

1 A

## LASERS: [LIGHT AMPLIFICATION BY STIMULATED EMISSION OF RADIATION]

Laser is a beam of light with unique properties such as

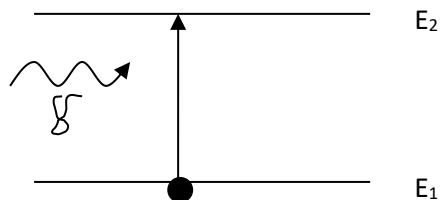
1. High degree of monochromaticity.
2. High power.
3. High intensity.
4. Coherence.
5. Directionality

### Interaction of radiation and matter

#### Induced absorption:

It is a process in which an atom at a lower level absorbs a photon to get excited to the higher level.

Let  $E_1$  and  $E_2$  be the energy levels in an atom and  $N_1$  and  $N_2$  be the number density in these levels respectively. Let  $U_\gamma$  be the energy density of the radiation incident.



$$\gamma = \frac{E_2 - E_1}{h}$$

Rate of absorption is proportional to the number of atoms in lower state and also on the energy density  $U_\gamma$ .

$$\text{Rate of absorption} = B_{12} N_1 U_\gamma$$

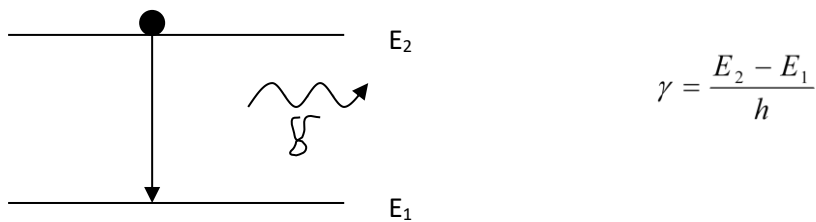
Here  $B_{12}$  is a constant known as Einsteins coefficient of spontaneous absorption.

**Spontaneous emission:**

It is a process in which ,atoms at the higher level voluntarily get excited emitting a photon. The rate of spontaneous emission representing the number of such deexcitations is proportional to number of atoms in the excited state.

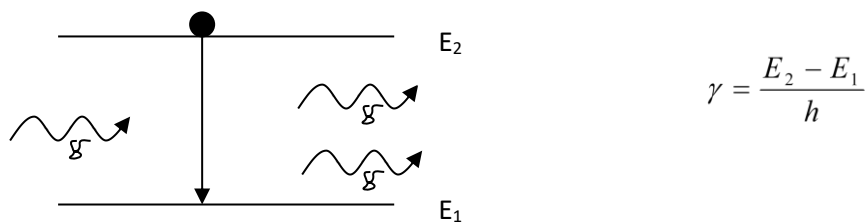
$$\text{Rate of spontaneous absorption} = A_{21} N_2$$

Here  $B_{12}$  is a constant known as Einsteins coefficient of spontaneous emission.



**Stimulated emission:**

In this process, an atom at the excited state gets deexcited in the presence of a photon of same energy as that of difference between the two states.



The number of stimulated emissions is proportional to the number of atoms in higher state and also on the energy density  $U_\gamma$ .

$$\text{Rate of stimulated emission} = B_{21} N_2 U_\gamma$$

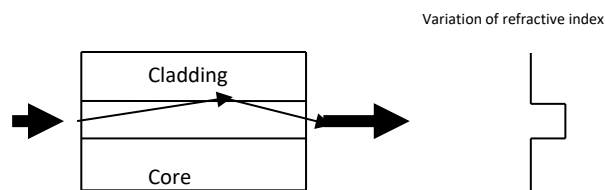
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1B

## Types:

### 1. Single mode fiber:

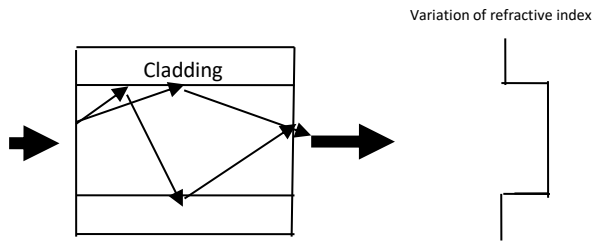
Core diameter is around 5-10  $\mu\text{m}$ . The core is narrow and hence it can guide just a single mode.



- No modal dispersion
- Difference between  $n_1$  &  $n_2$  is less. Critical angle is high. Low numerical aperture.
- Low Attenuation -0.35db/km
- Bandwidth -100GHz
- Preferred for short range

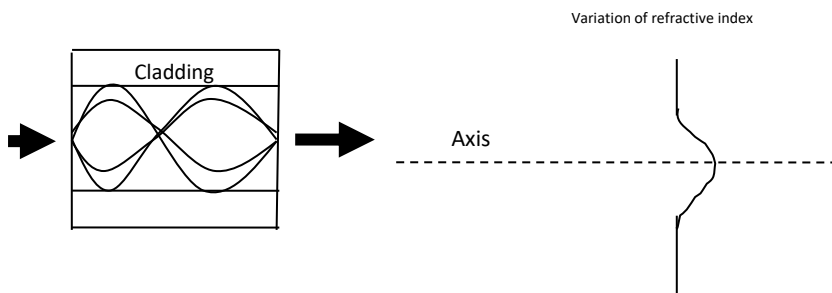
### Step index multimode fibre :

- Here the diameter of core is larger so that large number of rays can propagate. Core diameter is around 50.  $\mu\text{m}$ .
- High modal dispersion
- Difference between  $n_1$  &  $n_2$  is high. Low Critical angle. Large numerical aperture.
- Losses high
- Bandwidth -500MHz
- Allows several modes to propagate
- Preferred for Long range



**Graded index multimode fiber:**

In this type, the refractive index decreases in the radially outward direction from the axis and becomes equal to that of the cladding at the interface. Modes travelling close to the axis move slower where as the modes close to the cladding move faster. As a result the delay between the modes is reduced. This reduces modal dispersion.



