CBCS SCHEME

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BMATS101

First Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025

Mathematics – I for CSE Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Find the angle between the curves, $r = \dfrac{a}{1+\cos\theta}$ and $r = \dfrac{b}{1-\cos\theta}$.	6	L2	CO1
	b.	Find the pedal equations of the curve $r^m = a^m \cos(m\theta)$.	7	L2	CO1
	c.	Determine the radius of curvature of the curve $r^2 \sec(2\theta) = a^2$.	7	L2	CO1
		OR			
Q.2	a.	With usual notation prove that $\tan\phi = r\dfrac{d\theta}{dr}$.	8	L2	CO1
	b.	Show that tangents to the cardioid $r = a(1+\cos\theta)$ at the points $\theta = \dfrac{\pi}{3}$ and $\theta = \dfrac{2\pi}{3}$ are respectively parallel and perpendicular to the initial line.	7	L2	CO1
	c.	Using modern mathematical tool write a programme/code to plot $r = 2\lvert\cos 2\theta\rvert$.	5	L3	CO5
		Module – 2			
Q.3	a.	Expand $\sqrt{1+\sin 2x}$ as Maclaurin's series up to fourth degree terms.	6	L2	CO1
	b.	If $u = f(y-z, z-x, x-y)$, prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$.	7	L2	CO1
	c.	Compute $J = \dfrac{\partial(x,y,z)}{\partial(\rho,\phi,z)}$ for $x = \rho\cos\phi$, $y = \rho\sin\phi$ and $z = z$	7	L2	CO1
		OR			
Q.4	a.	If $u = e^{(ax+by)} f(ax-by)$, prove that $b\dfrac{\partial u}{\partial x} + a\dfrac{\partial u}{\partial y} = 2abu$.	8	L2	CO1
	b.	Prove that $x^2\dfrac{\partial u}{\partial x} + y^2\dfrac{\partial u}{\partial y} + z^2\dfrac{\partial u}{\partial z} = 0$ for $u = f\left(\dfrac{y-x}{xy}, \dfrac{z-x}{xz}\right)$	7	L2	CO1
	c.	Using modern mathematical tool write a programe/code to show that $u_{xx} + u_{yy} = 0$, given that $u = e^x(x\cos y - y\sin y)$.	5	L2	CO5
		Module – 3			
Q.5	a.	Solve $\left[y\left(1+\dfrac{1}{x}\right) + \cos y\right]dx + [x + \log x - x\sin y]dy = 0$	6	L2	CO2
	b.	Show that the curve $y^2 = 4a(x+a)$ is self-orthogonal.	7	L3	CO2
	c.	A 12-volts battery connected to a series circuit in which the inductance is $\dfrac{1}{2}$ henry and resistance is 10 ohms. Find the current 'i' if the initial current is zero.	7	L3	CO2

		OR			
Q.6	a.	Solve $x\frac{dy}{dx} + y = x^3y^6$.	6	L2	CO2
	b.	Find orthogonal trajectories of the family $r^n \cos n\theta = a^n$.	7	L3	CO2
	c.	Find the general solutions of the equations $(px - y)(py + x) = a^2P$ by reducing into Clairaut's form by taking $u = x^2$, $v = y^2$.	7	L2	CO2
		Module – 4			
Q.7	a.	Find remainder when $(349 \times 74 \times 36)$ is divided by 3.	6	L1	CO3
	b.	Solve linear Diophantine equations $13x + 17y = 5$.	7	L2	CO3
	c.	Solve the system of linear congruence $x \equiv 2 (\text{mod} 3)$, $x \equiv 3 (\text{mod} 5)$ and $x \equiv 2 (\text{mod} 7)$, using remainder theorem.	7	L2	CO3
		OR			
Q.8	a.	Find the last digit in 7^{126}.	6	L2	CO3
	b.	Solve $2x + 6y \equiv 1 (\text{mod} 7)$ $4x + 3y \equiv 2 (\text{mod} 7)$	7	L2	CO3
	c.	Find the remainder when 7^{121} is divisible by 13.	7	L2	CO3
		Module – 5			
Q.9	a.	Solve the system of equation by using Gauss-Jordan method. $x + y + z = 9$, $2x + y - z = 0$, $2x + 5y + 7z = 52$.	6	L2	CO4
	b.	For what values λ and μ the system of equations, $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ has (i) no solution (ii) a unique solution and (iii) Many solutions.	7	L2	CO4
	c.	Using power method, find the largest eigen value and corresponding vector of the matrix, $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.	7	L2	CO4
		OR			
Q.10	a.	Determine the rank of the matrix $A = \begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 92 & 93 & 94 & 95 & 96 \\ 93 & 94 & 95 & 96 & 97 \\ 94 & 95 & 96 & 97 & 98 \\ 95 & 96 & 97 & 98 & 99 \end{bmatrix}$.	8	L1	CO4
	b.	Using the Gauss-Seidel iteration method, solve the equation $27x + 6y - z = 85$, $6x + 15y + 2z = 72$, $x + y + 54z = 110$. Carry out four iterations.	7	L2	CO4
	c.	Using modern mathematical tool, write a program/code to find the largest eigen value of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ by power method.	5	L3	CO5

