difference between the two states.



$$\gamma = \frac{E_2 - E_1}{h}$$

The number of stimulated emissions is proportional to the number of atoms in higher state and also on the energy density $U_{\gamma.}$

$$\text{Rate of stimulated emission} = B_{21}\ N_2 U_\gamma$$

Here B_{21} is the constant known as Einsteins coefficient of stimulated emission.

Expression for Energy coefficients

At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$B_{12}\ N_1U_\gamma\ =\ A_{21}\ N_2\ +\ B_{21}\ N_2\ U_\gamma$$

$$U_\gamma = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

Rearranging this, we get

$$U_\gamma = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}N_1}{B_{21}N_2}-1}\right]$$

From Boltzmans law ,$\frac{N_1}{N_2} = e^{\frac{h\gamma}{kT}}$

Hence

$$U_\gamma = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}}e^{\frac{h\gamma}{kT}}-1}\right]$$

From Planck's radiation law,

$$U_\gamma = \frac{8\pi h\gamma^3}{c^3}\left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]}-1}\right]$$

Comparing these expressions, we get

$\frac{A_{21}}{B_{21}} = \frac{8\pi h\gamma^3}{c^3}$ and$\frac{B_{12}}{B_{21}} = 1$

$$\therefore U_\gamma = \frac{A}{B}\left[\frac{1}{e^{\frac{h\gamma}{kT}}-1}\right]$$

Conclusions

1.Rate of stimulated emission is directly proportional to wavelength

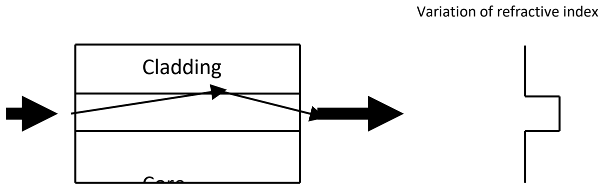
2. Rate of Induced absorption is equal to rate of Stimulated emission

5B

Types:

1. Single mode fiber:

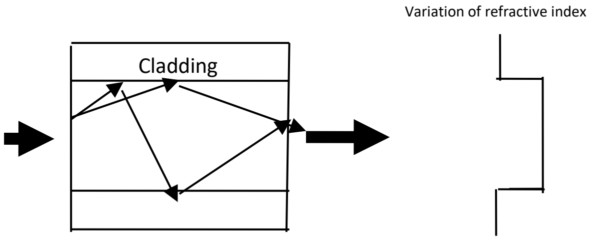
Core diameter is around 5-10 µm. The core is narrow and hence it can guide just a single mode.



- No modal dispersion
- Difference between n_1& n_2 is less.Critical angle is high. Low numerical aperture.
- Low Attenuation -0.35db/km
- Bandwidth -100GHz
- Preferred for short range

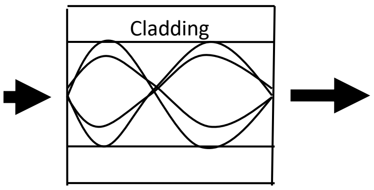
Step index multimode fibre :

- Here the diameter of core is larger so that large number of rays can propagate. Core diameter is around 50. µm.
- High modal dispersion
- Difference between n_1& n_2 is high. Low Critical angle. Large numerical aperture.
- Losses high
- Bandwidth -500MHz
- Allows several modes to propagate
- Preferred for Long range

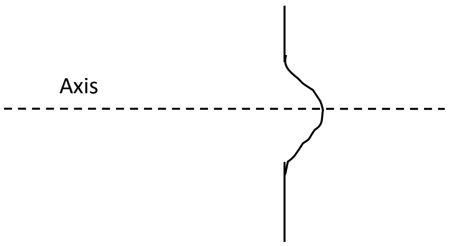


Graded index multimode fiber:

In this type, the refractive index decreases in the radially outward direction from the axis and becomes equal to that of the cladding at the interface. Modes travelling close to the axis move slower where as the modes close to the cladding move faster.As a result the delay between the modes is reduced. This reduces modal dispersion.