- Low modal dispersion
- High data carrying capacity.
- High cost
- Many modes propagate
- Bandwidth -10GHz

1C

$$\text{Attenuation coefficient } \alpha = \frac{10}{L} \log \left( \frac{P_{in}}{P_{out}} \right) = \frac{10}{0.5km} \log \left( \frac{0.1}{0.09} \right) = 0.9dB / km$$

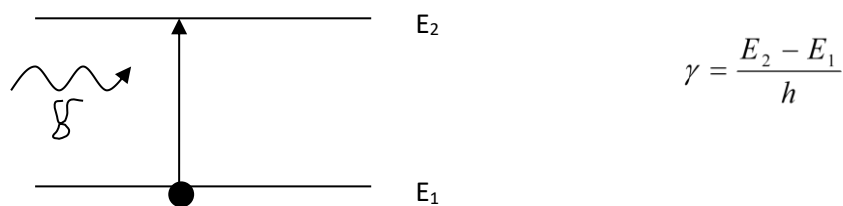
2A

**Expression for energy density:**

### Induced absorption:

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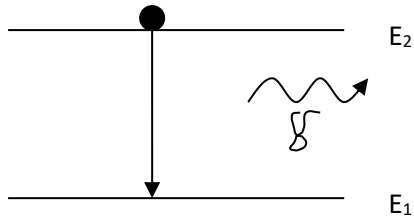
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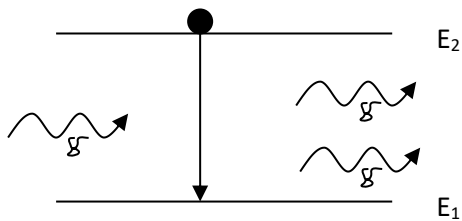
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$$\gamma = \frac{E_2 - E_1}{h}$$

**Stimulated emission:**

In this process, an atom at the excited state gets deexcited in the presence of a photon of same energy as that of difference between the two states.



$$\gamma = \frac{E_2 - E_1}{h}$$

The number of stimulated emissions is proportional to the number of atoms in higher state and also on the energy density U<sub>γ</sub>.

$$\text{Rate of stimulated emission} = B_{21} N_2 U_\gamma$$

Here B<sub>21</sub> is the constant known as Einsteins coefficient of stimulated emission.

At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$B_{12} N_1 U_\gamma = A_{21} N_2 + B_{21} N_2 U_\gamma$$

$$U_\gamma = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

Rearranging this, we get

$$U_{\gamma} = \frac{A_{21}}{B_{21}} \left[ \frac{1}{\frac{B_{12}N_1}{B_{21}N_2} - 1} \right]$$

From Boltzmann's law,  $\frac{N_1}{N_2} = e^{\frac{h\nu}{kT}}$

Hence

$$U_{\gamma} = \frac{A_{21}}{B_{21}} \left[ \frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{kT}} - 1} \right]$$

From Planck's radiation law,

$$U_{\gamma} = \frac{8\pi h \nu^3}{c^3} \left[ \frac{1}{e^{\left[\frac{h\nu}{kT}\right]} - 1} \right]$$

Comparing these expressions, we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3} \quad \text{and} \quad \frac{B_{12}}{B_{21}} = 1$$

$$\therefore U_{\gamma} = \frac{A}{B} \left[ \frac{1}{e^{\frac{h\nu}{kT}} - 1} \right]$$

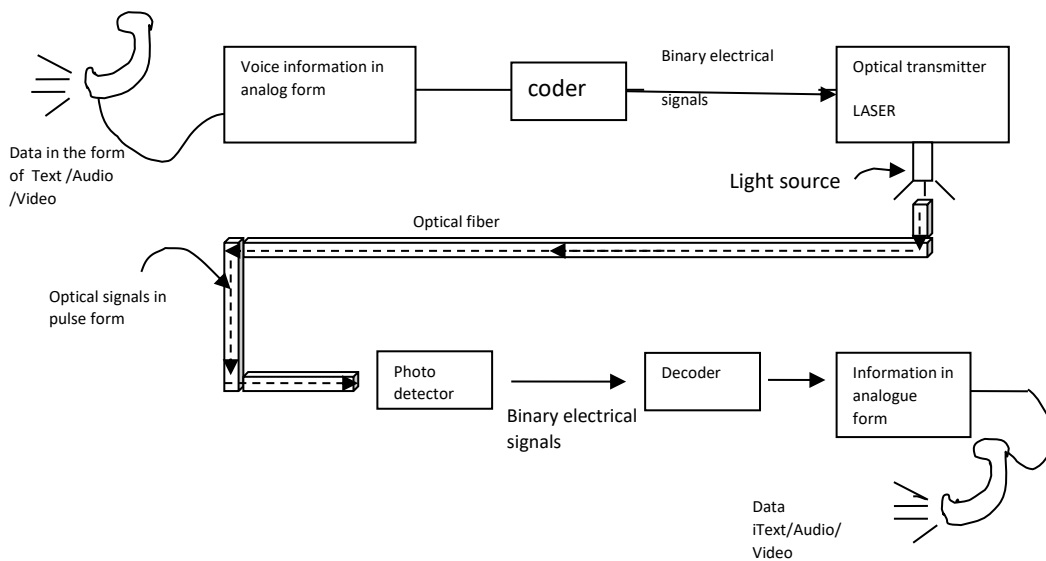
## Conclusions

1. Rate of stimulated emission is directly proportional to wavelength
2. Rate of Induced absorption is equal to rate of Stimulated emission

## 2B

### Point to point communication system using optical fibers

This system is represented through a block diagram as follows.