* * * * *

Q17. $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$

$r = a(1+\cos\theta)$

Taking log on both sides

$\log r = \log(a(1+\cos\theta))$

$\log r = \log a + \log(1+\cos\theta)$

differentiate with respect to θ, we get

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{a}.0 + \frac{1}{1+\cos\theta}(-\sin\theta)$$

$$\cot\phi_1 = \frac{-\sin\theta}{1+\cos\theta} \Rightarrow \cot\phi_1 = \frac{-\cancel{2}\sin\theta/2\,\cancel{\cos\theta/2}}{\cancel{2}\cancel{\cos^2}\theta/2}$$

$$\cot\phi_1 = -\tan\theta/2$$

$$\cancel{\cot}\,\phi_1 = \cancel{\cot}\left(\frac{\pi}{2}+\frac{\theta}{2}\right) \Rightarrow \boxed{\phi_1 = \frac{\pi}{2}+\frac{\theta}{2}}$$

$r = b(1-\cos\theta)$

Taking log on both sides.

$\log r = \log(b(1-\cos\theta))$

$\log r = \log b + \log(1-\cos\theta)$

Differentiate with respect to θ, we get,

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1-\cos\theta}\sin\theta$$

$$\cot\phi_2 = \frac{\cancel{2}\,\cancel{\sin\theta/2}\cos\theta/2}{\cancel{2}\,\cancel{\sin^2}\theta/2}$$

$$\cancel{\cot}\,\phi_2 = \cancel{\cot}\,\theta/2$$

$$\phi_2 = \theta/2$$

we know that $(\phi_1 - \phi_2) = \frac{\pi}{2} + \cancel{\frac{\theta}{2}} - \cancel{\frac{\theta}{2}} \Rightarrow \pi/2$

$\therefore$ The pair of orthogonal curves will intersect

(3) $r^m = a^m \cos n\theta$

Take log on both side

$\log r^m = \log(a^n \cos n\theta)$

$m \log r = \log a^m + \log \cos n\theta$

$m \log r = m \log a + \log \cos n\theta$

diff. w.r.t θ, we get

$$m \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta} (-\sin n\theta).m$$

$\cancel{r} \cot \phi = \frac{-\cancel{r} \sin n\theta}{\cos n\theta}$

$\cot \phi = -\tan n\theta$

$\cancel{\cot} \phi = \cancel{\cot}\left(\frac{\pi}{2} + n\theta\right) \Rightarrow \boxed{\phi = \frac{\pi}{2} + n\theta}$

w.K.T = $p = r \sin \theta$

$p = r \sin\left(\frac{\pi}{2} + n\theta\right)$

$\boxed{p = r \cos n\theta}$

w.K.T from given $r^n = a^n \cos n\theta$

$\cos n\theta = r^n / a^n$

$p = r \cdot \frac{r^n}{a^n}$

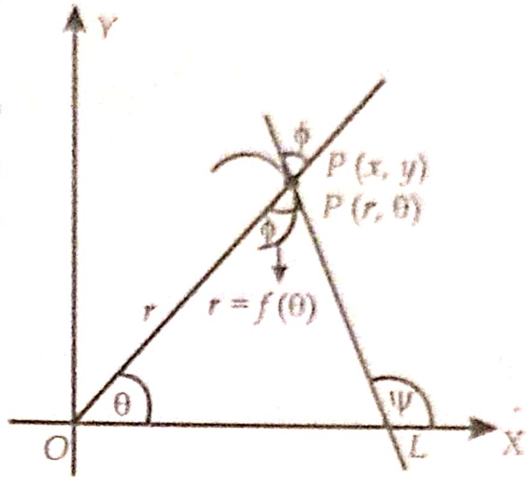
$\boxed{p a^n = r^{n+1}}$

1.21 Angle between radius vector and tangent

[Dec 18, 19, June 19, Sep 21, Apr 23]

Let $P(r, \theta)$ be any point on the curve $r = f(\theta)$.