Variation of refractive index



- Low modal dispersion
- High data carrying capacity.
- High cost
- Many modes propagate
- Bandwidth -10GHz

5C

$$\frac{N_g}{N_e} = e^{\frac{hc}{\lambda kT}}$$

$$T = 330K$$

$$\frac{N_g}{N_e} = 0.944\,x10^{30}$$

$$ln\frac{N_g}{N_e} = \frac{hc}{\lambda kT} lne_e$$

$\lambda = 6318\ x10^{-10}$m

6A

Carbon dioxide laser

It is a high efficient laser with power output in the range of few 100W to Kilowatt.

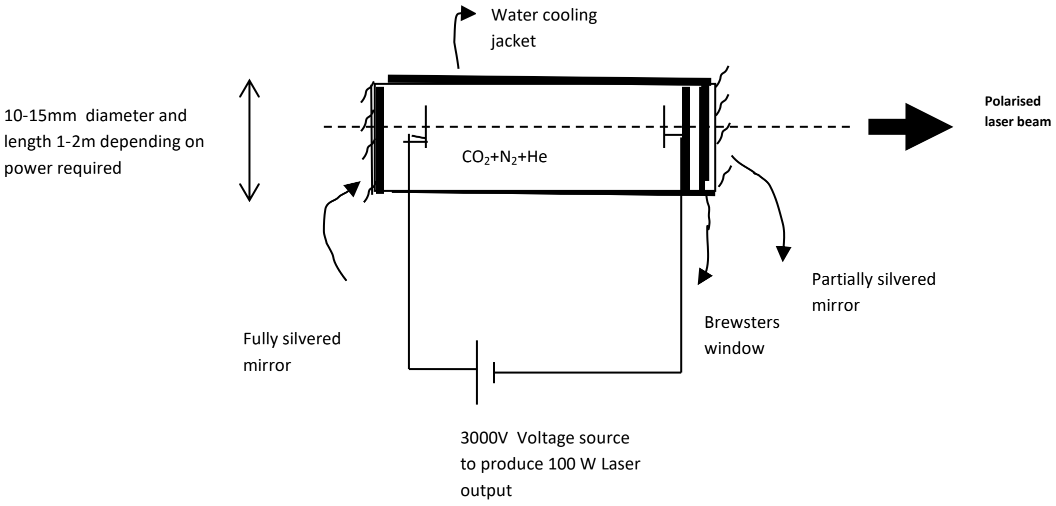
Construction

1.Active medium – Mixture of CO_2, N_2 and He in the ratio 1:2:8. Nitrogen absorbs energy from the pumping source efficiently.Helium gas conducts away the heat and also catalyses collisional deexcitation of CO_2 molecules.

2.The discharge tube consists of a glass tube of 10-15mm diameter with a coaxial water cooling jacket.

3.Partially reflecting and fully reflecting mirrors are mounted at the ends of the tube.

4.Optical pumping is achieved by electric discharge caused by applying potential difference of over 1000V.



Working:

1.CO_2 is a linear molecule and has three modes of vibration –Symmetric stretching (100), Asymmetric stretching (001) and bending (010).

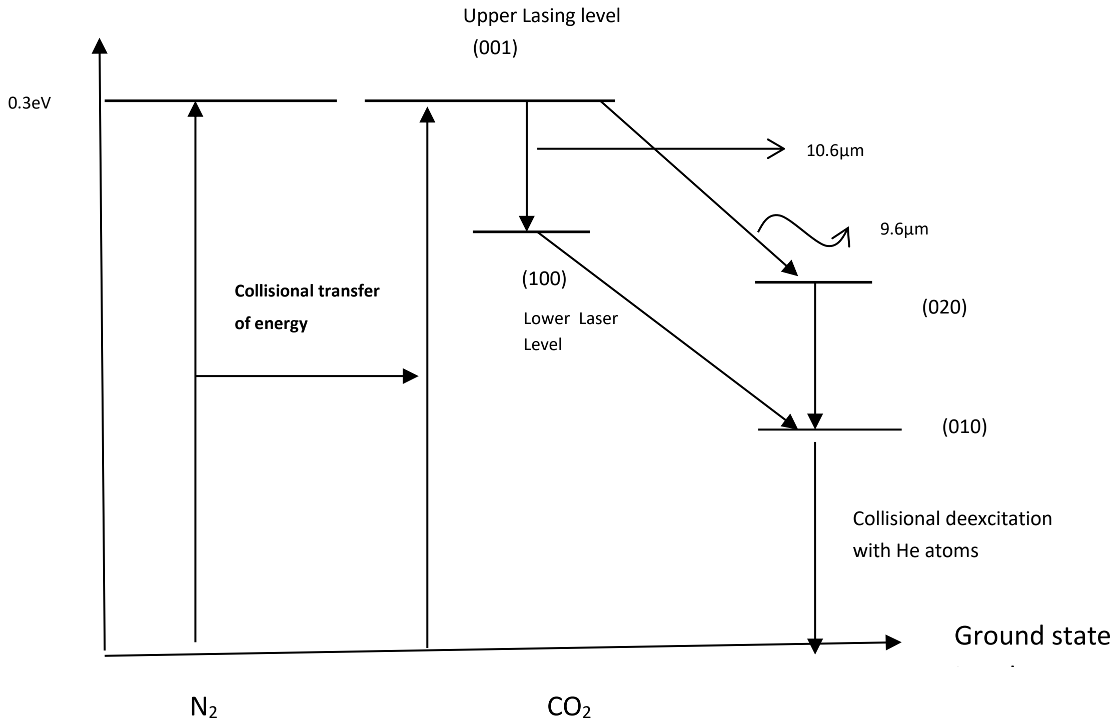
2. Asymmetric stretching (001) is the upper laser level which is a metastable state. (100) and (020) are the lower lasing states