The information in the form of voice/ picture/text is converted to electrical signals through the transducers such as microphone/video camera. The analog signal is converted in to binary data with the help of coder. The binary data in the form of electrical pulses are converted in to pulses of optical power using Semiconductor Laser. This optical power is fed to the optical fiber. Only those modes within the angle of acceptance cone will be sustained for propagation by means of total internal reflection. At the receiving end of the fiber, the optical signal is fed in to a photo detector where the signal is converted to pulses of current by a photo diode. Decoder converts the sequence of binary data stream in to an analog signal . Loudspeaker/CRT screen provide information such as voice/ picture.

2C

$$d \cdot \sin \theta = n\lambda$$

$$\lambda = \frac{5.05 \times 10^{-5} \times \sin 1.48}{2} = 6.52 \times 10^{-7} \text{ m}$$

3A



## HEISENBERG'S UNCERTAINTY PRINCIPLE:

The position and momentum of a particle cannot be determined accurately and simultaneously. The product of uncertainty in the measurement of position ( $\Delta x$ ) and momentum ( $\Delta p$ ) is always greater than or equal to  $\frac{h}{2\pi}$ .

$$\boxed{(\Delta x) \cdot (\Delta p) \geq \frac{h}{4\pi}}$$

This uncertainty is not due to discrepancy with the apparatus or with the method of measurement, but because of the very wave nature of the object. This uncertainty persists as long as matter possesses wave nature.

### Different forms of Heisenberg's Principle:

$$(\Delta x) \cdot (\Delta p) \geq \frac{h}{4\pi}$$

$$(\Delta L) \cdot (\Delta \theta) \geq \frac{h}{4\pi}$$

$$(\Delta E) \cdot (\Delta t) \geq \frac{h}{4\pi}$$

Here  $\Delta L$  is the uncertainty in angular momentum

$\Delta \theta$  is the uncertainty in the measurement of angular displacement

$\Delta E$  is the uncertainty in the measurement of energy

$\Delta t$  is the uncertainty in the measurement of time interval during which the particle exists in the state E

### Physical Significance:

1. It introduces the concept of probability.
2. It can be used to find life time of electrons in an excited state.

3.It can be used to show that electrons do not exist inside the nucleus.

**TO SHOW THAT ELECTRON DOES NOT EXIST INSIDE THE NUCLEUS:**

We know that the diameter of the nucleus is of the order of  $10^{-14}$ m.If the electron is to exist inside the nucleus, then the uncertainty in its position  $\Delta x$  cannot exceed the size of the nucleus

$$\Delta x = 5 \times 10^{-15} \text{ m}$$

Now the uncertainty in momentum is

$$\Delta x = 5 \times 10^{-15} \text{ m}$$

$$\Delta P = \frac{h}{4\pi x \Delta x} = 0.1 \times 10^{-19} \text{ kg.m / s}$$

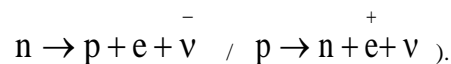
Then the momentum of the electron can atleast be equal to the uncertainty in momentum.

$$P \approx \Delta P = 0.1 \times 10^{-19} \text{ kg.m / s}$$

Now the energy of the electron with this momentum supposed to be present in the nucleus is given by (for small velocities - non-relativistic-case)

$$E = \frac{P^2}{2m} = 5.5 \times 10^{-11} \text{ J} = 343 \text{ MeV}$$

The beta decay experiments have shown that the kinetic energy of the beta particles (electrons) is only a fraction of this energy. This indicates that electrons do not exist within the nucleus. They are produced at the instant of decay of nucleus (



3B

**Time independent Schrödinger equation**

A matter wave can be represented in complex form as

$$\Psi = A \sin kx(\cos wt + i \sin wt)$$

$$\Psi = A \sin kxe^{iwt}$$

Differentiating wrt x

$$\frac{d\Psi}{dx} = kA \cos kxe^{iwt}$$

$$\frac{d^2\Psi}{dx^2} = -k^2 A \sin kxe^{iwt} = -k^2 \Psi \dots\dots\dots (1)$$

From debroglie's relation

$$\frac{1}{\lambda} = \frac{h}{mv} = \frac{h}{p}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$$

$$k^2 = 4\pi^2 \frac{p^2}{h^2} \dots\dots\dots (2)$$

Total energy of a particle E = Kinetic energy + Potential Energy

$$E = \frac{1}{2} m v^2 + V$$

$$E = \frac{p^2}{2m} + V$$

$$p^2 = (E - V)2m$$

Substituting in (2)

$$k^2 = \frac{4\pi^2 (E - V)2m}{h^2}$$

∴ From (1)

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m(E - V)\Psi}{h^2} = 0$$

3C

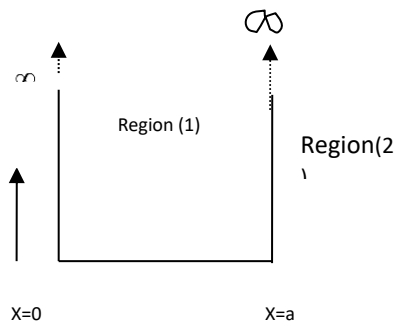
$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19}}} = 1.22 \times 10^{-10} m$$

$$m = \frac{0.5 \times 10^6 \times 1.6 \times 10^{-19}}{(3 \times 10^8)^2} = 9.1 \times 10^{-31} kg$$

4A

**Particle in an infinite potential well problem:**

Consider a particle of mass m moving along X-axis in the region from X=0 to X=a in a one dimensional potential well as shown in the diagram. The potential energy is assumed to be zero inside the region and infinite outside the region.