$\therefore \quad X\hat{O}P = \theta$ and $OP = r$.



Let PL be the tangent to the curve at P subtending an angle ψ with the positive direction of the initial line (x–*axis*) and ϕ be the angle between the radius vector OP and the tangent PL. That is $O\hat{P}L = \phi$.

From the figure we have,

$$\psi = \phi + \theta$$

(*Recall from geometry that, an exterior angle is equal to the sum of the interior opposite angles*)

$$\Rightarrow \qquad \tan\psi = \tan(\phi + \theta)$$

or

$$\tan\psi = \frac{\tan\phi + \tan\theta}{1 - \tan\phi\tan\theta} \qquad \ldots (1)$$

Let (x, y) be the cartesian coordinates of P so that we have,

$$x = r\cos\theta, \quad y = r\sin\theta$$

Since r is a function of θ, we can as well regard these as parametric equations in terms of θ.

We also know from the geometrical meaning of the derivative that

$$\tan\psi = \frac{dy}{dx} = \text{slope of the tangent } PL$$

ie., $\tan\psi = \dfrac{dy}{d\theta} \Big/ \dfrac{dx}{d\theta}$ since x and y are functions of θ.

ie., $$\tan\psi = \frac{\dfrac{d}{d\theta}(r\sin\theta)}{\dfrac{d}{d\theta}(r\cos\theta)} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta} \quad \text{where } r' = \frac{dr}{d\theta}$$

(*We try to correlate this expression with the already existing expression for* $\tan\psi$ *in* (1). *Observe that the positive term in the denominator of* (1) *is equal to* 1)

Dividing both the numerator and denominator by $r'\cos\theta$ we have,

$$\tan\psi = \frac{\dfrac{r\cos\theta}{r'\cos\theta} + \dfrac{r'\sin\theta}{r'\cos\theta}}{\dfrac{-r\sin\theta}{r'\cos\theta} + \dfrac{r'\cos\theta}{r'\cos\theta}}$$

ie.,

$$\tan\psi = \frac{\dfrac{r}{r'} + \tan\theta}{1 - \dfrac{r}{r'}\cdot\tan\theta} \qquad \ldots (2)$$

Comparing equations (1) and (2) we have,

$$\tan\phi = \frac{r}{r'} = \frac{r}{\left(\dfrac{dr}{d\theta}\right)} \quad \text{or} \quad \boxed{\tan\phi = r\left(\frac{d\theta}{dr}\right)}$$

Equivalently we can write in the form,

$$\frac{1}{\tan\phi} = \frac{1}{r}\left(\frac{dr}{d\theta}\right) \quad \text{or} \quad \boxed{\cot\phi = \frac{1}{r}\left(\frac{dr}{d\theta}\right)}$$

Note : *A question format :- Prove with usual notations,* $\tan\phi = r\dfrac{d\theta}{dr}$

1.22 Length of the perpendicul

2.b $r = a(1+\cos\theta) \rightarrow ①$

W.K.T $\frac{dy}{dx} = \frac{r'(\theta)\sin\theta + r(\theta)\cos\theta}{r'(\theta)\cos\theta - r(\theta)\sin\theta}$

diff Eq ① w.r.t "θ"

$r'(\theta) = \frac{dr}{d\theta} = -a\sin\theta$

at $\theta = \pi/3$

$r = a(1+\cos\theta)$

$= a(1+\cos(\pi/3))$

$= a(1+1/2)$

$r = \frac{3a}{2}$

$r'(\theta) = -a\sin\theta$

$r'(\pi/3) = -\frac{\sqrt{3}a}{2}$

$$\frac{dy}{dx} = \frac{-a\times\frac{\sqrt{3}}{2}\sin\pi/3 + \frac{3a}{2}\cos\pi/3}{-a\times\frac{\sqrt{3}}{2}\cos\pi/3 - \frac{3a}{2}\sin\pi/3} = 0$$

Since the slope at $\theta = \pi/3$ is 0, the tangent is parallel to the initial line