3.During electric discharge, the electrons released due to ionisation excite N_2 molecules to its first vibrational level which is close to upper lasing level of CO_2.

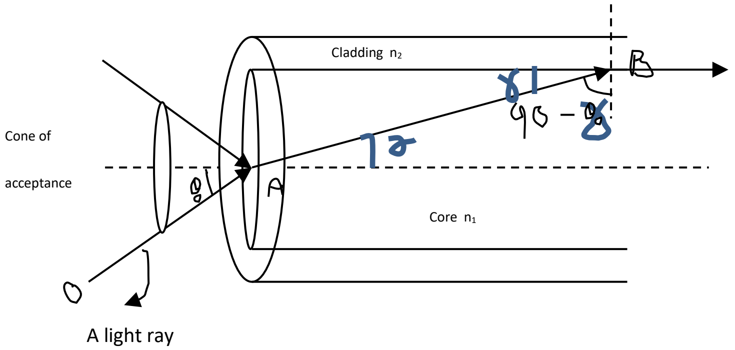
4.N_2 molecules undergo collisions with CO_2 molecules and excite them to (001). This results in population inversion.

5.Lasing transition occurs between (001) and (100) emitting at 10.6µm and (001) to (020) emitting at 9.6µm

6. CO_2 molecules deexcite to ground state through collisions with Helium atom.



6B



Expression for condition for propagation :

Consider a light ray falling in to the optical fibre at an angle of incidence θ_o equal to acceptance angle. Let n_0 be the refractive index of the surrounding medium .

Let n_1 be the refractive index of the core.

Let n_2 be the refractive index of the cladding.

From Snell's Law:

For the ray OA $\quad n_0 \sin\theta_0 = n_1 \sin r = n_1\left(\sqrt{1-\cos^2 r}\right)$………… (1)

For the ray AB $\quad n_1 \sin(90 - r) = n_2 \sin 90$

$$n_1 \cos r = n_2$$

$\cos r = \frac{n_2}{n_1}$

[here the angle of incidence is $(90 - \theta_1)$ for which angle of refraction is 90^0].

Substituting for cosr in equation (1)

$$n_0 \sin\theta_0 = n_1\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$\sin\theta_0 = Sin\theta\,\frac{\sqrt{n_1^2-n_2^2}}{n_0}$$

If the medium surrounding the fiber is air then $n_0 = 1$,

Numerical aperture = $\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$

The total internal reflection will take place only if the angle of incidence $\theta_i < \theta_0$

$$\therefore \sin\theta_i < \sin\theta_0$$

$$\boldsymbol{\sin\theta_i < \sqrt{n_1^2 - n_2^2}}$$

This is the condition for propagation.

6C

$$\alpha = \frac{10}{L} log_{10}\left(\frac{P_{in}}{P_{Out}}\right)$$
$$\left(\frac{P_{in}}{P_{out}}\right) = 10^{\frac{\alpha L}{10}} = 2.75$$

$$P_{out} = \frac{P_{in}}{2.75}$$

7A

Gauss divergence theorem:

Statement: The volume integral of the divergence of a vector function ‘F’ over a volume ‘V’ is equal to the surface integral of the normal component of the vector function ‘F’ over the surface enclosing the volume V.