Applying, Schrodingers equation for region (1) as particle is supposed to be present in region (1)

$$\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 mE\psi}{h^2} = 0 \because V = 0$$

$$\text{But } k^2 = \frac{8\Pi^2 mE}{h^2}$$

$$\therefore \frac{d^2\Psi}{dx^2} + k^2\Psi = 0$$

Auxiliary equation is  $(D^2 + k^2)x = 0$

Roots are  $D = +ik$  and  $D = -ik$

The general solution is

$$\begin{aligned} x &= Ae^{ikx} + Be^{-ikx} \\ &= A(\cos kx + i \sin kx) + B(\cos kx - i \sin kx) \\ &= (A + B) \cos kx + i(A - B) \sin kx \\ &= C \cos kx + D \sin kx \end{aligned}$$

The boundary conditions are

$$1. \text{ At } x=0, \Psi = 0 \therefore C = 0$$

$$2. \text{ At } x=a, \Psi = 0$$

$$D \sin ka = 0 \Rightarrow ka = n\Pi \dots\dots\dots(2)$$

where  $n = 1, 2, 3, \dots$

$$\therefore \Psi = D \sin\left(n \frac{\Pi}{a}\right)x$$

From (1) and (2) Eigen value  $E = \frac{n^2 h^2}{8ma^2}$

**To evaluate the constant D:**

Normalisation: For one dimension

$$\int_0^a \Psi^2 dx = 1$$

$$\int_0^a D^2 \sin^2\left(\frac{n\Pi}{a}x\right) dx = 1$$

But  $\cos 2\theta = 1 - 2\sin^2 \theta$

$$\int_0^a D^2 \frac{1}{2} (1 - \cos 2\left(\frac{n\Pi}{a}x\right)) dx = 1$$

$$\int_0^a \frac{D^2}{2} dx - \int_0^a \frac{1}{2} \cos 2\left(\frac{n\Pi}{a}x\right) dx = 1$$

$$\frac{D^2 a}{2} - \left[ \sin 2\left(\frac{n\Pi}{a}x\right) \frac{x}{2} \right]_0^a = 1$$

$$D^2 \frac{a}{2} - 0 = 1$$

$$D = \sqrt{\frac{2}{a}}$$

Eigen function  $\therefore \Psi_n = \sqrt{\frac{2}{a}} \sin\left(n \frac{\Pi}{a}x\right)$

4B

**Debroglie's theory:**

**Statement:** By the law of symmetry of nature, a particle exhibits wave properties in addition to its particle properties.

The wavelength of the group of waves associated with particle of mass  $m$  moving with a velocity  $v$  is given by the expression

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \frac{h}{\sqrt{2mE}}$$

where  $h$  is the Planck's constant

### **Properties of Matter waves:**

1. Matter waves represent the probability density variation in a region.
2. A matter wave in complex form is written as  $\Psi = A \sin kx(\cos \omega t + i \sin \omega t)$ . It is obtained as general solution to Schrodinger's equation.
3. Matter waves are neither transverse nor longitudinal and their velocity is equal to that of Particle.
4. They propagate as group of waves.

### **Derivation:**

$$E = mc^2$$

$$E = hf$$

$$hf = mc^2$$

$$p = mc = \frac{hf}{c} = \frac{h}{\frac{c}{f}} = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{mv}$$

$$E = \frac{n^2 h^2}{8mL^2}$$

$$n = 1$$

$$L = 1 \times 10^{-10} \text{ m}$$

$$E_1 = 6 \times 10^{-18} \text{ J} = 37.6 \text{ eV}$$

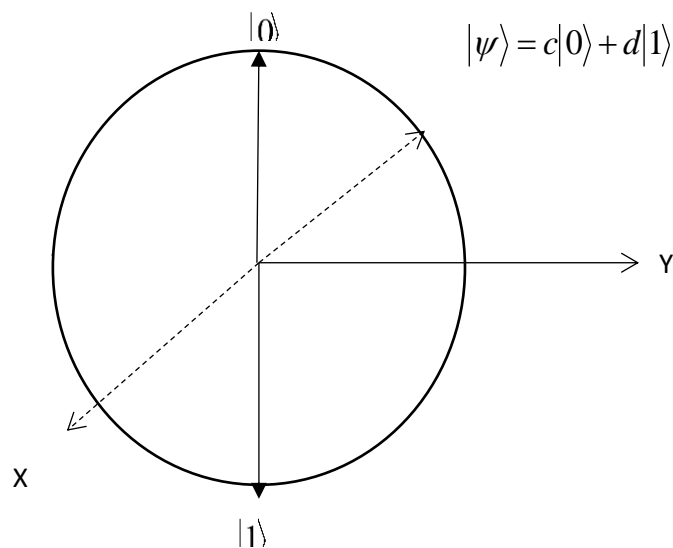
$$n = 2, E_2 = 24 \times 10^{-18} \text{ J} = 150.4 \text{ eV}$$