Now at $\theta = \frac{2\pi}{3}$

$r = a(1+\cos\theta)$

$r = a(1+\cos(2\pi/3))$

$r = \frac{a}{2}$

$r'(\theta) = -a\sin\theta$

$r'(2\pi/3) = -\frac{a\sqrt{3}}{2}$

$$\frac{dy}{dx} = \frac{-a\times\sqrt{3}/2\sin 2\pi/3 + a/2\cos 2\pi/3}{-a\times\sqrt{3}/2\cos 2\pi/3 - a/2\sin 2\pi/3} =$$

$$\frac{dy}{dx} = \infty$$

The slope is infinite, meaning the tangent is perpendicular to the initial line

$\therefore$ the tangents to the Cardioid $r = a(1+\cos\theta)$ at $\theta = \pi/3$ and $\theta = 2\pi/3$ are parallel and perpendi-cular to the initial line.

Four leaved Rose: $r = 2|\cos 2\theta|$

```
from pylab import *
theta = linspace (0, 2* pi, 1000)
r = 2 * abs (cos (2* theta))
polar (theta, r, 'r')
show()
```

[6] *Using Maclaurin's series, prove that* $\sqrt{1+\sin 2x} = 1+x-\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}\cdots$

[June, Dec 18, Sep 20, 21, Apr 23]

☞ We have, $y(x) = y(0)+xy_1(0)+\frac{x^2}{2!}y_2(0)+\ldots$

Let, $y = \sqrt{1+\sin 2x} = \sqrt{\cos^2 x+\sin^2 x+2\sin x\cos x}$

$= \sqrt{(\cos x+\sin x)^2} = \cos x+\sin x$

Hence, $y = \cos x + \sin x$ $\quad\therefore y(0) = 1$

$y_1 = -\sin x + \cos x$ $\quad\therefore y_1(0) = 1$

$y_2 = -\cos x - \sin x = -y$ *ie.,* $y_2 = -y$ $\quad\therefore y_2(0) = -1$

$\therefore$ $y_3 = -y_1$ $\quad\therefore y_3(0) = -1$

$y_4 = -y_2$ $\quad\therefore y_4(0) = 1$

Thus by substituting these values in the expansion of $y(x)$ we get,

$$\boxed{\sqrt{1+\sin 2x} = 1+x-\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}\cdots}$$

Remark : *We have used a technique to simplify the given function leading to a very simple form, there by the problem is completed easily.*

7m/8m Problem:-

1) If $u = f(x-y, y-z, z-x)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

Soln:- $r = x-y$ $\quad s = y-z$ $\quad t = z-x$.

$\frac{\partial r}{\partial x} = 1$ $\quad \frac{\partial s}{\partial x} = 0$ $\quad \frac{\partial t}{\partial x} = -1$

$\frac{\partial r}{\partial y} = -1$ $\quad \frac{\partial s}{\partial y} = 1$ $\quad \frac{\partial t}{\partial y} = 0$

$\frac{\partial r}{\partial z} = 0$ $\quad \frac{\partial s}{\partial z} = -1$ $\quad \frac{\partial t}{\partial z} = 1$

$\rightarrow \frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial x}$

$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}(1) + \frac{\partial u}{\partial s}(0) + \frac{\partial u}{\partial t}(-1)$

$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} - \frac{\partial u}{\partial t} \rightarrow (1)$

$\rightarrow \frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial y}$

$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}(-1) + \frac{\partial u}{\partial s}(1) + \frac{\partial u}{\partial t}(0)$

$$\frac{\partial u}{\partial y} = -\frac{\partial u}{\partial r} + \frac{\partial u}{\partial s} \rightarrow (2)$$

$$\rightarrow \frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial z} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial z} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial z}$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}(0) + \frac{\partial u}{\partial s}(-1) + \frac{\partial u}{\partial t}(1)$$

$$\frac{\partial u}{\partial z} = -\frac{\partial u}{\partial s} + \frac{\partial u}{\partial t} \rightarrow (3)$$

substitute Eq^n (1), (2), (3) in the problem

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

$$\cancel{\frac{\partial u}{\partial r}} - \cancel{\frac{\partial u}{\partial t}} - \cancel{\frac{\partial u}{\partial r}} + \cancel{\frac{\partial u}{\partial s}} - \cancel{\frac{\partial u}{\partial s}} + \cancel{\frac{\partial u}{\partial t}} = 0$$

$$0 = 0.$$

C.