Explanation:

Consider a Gaussian surface enclosing a charge Q with a charge density ρ_v.

Then $Q = \int_v \int \int \quad \rho dv$

From Gauss law, total charge enclosed is

$$\therefore \boldsymbol{Q} = \int \int \vec{\boldsymbol{D}} \bullet \boldsymbol{ds}$$

From differential form of Gauss law, $\nabla \bullet \vec{D} = \rho_v$

$$\therefore Q = \iint \int_v \nabla \bullet \vec{D} dv = \int_s \vec{D} \bullet ds$$

$$\boxed{\therefore \int_v \nabla \bullet \vec{D} dv = \int_s \vec{D} \bullet ds}$$

7B

Wave equation for electric field:

Consider the equation $\boldsymbol{\nabla x \vec{E}} = -\frac{\boldsymbol{\partial \vec{B}}}{\boldsymbol{\partial t}}$

$$= -\boldsymbol{\mu} \frac{\boldsymbol{\partial \vec{H}}}{\boldsymbol{\partial t}} \because \vec{\boldsymbol{B}} = \boldsymbol{\mu \vec{H}}$$

Taking curl on both sides

$$\boldsymbol{\nabla x}(\boldsymbol{\nabla x \vec{E}}) = -\boldsymbol{\mu} \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}} \left(\boldsymbol{\nabla x \vec{H}} \right) \text{............(1)}$$

Also $\nabla x(\nabla x\vec{E}) = \nabla\left(\nabla \bullet \vec{E}\right) - \nabla^2\vec{E} = \nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2\vec{E}$ …………(2)

From (1) and (2) ,

$$\boldsymbol{\nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}\left(\nabla x\vec{H}\right)}$$

$$\boldsymbol{\because \nabla \bullet \vec{D} = \rho}$$

$$\boldsymbol{\nabla \bullet \varepsilon\vec{E} = \rho}$$

$$\boldsymbol{\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon}}$$

$$\boldsymbol{\nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}\left(\vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t}\right)}$$

$$\boldsymbol{\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = \mu\frac{\partial J}{\partial t} + \nabla\left(\frac{\rho}{\varepsilon}\right)}$$

This is the characteristic form of a wave equation .The solution to this equation represents a wave .

For a free space where there are no charges (ρ=0), no currents (J=0).

$$\boldsymbol{\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} = 0}\text{……………(3)}$$

7C

$\nabla \bullet \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 6x^2 + 10y + 9$ =25

8A

1.$\nabla \bullet \vec{D} = \rho$

Statement of Gauss law:

The total electric flux over a closed surface $\left(\oint\!\!\oint_S \vec{D} \bullet \vec{ds}\right)$is equal to the charge enclosed by the surface.

2.$\nabla \bullet \vec{B} = 0$

Gauss law for magnetic fields:

Magnetic flux lines always form a closed loop. If a closed surface is imagined in a magnetic field, for every flux line that enters the surface, there must always be a flux line emerging out of the surface.

Therefore total flux over the closed surface = 0.

3. $\boldsymbol{\nabla x\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$ **(Modified amperes law)**

.The magneto motive force around a closed path is equal to the conduction current $(\vec{J})$ plus the rate of change of electric displacement $(\vec{D})$ through the surface bounded by the path.

4**.**$\boldsymbol{\nabla x\vec{E}} = -\frac{\partial \vec{\boldsymbol{B}}}{\partial \boldsymbol{t}}$ **(Faradays law)**

Faradays law of electromagnetic induction:

According to this law, the magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux through it. The induced emf will be in a direction which opposes the change which causes it.

Induced emf e = -$\frac{d\phi}{dt}$ where Φ is the flux linking with the circuit.

8B

From the equation of continuity

$\nabla \bullet \vec{J} = -\frac{\partial \rho}{\partial t}$where ρ is the charge density

$$\nabla \bullet \vec{J} = -\frac{\partial}{\partial t}\left(\nabla \bullet \vec{D}\right) = \nabla \bullet \frac{\partial \vec{D}}{dt}$$

$$\Rightarrow \nabla \bullet \left(\vec{J} + \frac{\partial \vec{D}}{dt}\right) = 0$$

Thus although $\nabla \bullet \vec{J}$ is not zero but the divergence of $\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$ is always zero .Hence Maxwell made the assumption that the term $\vec{J}$ in amperes law must be replaced by $\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$.