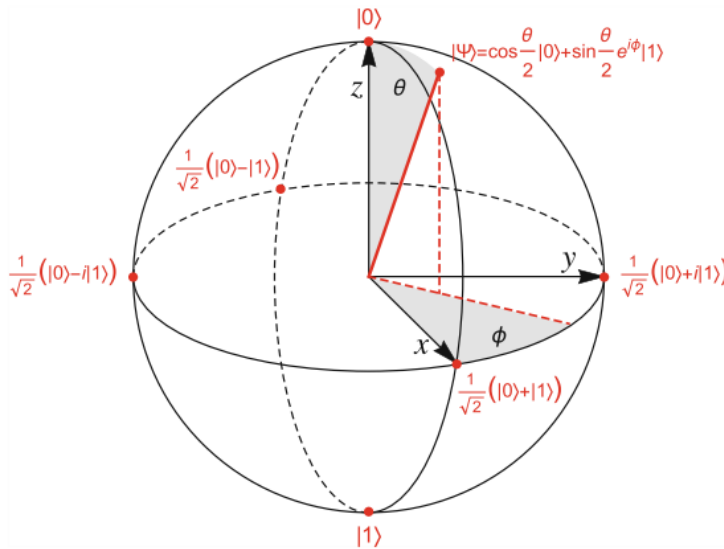
$$n = 3, E_3 = 54 \times 10^{-18} \text{ J} = 338.4 \text{ eV}$$

5A

## BLOCK SPHERE

It represents a sphere with all the points on its surface correspond to state vectors in Hilbert space. The vector drawn to any point on the surface from the centre represents a state. In the diagram,  $|\psi\rangle = c|0\rangle + d|1\rangle$  is a superposed state.  $|0\rangle$  and  $|1\rangle$  are represented along + Z and - Z axes.



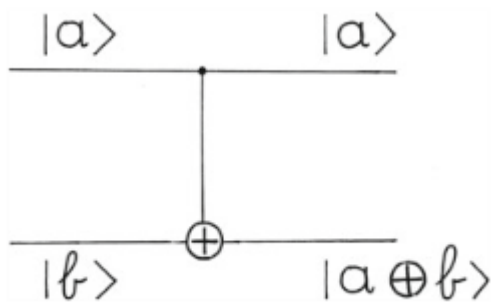


**BIT** : A light bulb can be either in the ON and OFF state and thus serves as a storage device for a single bit of information. 0 and 1 denote the value of a bit.

## 5B

### CONTROLLED NOT GATE

a and b are two inputs. a is called control qubit and b the target qubit. Target qubit flips if and only if a = 1. If a = 0, the second qubit remains unchanged.



Input	Output
00>	00>
01>	01>
10>	11>
11>	10>

## 5C



$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$X|0\rangle = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$ax1 + bx0 = 0 \Rightarrow a = 0$$

$$cx1 + dx0 = 1 \Rightarrow c = 1$$

$$X|1\rangle = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$ax0 + bx1 = 1 \Rightarrow b = 1$$

$$cx0 + dx1 = 0 \Rightarrow d = 0$$

**6A**

## PAULI MATRICES

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\sigma_x$  is a classical not gate. When operated on a state vector say  $|0\rangle$ , it flips to  $|1\rangle$

$$\sigma_x|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0x1 + 1x0 \\ 1x1 + 0x0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\sigma_x|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0x0 + 1x1 \\ 1x0 + 0x1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\sigma_y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0x1 + (-ix0) \\ ix1 + 0x0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix}$$

$$\sigma_y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0x0 + (-ix1) \\ ix0 + 0x1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix}$$

$$\sigma_z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \times 1 + 0 \times 0 \\ 0 \times 1 + -1 \times 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\sigma_z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \times 0 + 0 \times 1 \\ 0 \times 0 + -1 \times 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

6B

Classical computer	Quantum computer
1. Data is stored 0's and 1's represented by capacitor / transistor/etc at LOW / HIGH voltage	1. Data is stored as 0's (lower energy state), 1's (upper energy state) and linear combination of these states occupied by photons/electrons/atoms/ nuclei / ions
2. Processing performed through logic gates	2. superposition allows for exponentially many quantum states at once
3. A bit is either in 0 or 1 state	QUBIT can 0 or 1 or superposed state
4. Quantum entanglement not possible	3. Quantum entanglement applicable Two particles that are too far apart can be strongly correlated.
5. Classical computer conduct sequential operations	5. Quantum computers can do $2^n$ operations at a time
6 Classical gates are irreversible	6. Quantum gates are reversible
7. Number of BITS needed for memory is linear function of number of numbers	7 Number of QUBITS needed for memory is logarithmic function of number of numbers

6C

$$|0\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$I|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$I|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

7A

**Failures of Classical free electron theory:**

1. Prediction of low specific heats for metals:

Classical free electron theory assumes that conduction electrons are classical particles similar to gas molecules. Hence, they are free to absorb energy in a continuous manner. Hence metals possessing more electrons must have higher heat content. This resulted in high specific heat given by the expression  $C_v = 10^{-4} R$ .

This was contradicted by experimental results which showed low specific heat for metals.

2. Temperature dependence of electrical conductivity:

From the assumption of kinetic theory of gases

$$\frac{3}{2}kT = \frac{1}{2}mv^2$$
$$\therefore v \propto \sqrt{T}$$

Also mean collision time  $\tau$  is inversely proportional to velocity,

$$\tau \propto \frac{1}{v}$$
$$\tau \propto \frac{1}{\sqrt{T}}$$
$$\therefore \sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto \frac{1}{\sqrt{T}}$$

However experimental studies show that  $\sigma \propto \frac{1}{T}$

3. Dependence of electrical conductivity on electron concentration:

As per free electron theory,  $\sigma \propto n$

The electrical conductivity of Zinc and Cadmium are  $1.09 \times 10^7$  /ohm m and  $.15 \times 10^7$  /ohm m respectively which are very much less than that for Copper and Silver for which the values are  $5.88 \times 10^7$  /ohm m and  $6.2 \times 10^7$  /ohm m. On the contrary, the electron concentration for zinc and cadmium are  $13.1 \times 10^{28} /m^3$  and  $9.28 \times 10^{28} /m^3$  which are much higher than that for Copper and Silver which are  $8.45 \times 10^{28} /m^3$  and  $5.85 \times 10^{28} /m^3$ .