Sol: $x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$

$\frac{\partial x}{\partial \rho} = \cos\phi$ $\quad \frac{\partial y}{\partial \rho} = \sin\phi$ $\quad \frac{\partial z}{\partial \rho} = 0$

$\frac{\partial x}{\partial \phi} = -\rho\sin\phi$ $\quad \frac{\partial y}{\partial \phi} = \rho\cos\phi$ $\quad \frac{\partial z}{\partial \phi} = 0$

$\frac{\partial x}{\partial z} = 0$ $\quad \frac{\partial y}{\partial z} = 0$ $\quad \frac{\partial z}{\partial z} = 1$

$$J = \frac{\partial(x, y, z)}{\partial(\rho, \phi, z)} = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial \rho} & \frac{\partial z}{\partial \phi} & \frac{\partial z}{\partial z} \end{vmatrix}$$

$$= \begin{vmatrix} \cos\phi & -\rho\sin\phi & 0 \\ \sin\phi & \rho\cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \cos\phi\,[\rho\cos\phi] + \rho\sin\phi\,[\sin\phi]$$

$$= \rho\cos^2\phi + \rho\sin^2\phi$$

$$= \rho\,[\sin^2\phi + \cos^2\phi]$$

$$\boxed{J = \rho}$$

imp

② If $u = e^{ax+by} f(ax-by)$ prove that $b \cdot \frac{\partial u}{\partial x} + a \cdot \frac{\partial u}{\partial y} = 2abu$.

Sol^n:- $u = e^{ax+by} f(ax-by)$

partial diff. w.r.t 'x'

$$\frac{\partial u}{\partial x} = e^{ax+by} f'(ax-by) \cdot a + f(ax-by) e^{ax+by} \cdot a$$

$$\boxed{\text{partial diff. w.r.t 'y'} \quad \frac{\partial u}{\partial y} = e^{ax+by}}$$

$$\frac{\partial u}{\partial x} = e^{ax+by} f(ax-by) a + au \rightarrow ①$$

partial diff. w.r.t y

$$\frac{\partial u}{\partial y} = e^{ax+by} f'(ax-by) \cdot (-b) + f(ax-by) \cdot e^{ax+by} \cdot b$$

$$\frac{\partial u}{\partial y} = -b e^{ax+by} \cdot f'(ax-by) + ub \rightarrow ②$$

Substitute Eq^n ① and ② in the given problem

$$b \cdot \frac{\partial u}{\partial x} + a \frac{\partial u}{\partial y} = 2abu.$$

$$= b \cdot \left[e^{ax+by} f'(ax-by) a + au\right] + a\left[\left[-b e^{ax+by} \cdot f'(ax-by) + bu\right] = 2abu\right.$$

$$= b e^{ax+by} \cdot \cancel{f'(ax-by)} a + bau - ab e^{ax+by} \cdot \cancel{f'(ax-by)} + abu = 2abu$$

$$= abu + abu = 2abu$$

$$= 2abu = 2abu.$$

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4.b $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0 \rightarrow (1)$

$u = f\left[\frac{y-x}{xy}, \frac{z-x}{xz}\right]$

$u \rightarrow f(r, s, t) \rightarrow (x, y, z)$

$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial x}$

$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial y}$

$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial z} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial z} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial z}$

$r = \frac{y-x}{xy}, \quad s = \frac{z-x}{xz}, \quad t = 0$

$\cancel{\frac{\partial r}{\partial x} =} \quad r = \frac{y}{xy} - \frac{x}{xy} \quad s = \frac{z}{xz} - \frac{x}{xz}$

$r = \frac{1}{x} - \frac{1}{y}, \quad s = \frac{1}{x} - \frac{1}{z}$

$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\left[\frac{-1}{x^2}\right] + \frac{\partial u}{\partial s}\left[\frac{-1}{x^2}\right] + \frac{\partial u}{\partial t}[0]$

$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\left[\frac{1}{y^2}\right] + \frac{\partial u}{\partial s}[0] + \frac{\partial u}{\partial t}[0]$

$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}[0] + \frac{\partial u}{\partial s}\left[\frac{1}{z^2}\right] + \frac{\partial u}{\partial t}[0]$