∴**For time varying fields** $\boldsymbol{\nabla x \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$

The term $\left(\frac{\partial \vec{D}}{\partial t}\right)$ is known as displacement current density . It represents rate of change of Electric flux density. .It is clear from the above expression that the rate of change of electric displacement produces magnetic field just as the conduction current ($\vec{J}$)does.

8C

$$\boldsymbol{\nabla x A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2 + y + 2z & x - 5y^3 - 2z & 2x - 2y + 3z^2 \end{vmatrix}}$$

$$\boldsymbol{\nabla x A = i(2-2) - j(2-2) + z(1-1) = 0}$$

9A

Conductivity of Intrinsic semiconductors:

Electron current $I_e = n_h e A v_d(e)$

Hole current $I_h = n_h eAv_d(h)$

current density $J = \frac{I}{A} = \frac{I_e+I_h}{A} = n_h ev_d(h) + n_e ev_d(e) = \sigma E$

But drift velocity V_d = μE=μ.J/σ μ = V_d/E J = σ . E

Using (1), σ = n_e e μ_e + n_h e μ_h

In an intrinsic semiconductor, number of holes is equal to number of electrons.

$\sigma e_{e_{hole_{int}}}$

n_e is the electron concentartion

n_p is the hole concentration

μ_e is the mobility of electrons

μ_h is the mobility of holes

Law of mass action:

The product of electron and hole concentration is constant at any given temperature independent of Fermi energy.This principle is used to calculate hole and electron densities.

$$n_e n_h = 4\left(\frac{kT}{2\pi h^2}\right)^3 (m_e^* m_h^*)^{\frac{3}{2}}\, e^{-\frac{E_g}{kT}} = a \quad cons\,tan\,t$$

9B

FOUR PROBE METHOD

Four Probe method permits measurements of resistivity in samples having a wide variety of shapes, including the resistivity of small volumes within bigger pieces of semiconductor. The basic model for all these measurements is indicated in Figure. Four sharp probes are placed on a flat surface of the material to be measured, current is passed through the two outer electrodes, and the floating potential is measured across the inner pair. If the flat surface on which the probes rest is adequately large and the crystal is big the semiconductor may be considered to be a semi-infinite volume. To prevent minority carrier injection and make good contacts, the surface on which the probes rest, maybe mechanically lapped.

The experimental circuit used for measurement is illustrated schematically in Figure. A nominal value of probe spacing which has been found satisfactory is an equal distance of 2.0 mm between adjacent probes. This permit measurement with reasonable current of n-type or p-type semiconductor from 0.001 to 50 ohm. cm.

In order to use this four probe method in semiconductor crystals or slides itis necessary to assume that :

1. The resistivity of the material is uniform in the area of measurement.
2. If there is minority carrier injection into the semiconductor by the current - carrying electrodes most of the carriers recombine near the electrodes so that their effect on the conductivity is negligible. (This means that the measurements should be made on surface which have a high recombination rate, such as mechanical lapped surfaces).
3. The surface on which the probes rest is flat with no surface leakage.
4. The four probes used for resistivity measurements contact the surface at points thatlie in a straight line.
5. The diameter of the contact between the metallic probes and the semiconductor should be small compared to the distance between probes.
6. The boundary between the current-carrying electrodes and the bulk material ishemispherical and small in diameter.
7. The surfaces of the semiconductor crystal may be either conducting or non-conducting.
   (a) A conducting boundary is one on which a material of much lower resistivity than semiconductor (such as copper) has been plated.
   (b) A non-conducting boundary is produced when the surface of the crystal is incontact with an insulator.