These examples indicate that  $\sigma \propto n$  does not hold good.

4. Mean free path, mean collision time found from classical theory are incorrect.

**Quantum free electron theory:**

**Quantum free electron theory:**

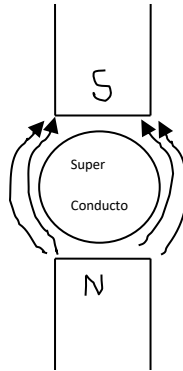
**Assumptions:**

1. The energy of conduction electrons in a metal is quantized.
2. The distribution of electrons amongst various energy levels is according to Pauli's exclusion principle and Fermi – Dirac statistical theory.
3. The average kinetic energy of an electron is equal to  $\frac{3}{5} E_F$
4. The attraction between the electrons and ions, the repulsion between electrons are ignored.

## 7B

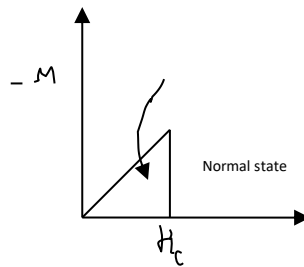
When a superconductor is placed in a magnetic field, it expels the magnetic flux out of its body and behaves like a diamagnet. This effect is known as **Meissener's effect**.

Using the experiments on superconducting cylinders in presence of small magnetic fields it was demonstrated that as temperature is lowered to  $T_c$ , the magnetic flux inside the superconductor is suddenly and completely expelled as the specimen becomes superconductor as shown in the diagram. This effect is reversible.



**Types of superconductors**

**Type 1 Superconductors:**



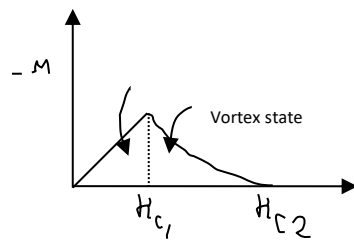
These are pure superconductors.

When kept in magnetic field, initially they continue to exhibit superconductivity and the negative magnetic moment increases. At critical magnetic field there is a sharp transition to normal state due to the penetration of magnetic flux lines. The transition is sharp.

These possess low critical magnetic fields. Their critical temperatures also low. They are generally pure metals.

Ex: Al, Pb

**Type 2 superconductor:**



These are generally alloys.

When kept in magnetic field, initially they continue to exhibit superconductivity and the negative magnetic moment increases. At lower critical magnetic field  $H_{c1}$ , the flux lines start penetrating. As the magnetic field is increased, the super conductivity coexists with magnetic field and this phase is known as mixed state(vortex state). At higher critical magnetic field  $H_{c2}$ , the penetration is complete and the material transforms to normal state. They possess higher critical magnetic fields. Their critical temperatures are high.

Ex:  $Nb_3Ge$ ,  $YBa_2Cu_3O_7$

7C

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Lead has superconducting transition temperature of 7.26 K. If the initial field at 0K is  $50 \times 10^3 \text{ Am}^{-1}$  Calculate the critical field at 6k.

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$$\begin{aligned}
 H_c &= H_o \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right] \\
 &= 50 \times 10^3 \left( 1 - \left( \frac{6}{7.26} \right)^2 \right) \\
 &= 15849 \text{ A/m}
 \end{aligned}$$

8A

**Fermi probability factor:** It represents the probability of occupation of an energy level.

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$$

To show that energy levels below Fermi energy are completely occupied:

For  $E < E_F$ , at  $T = 0$ ,

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1} = 1$$

To show that energy levels above Fermi energy are empty:

For  $E > E_F$ , at  $T=0$

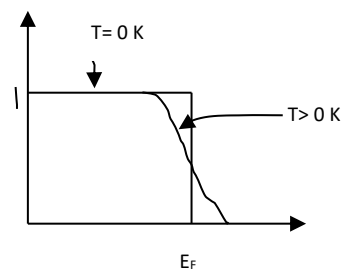
$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1} = 0$$

At ordinary temperatures, for  $E = E_F$ ,

$$f(E) = \frac{1}{2}$$

Fermi energy for  $T > 0$  K,

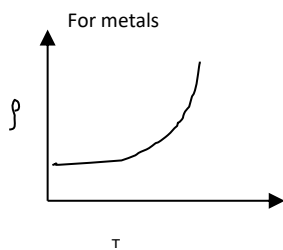
$$E_f = E_{f0} \left[ 1 - \frac{\pi^2}{12} \left( \frac{kT}{E_F} \right)^2 \right]$$



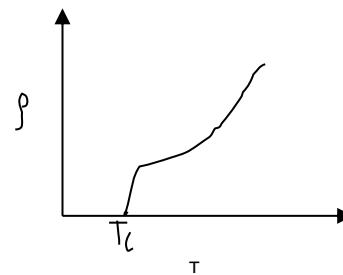
## SUPER CONDUCTIVITY:[KAMERLINGH IN 1914]

It is a phenomenon in which some materials lose their resistance completely below certain temperature.