Eq ① $\Rightarrow$

$$x^2\left[-\frac{1}{x^2}\frac{\partial u}{\partial r}-\frac{1}{x^2}\frac{\partial u}{\partial s}\right]+y^2\left[\frac{1}{y^2}\frac{\partial u}{\partial r}\right]+z^2\left[\frac{1}{z^2}\frac{\partial u}{\partial s}\right]=0$$

$$-\cancel{\frac{\partial u}{\partial r}}-\cancel{\frac{\partial u}{\partial s}}+\cancel{\frac{\partial u}{\partial r}}+\cancel{\frac{\partial u}{\partial s}}=0$$

$$=0$$

LAB 6: Finding partial derivatives and Jacobian of functions of several variables.

1. Prove that mixed partial derivatives, $u_{xy} = u_{yx}$ for $u = exp(x)(x\cos(y) - y\sin(y))$.

```
from Sympy import *
x, y = Symbols ('x y')
u = exp (x) * (x * cos(y) - y * sin(y))
dux = diff (u, x)
duy = diff (u, y)
duxy = diff (dux, y)
duyx = diff (duy, x)
if duxy == duyx:
    print ('Mixed partial derivatives are equal')
else:
    print ('Mixed partial derivatives are not equal')
```

[16] *Solve* : $[y(1+1/x)+\cos y]dx+[x+\log x-x\sin y]dy = 0$ **[Jan 21]**

☞ Let $M = y(1+1/x)+\cos y$ and $N = x+\log x - x\sin y$

$\therefore \quad \dfrac{\partial M}{\partial y} = 1+1/x-\sin y$ and $\dfrac{\partial N}{\partial x} = 1+1/x-\sin y$

Since, $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$, the given equation is exact.

The solution is $\int M\,dx + \int N(y)\,dy = c$

ie., $\int[y(1+1/x)+\cos y]dx+\int 0\;dy = c$

Thus, $\boxed{y\,(x + \log x) + x\cos y = c}$ is the required solution.

[29] *Show that the family of parabolas* $y^2 = 4a(x+a)$ *is self orthogonal.*

[June, Dec 2018, Jan 21, Apr 23]

☞ Consider $y^2 = 4a(x+a)$ $\quad\ldots(1)$

Differentiating w.r.t x, we have,

$$2y\frac{dy}{dx} = 4a \quad \therefore a = \frac{yy_1}{2} \quad \text{where } y_1 = \frac{dy}{dx}$$

Substituting this value of 'a' in (1) we have,

$$y^2 = 2yy_1\left(x + \frac{yy_1}{2}\right)$$

or, $$y = 2xy_1 + yy_1^2 \qquad \ldots(2)$$

This is the DE of the given family.

Now replacing y_1 by $-1/y_1$, (2) becomes,

$$y = 2x\left(\frac{-1}{y_1}\right) + y\left(\frac{-1}{y_1}\right)^2 \quad \text{or} \quad y = \frac{-2x}{y_1} + \frac{y}{y_1^2}$$

That is, $$yy_1^2 + 2xy_1 = y \qquad \ldots (3)$$

(3) is the DE of the orthogonal family which is same as (2) being the DE of the given family.

Thus the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal.

3. $x\frac{dy}{dx} + y = x^3y^6$

$\Rightarrow x\frac{dy}{dx} + y = x^3y^6 \longrightarrow (1)$

$\div$ (1) by xy^6

$$\frac{1}{y^6}\frac{dy}{dx} + \frac{1}{xy^5} = x^2 \longrightarrow (2)$$

Put $\frac{1}{y^5} = t$

$$\frac{-5}{y^6}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{1}{y^6}\frac{dy}{dx} = \frac{-1}{5}\frac{dt}{dx}$$

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eq (5) b/c →

$$\frac{-1}{5}\frac{dt}{dx} + \frac{t}{x} = x^2.$$

x'ly by (-5)

$$\frac{dt}{dx} - \frac{5t}{x} = -5x^2$$

$$P = \frac{-5}{x} \qquad Q = -5x^2.$$

$$I.f = e^{\int P dx} = e^{\int \frac{-5}{x} dx} = e^{-5\log x}$$

$$I.f = \frac{1}{x^5}.$$

$$t\, I.f = \int Q\, I.f\, dx + c$$

$$\left(\frac{1}{y^5}\right)\left(\frac{1}{x^5}\right) = \int -5\cancel{x^2} \frac{1}{\cancel{x^5}\,3} dx + c$$