PROCEDURE:

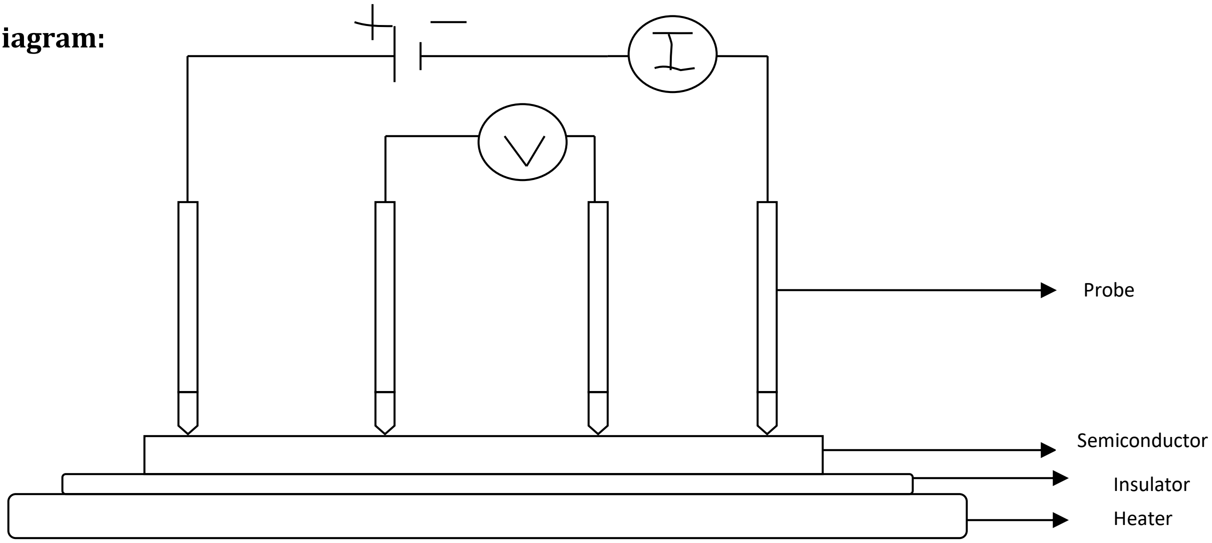
1. The semiconductor sample is placed in the four probe fixture and the probes are in contact with the semiconductor sample. Turn on the instrument and set the constant current to flow through the semiconductor. Note down the current reading (I).
2. Turn on the oven. The temperature begins to increase.
1. Note down the probe output voltage at regular intervals of say 5^0C beginning from 320K to 380K.
2. The resistivity of the semiconductor is found by the expression

$$\rho = \frac{V \times 2\pi \times S}{I \times F}$$

5. Plot the graph of ln ρ against 1/T and find the slope.
6. Energy gap of the semiconductor is calculated using the relation

$$E_g = 2 \times k \times slope \quad eV$$

Diagram:



TABULATION & OBSERVATION:

Temperature T ( )	1/T ( )	Probe output voltage	Resistivity ρ( )	ln ρ

Observations:

Average distance between the probes S =0.2cm

Correction factor F =5.89

The resistivity of the semiconductor is found by the expression

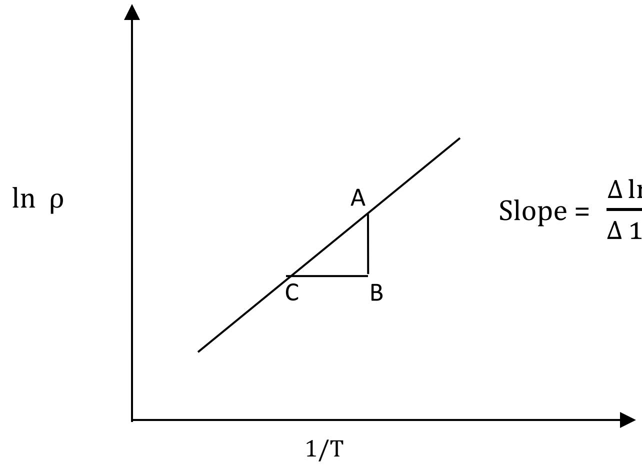
$$\rho = \frac{V \times 2\pi \times S}{I \times F}$$

V is the voltage measured between the inner probes
S is the distance between the probes in cm.
F is the correction factor (0.89)
I is the current through the semiconductor