### Temperature dependence of resistivity:

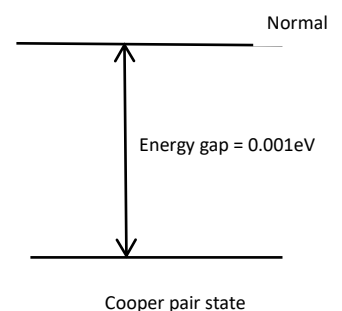
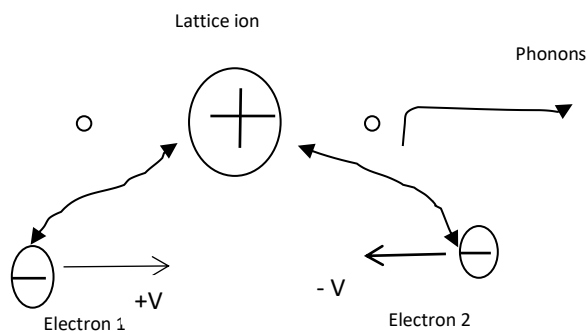


For superconductors



### BCS Theory :[Bardeen , Cooper, Schrieffer]

1. When the temperature of the material is reduced below critical temperature, electrons attain lower energy state than the normal energy creating an energy gap of few milli electron volt.
2. Positively charged lattice ion attracts a pair of electrons with equal and opposite spin and momentum through a feeble attractive interaction known as electron-lattice-electron interaction constituting cooper pairs.
3. Cooper pairs interact through exchanging Phonons.
4. All the cooper pairs are in same energy state and possess common wavefunction and Energy.
5. When a potential difference applied, the current is constituted by flow of cooper pairs and are not scattered as the energy required to break it up is large enough. This reduces the resistance.



6. When the temperature / magnetic field is increased beyond critical limit, cooper pairs breakup and normal state is restored.



8C

$$f(E) = \frac{1}{e^{\frac{E-E_f}{kT}} + 1} = \frac{1}{e^{\frac{0.2 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 200}} + 1} = \frac{1}{2208 + 1} = 0.000009$$

9A

Timing is the choice of positioning an event in the time duration of an animation.

Timing of action refers to

Linear motion refers to motion in a straight line, always in the same direction. An object moving with linear motion might speed up or slow down as it follows a linear path. A heavy ball rolling on a table or incline is an example of linear motion. The ball is rotating, but its center of gravity follows a linear path. A heavy ball rolling on a table or incline is an example of linear motion. The ball is rotating, but its center of gravity follows a linear path.

When uniform motion occurs, the net force on the object is zero. Net force is the total of all forces added up. There might be several forces acting on the object, but when both the magnitude and direction of the forces are added up, they add up to zero. Uniform motion is a type of linear motion with constant speed and no acceleration or deceleration. The object moves the same distance between consecutive frames. The longer the distance between frames, the higher the speed.

When motion is accelerating or decelerating, we refer to this type of motion as a slow in or slow out. This type of motion is sometimes called ease in or ease out. In this book, we use the hyphenated forms slow-in and slow-out for easier understanding.

1. Slow in, ease in—The object is slowing down, often in preparation for stopping.
2. Slow out, ease out—The object is speeding up, often from a still position.

The term slow out can be confusing, since it essentially means “speed up.” one can think of slow out as the same as ease out, as in easing out of a still position and speeding up to full speed.

For example, a ball rolling down an incline or dropping straight down is slowing out, as it goes from a still position or slow speed to a fast speed. A ball rolling up an incline is slowing in.

9B

The field of statistics is divided into two major divisions: descriptive and inferential. Each of these segments is important, offering different techniques that accomplish different objectives. Descriptive statistics describe what is going on in a population or data set. Inferential statistics, by contrast, allow scientists to take findings from a sample group and generalize them to a larger population. The two types of statistics have some important differences.

**Descriptive Statistics**

Descriptive statistics is the type of statistics that probably springs to most people’s minds when they hear the word “statistics.” In this branch of statistics, the goal is to describe. Numerical measures are

used to tell about features of a set of data. There are a number of items that belong in this portion of statistics, such as:

- The average, or measure of the center of a data set, consisting of the mean, median, mode, or midrange
- The spread of a data set, which can be measured with the range or standard deviation
- Overall descriptions of data such as the five number summary
- Measurements such as skewness and kurtosis
- The exploration of relationships and correlation between paired data
- The presentation of statistical results in graphical form

These measures are important and useful because they allow scientists to see patterns among data, and thus to make sense of that data. Descriptive statistics can only be used to describe the population or data set under study: The results cannot be generalized to any other group or population.

#### Types of Descriptive Statistics

There are two kinds of descriptive statistics that social scientists use:

Measures of central tendency capture general trends within the data and are calculated and expressed as the mean, median, and mode. A mean tells scientists the mathematical average of all of a data set, such as the average age at first marriage; the median represents the middle of the data distribution, like the age that sits in the middle of the range of ages at which people first marry; and, the mode might be the most common age at which people first marry.

Measures of spread describe how the data are distributed and relate to each other, including:

- The range, the entire range of values present in a data set
- The frequency distribution, which defines how many times a particular value occurs within a data set
- Quartiles, subgroups formed within a data set when all values are divided into four equal parts across the range
- Mean absolute deviation, the average of how much each value deviates from the mean
- Variance, which illustrates how much of a spread exists in the data
- Standard deviation, which illustrates the spread of data relative to the mean

Measures of spread are often visually represented in tables, pie and bar charts, and histograms to aid in the understanding of the trends within the data.

#### Inferential Statistics

Inferential statistics are produced through complex mathematical calculations that allow scientists to infer trends about a larger population based on a study of a sample taken from it. Scientists use inferential statistics to examine the relationships between variables within a sample and then make generalizations or predictions about how those variables will relate to a larger population.