$$\frac{1}{x^5y^5} = -5 \int \frac{1}{x^3} dx + c$$

$$\frac{1}{x^5y^5} = -5 \left\{\frac{x^{-3+1}}{-3+1}\right\} \cancel{dx} + c$$

$$\frac{1}{x^5y^5} = \cancel{-}5 \left(\frac{x^{-2}}{\cancel{-}2}\right) + c$$

$$\boxed{\frac{1}{x^5y^5} = \frac{5}{2x^2} + c}$$

6.c $(px-y)(py+x)=a^2p \rightarrow (1)$

$u=x^2, \quad v=y^2$

$u=x^2 \qquad \frac{dv}{dy}=2y$

$\frac{du}{dx}=2x$

$dx=\frac{du}{2x} \qquad dy=\frac{dv}{2y}$

$p=\frac{dy}{dx}=\frac{dv/2y}{du/2x}=\frac{dv}{2y}\times\frac{2x}{du}$

$p=\frac{x}{y}\frac{dv}{du}$

$p=\frac{x}{y}P$

$u=x^2 \qquad v=y^2$

$x=\sqrt{u} \qquad y=\sqrt{v}$

$p=\frac{\sqrt{u}}{\sqrt{v}}P$

$\left[\frac{\sqrt{u}}{\sqrt{v}}P\sqrt{u}-\sqrt{v}\right]\left[\frac{\sqrt{u}}{\cancel{\sqrt{v}}}P.\cancel{\sqrt{v}}+\sqrt{u}\right]=a^2\frac{\sqrt{u}}{\sqrt{v}}P$

$\left[\frac{uP-v}{\sqrt{v}}\right]\sqrt{u}\left[P+1\right]=a^2\frac{\sqrt{u}}{v}P$

$\frac{\cancel{\sqrt{u}}}{\cancel{\sqrt{v}}}\left[uP-v\right]\left[P+1\right]=a^2P\frac{\cancel{\sqrt{u}}}{\cancel{\sqrt{v}}}$

$(uP-v)(P+1)=a^2P$

$y=px+f(p)$

$uP-v=\frac{a^2P}{P+1}$

$v=uP-\frac{a^2P}{P+1} \Rightarrow \boxed{y^2=Px^2-\frac{a^2P}{P+1}}$

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18 $349 \times 74 \times 36$ - - - (3)

$\Rightarrow$

$$3\overline{)349}(116 \qquad 3\overline{)74}(24 \qquad 3\overline{)36}(12$$

349 ÷ 3: 3 | 4 | 3 | 19 | 18 | 1

74 ÷ 3: 6 | 14 | 12 | 2

36 ÷ 3: 36 | 0

$349 \equiv 1 \pmod 3$ $\quad 74 \equiv 2 \pmod 3$ $\quad 36 \equiv 0 \pmod 3$

$349 \times 74 \times 36 \equiv 1 \times 2 \times 0 \pmod 3$

$349 \times 74 \times 36 \equiv 0 \pmod 3$

Remainder = 0 //

* Problem : (CRT)

Imp! 1. Solve the problem by using CRT;

$x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$, $x \equiv 2 \pmod 7$

$\Rightarrow$ $x \equiv 2 \pmod 3$ $x \equiv 3 \pmod 5$ $x \equiv 2 \pmod 7$

$x = a_1 \pmod{m_1}$, $x \equiv a_2 \pmod{m_2}$, $x \equiv a_3 \pmod{m_3}$

$a_1 = 2$; $m_1 = 3$, $a_2 = 3$; $m_2 = 5$; $a_3 = 2$; $m_3 = 7$.

$X \equiv [a_1 M_1 M_1^{-1} + a_2 M_2 M_2^{-1} + a_3 M_3 M_3^{-1}] \bmod (M) \rightarrow (1)$

$M = m_1 \times m_2 \times m_3$

$M = 3 \times 5 \times 7$

$M = 105$ //

$M_1 = \frac{M}{m_1} = \frac{105}{3} = 35$

$M_2 = \frac{M}{m_2} = \frac{105}{5} = 21$

$M_3 = \frac{M}{m_3} = \frac{105}{7} = 15$

- $M_1 \times M_1^{-1} \equiv 1 \pmod{m_1}$

$35 \times M_1^{-1} \equiv 1 \pmod{3}$

$\therefore M_1^{-1} = 2$

- $M_2 \times M_2^{-1} \equiv 1 \pmod{m_2}$

$21 \times M_2^{-1} = 1 \pmod{5}$.