Energy gap of the semiconductor is calculated using the relation

$$E_g = 2 \times k \times slope \qquad eV$$

NATURE OF GRAPH:



$$\text{Slope} = \frac{\Delta \ln \rho}{\Delta\, 1/T}$$

9C

$$k = \frac{10^{-6} d t_{\frac{1}{2}}}{0.693 \varepsilon_o A R}$$

$$k = 4.3$$

$$d = 1.02x10^{-3}$$

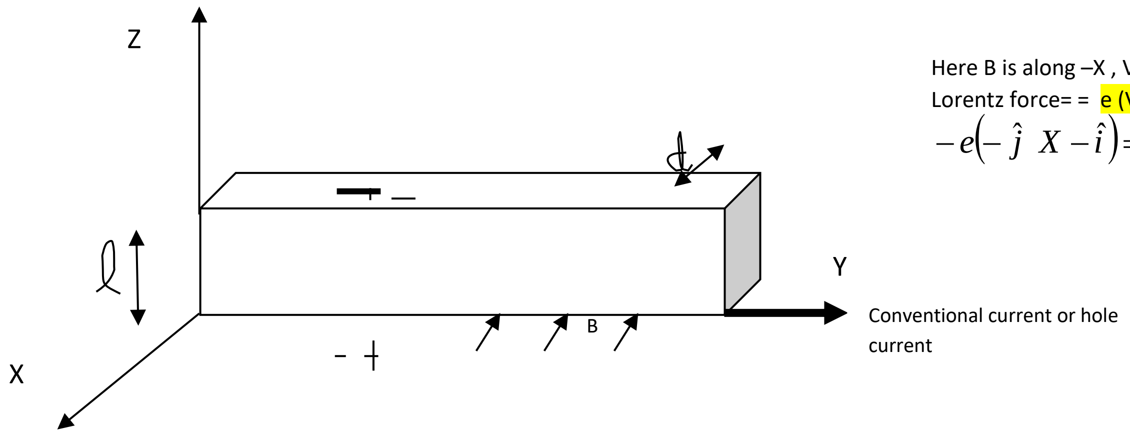
$$t = 7$$

$$R = 110000\Omega$$

$$\boldsymbol{A = 9.3x10^{-4} m^2}$$

10A

Hall effect: When a conductor carrying current is placed in magnetic field, an electric field is produced inside the conductor in a direction normal to both current and the magnetic field.



Here B is along –X , V is along –Y axis

Lorentz force= = e (VXB)

$$-e\left(-\hat{j}\ X\ -\hat{i}\right)=+\hat{k}$$

Consider a rectangular slab of an n type semiconductor carrying a current I along + X axis. Magnetic field B is applied along –Z direction. Now according to Fleming's left hand rule, the Lorentz force on the electrons is along +Y axis. As a result the density of electrons increases on the upper side of the material and the lower side becomes relatively positive. The develops a potential V_H-Hall voltage between the two surfaces. Ultimately, a stationary state is obtained in which the current along the X axis vanishes and a field E_Y is set up.

Expression for electron concentration:

At equilibrium, Lorentz force is equal to force due to applied electric field

$$Bev = -e\,E_H$$

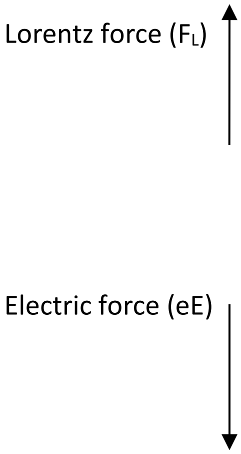
Hall Field $E_H = Bv$

$$Current\ \ density\ \ J = I/A = neAV/A = -n_e ev$$

$$v = \frac{J}{n_e e}$$

Hence

$$\frac{E_H}{JB} = -\frac{1}{n_e e} = R_H$$



R_H is known as Hall coefficient. It is negative for n type and p type for

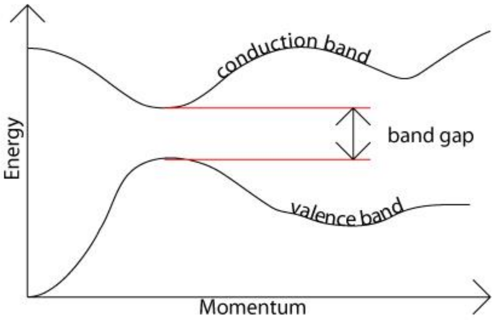
Hall voltage $V_H = E_H.l = R_H JBl$ $\qquad \because E = \frac{dv}{dx}$

10B

DIRECT GAP SEMICONDUCTOR

In a **direct band gap semiconductor**, the top of the valence band and the bottom of the conduction band occur at the same value of momentum

A photon of energy E_g, where E_g is the band gap energy, can produce an electron-hole pair in a direct band gap semiconductor quite easily, because the electron does not need to be given very much momentum. However, an electron must also undergo a significant change in its momentum for a photon of energy E_g to produce an electron-hole pair in an indirect band gap semiconductor. This is possible, but it requires such an electron to interact not only with the photon to gain energy, but also with a lattice vibration called a phonon in order to either gain or lose momentum.