It is usually impossible to examine each member of the population individually. So scientists choose a representative subset of the population, called a statistical sample, and from this analysis, they are able to say something about the population from which the sample came. There are two major divisions of inferential statistics:

- A confidence interval gives a range of values for an unknown parameter of the population by measuring a statistical sample. This is expressed in terms of an interval and the degree of confidence that the parameter is within the interval.
- Tests of significance or hypothesis testing where scientists make a claim about the population by analyzing a statistical sample. By design, there is some uncertainty in this process. This can be expressed in terms of a level of significance.

Techniques that social scientists use to examine the relationships between variables, and thereby to create inferential statistics, include linear regression analyses, logistic regression analyses, ANOVA, correlation analyses, structural equation modeling, and survival analysis. When conducting research using inferential statistics, scientists conduct a test of significance to determine whether they can generalize their results to a larger population. Common tests of significance include the chi-square and t-test. These tell scientists the probability that the results of their analysis of the sample are representative of the population as a whole.

9C

$$\text{Angle of acceptance} = \tan^{-1}(R/D) = \tan^{-1}\left(\frac{0.004}{.031}\right) = 6.6 \text{ degree}$$

$$\text{numerical aperture} = \sin \theta_A = 0.1$$

10A

### **Jumping**

A jump is an action where the character's entire body is in the air, and both the character's feet leave the ground at roughly the same time. A jump action includes a takeoff, free movement through the air, and a landing.

### **Parts of Jump**

A jump can be divided into several distinct parts.

**Crouch**—A squatting pose taken as preparation for jumping.

**Takeoff**—Character pushes up fast and straightens legs with feet still on the ground. The distance from the character's center of gravity (CG) in the crouch to the CG when the character's feet are just about to leave the ground is called the push height. The amount of time (or number of frames) needed for the push is called the push time.

### **In the air**

—Both the character's feet are off the ground, and the character's center of gravity (CG) moves in a parabolic arc as any free-falling body would. First it reaches an apex, and then falls back to the ground at the same rate at which it rose. The height to which the character jumps, called the jump height, is measured from the CG at takeoff to the CG at the apex of the jump. The amount of time the character is in the air from takeoff to apex is called the jump time. If the takeoff pose and the landing pose are similar, then the jump height and jump time are about the same going up as they are going down.

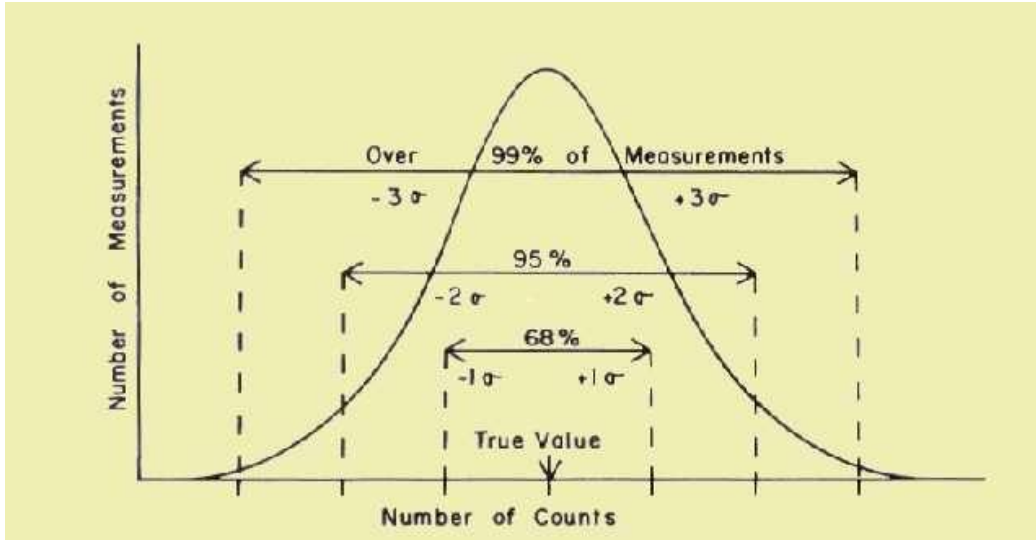
**Landing** Character touches the ground and bends knees to return to a crouch. The distance from the character's CG when her feet hit the ground to the point where the character stops crouching is called the stop height. The stop height is not always exactly the same as the push height.

10B

### **Normal Distribution and Bell Curves**

A bell curve is a common type of distribution for a variable, also known as the normal distribution. The term "bell curve" originates from the fact that the graph used to depict a Normal Distribution consists of a symmetrical bell-shaped curve.

The highest point on the curve, or the top of the bell, represents the most probable event in a series of data (its Mean, Mode and Median in this case), while all other possible occurrences are symmetrically distributed around the mean, creating a downward-sloping curve on each side of the peak. The width of the bell curve is described by its Standard Deviation.



The term "bell curve" is used to describe a graphical depiction of a normal probability distribution, whose underlying standard deviations from the mean create the curved bell shape. A standard deviation is a measurement used to quantify the variability of data dispersion, in a set of given values around the mean. The mean, in turn, refers to the average of all data points in the data set or sequence and will be found at the highest point on the bell curve.

10C

$$\text{Base distance} = \text{Total distance} / (\text{Last frame no} - 1)^2 = 25 / [6 - 1]^2 = 1\text{m}$$

Frame No	Frame multiplier	Distance covered in Nth frame	Distance covered TILL Nth frame
1	.....	base distance X frame multiplier	
2	1	1m	1
3	3	1 x 3=3m	4
4	5	1 x 5=5m	9
5	7	1 x 7=7m	16
6	9	1 x 9=9 m	25