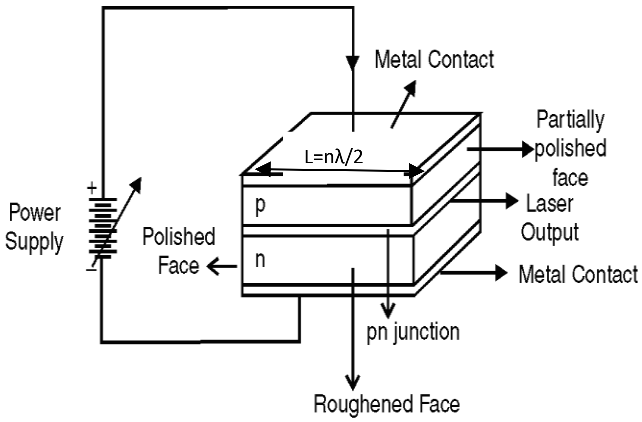
Gallium – Arsenide Semiconductor laser :

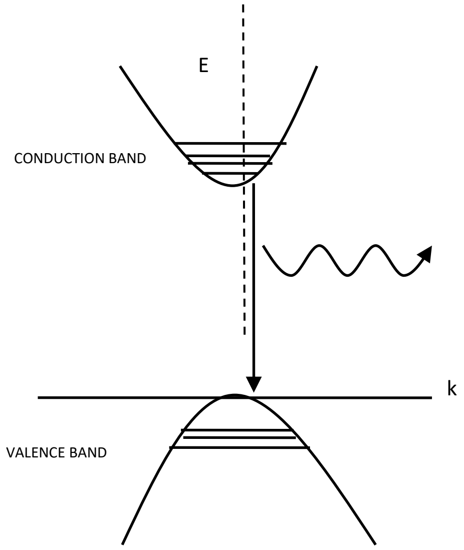
It is the only device which can be used for amplification in the infrared and optical ranges.

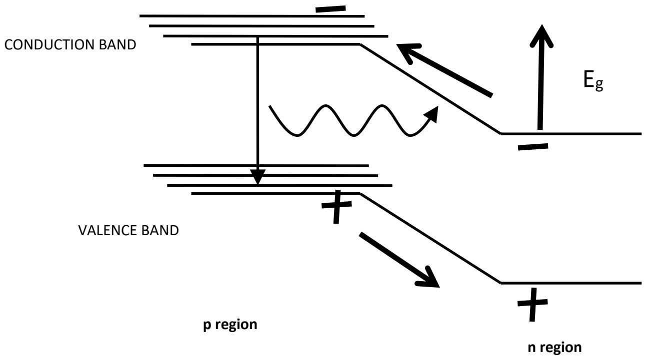
CONSTRUCTION

Gallium Arsenide is heavily doped with Tellurium (n side) and Zinc (P side) to a concentration of 10^{19} atoms /cm^3. Resonant cavity is formed by polishing the end faces of Junction diode.

Amplification is possible if the population of the valence and conduction bands could be inverted as shown in the diagram.







WORKING

The first laser action was observed in a GaAs junction(8400Å) which is a direct gap semiconductor. When a heavily doped junction is forward biased, electrons from n side are injected into p side causing population inversion. They combine with holes on the P side releasing photons. The junction region is the active region .The optical cavity is formed by the faces of the crystal itself which are taken on the cleavage plane and are then polished. The wavelength of the radiation depends on temperature. The wavelength of laser increases as the energy gap decreases. The frequency can be increased to the optical region by alloying with phosphor according to the relation $Ga(As)_{1-x}P_x$.

If E_g is the energy gap, then $E_g = eV_{forward} = \frac{hc}{\lambda}$

10C

$$\therefore \sigma \quad = n_e e \mu_e + n_{hole} e \mu_{hole}$$

$$2.12 = 2.37\text{x}10^{19}\text{x}1.6\text{x}10^{-19}(0.38 + \mu_{hole})$$

$$\mu_{hole} = -0.18 m^2 V^{-1} s^{-1}$$