$\therefore M_2^{-1} = 1$

- $M_3 \times M_3^{-1} \equiv 1 \pmod{m_3}$

$15 \times M_3^{-1} \equiv 1 \pmod{7}$.

$M_3^{-1} = 1$

eq (1) becomes

$$X \equiv [2 \times \overset{2\,\times 4}{\underset{140}{35}} \times 2 + 3 \times 21 \times 1 + 2 \times 15 \times 1] \bmod (105)$$

$$X \equiv [140 + 63 + 30] \bmod (105)$$

$$X \equiv 233 \bmod (105)$$

$$\begin{array}{r} 2 \\ 105\overline{)\,233} \\ \underline{210} \\ 23 \end{array}$$

$$X \equiv 23 \bmod (105)$$

(ii) $7^2 = 49 \equiv 9 \pmod{10}$

$\Rightarrow \quad (7^2)^2 \equiv 81 \pmod{10}$ or $7^4 \equiv 1 \pmod{10}$

$\therefore \quad (7^4)^{31} = 1^{31} \pmod{10}$ or $7^{124} \equiv 1 \pmod{10}$

Now, $7^2 \times 7^{124} \equiv 9 \pmod{10}$ or $7^{126} \equiv 9 \pmod{10}$

Thus the last digit in 7^{126} is 9.

* Problem :

1. Solve the system of eq^n :

$$2x + 6y \equiv 1 \pmod 7 \quad \& \quad 4x + 3y \equiv 2 \pmod 7$$

① ②

$\Rightarrow \quad ax + by \equiv r \pmod m \qquad cx + dy \equiv s \pmod m$

$\therefore \quad a = 2, \; b = 6, \; c = 4, \; d = 3, \; m = 7.$

$\gcd(a, b, m) / c$

$\rightarrow \gcd(a, b, m) \Rightarrow \gcd(2, 6, 7).$ [largest & smallest no/.]

$$
\begin{array}{l}
2\,)\,7\,(\,3 \\
6 \\
\hline
1\,)\,2\,(\,2 \\
2 \\
\hline
0
\end{array}
$$

$\therefore \quad \gcd(2, 6, 7) = 1.$

$\gcd(a,b,m)/c \Rightarrow \gcd(2, 6, 7)/4 = 1/4$

$\gcd(ad - bc, m) = 1$

$\gcd(6 - 24, 7) = 1.$

$\gcd(-18, 7) = 1$

$\therefore \gcd = 1$

$1 = 1.$

$7)18(2$
14
$4)7(1$
4
$3)4(1$
3
$1)3(3$
3
0

$2x + 6y \equiv 1 \pmod 7 \times 2 \rightarrow (1)$

$4x + 3y \equiv 2 \pmod 7 \times 1$

$\rightarrow \cancel{4x} + 12y \equiv 2 \pmod 7$

$\quad \cancel{4x} + 3y \equiv 2 \pmod 7$

$\quad - \quad - \quad -$

$9y \equiv 0 \pmod 7$

$9y = 0$

$\boxed{y = 0}$

$\boxed{\therefore y \equiv 0 \pmod 7}$

Sub $y = 0$ in (1)

$2x \equiv 1 \pmod 7$

$2x - 1 = 7k.$

$x = \dfrac{7k+1}{2}$

$k = 1, \ \boxed{x = 4}$

$\boxed{x \equiv 4 \pmod 7}$

① find the remainder by using fermat's little theorem $7^{121} \pmod{13}$.

sol:- $7^{121} \pmod{13}$.

$a = 7$, $p = 13$

$a^{p-1} \equiv 1 \pmod{p}$

$a^{13-1} \equiv 1 \pmod{13}$

$7^{12} \equiv 1 \pmod{13}$

$(7^{12})^{10} \equiv 1^{10} \pmod{13}$

$7^{120} \equiv 1 \pmod{13}$

multiply 7^1

$7^{120} \cdot 7^1 \equiv 1 \cdot 7^1 \pmod{13}$

$7^{121} \equiv 7 \pmod{13}$

remainder = 7

$12 \overline{)\,121\,}(\,10$

$\underline{120}$

01

$$a_1''x = d_1'' \; ; \; b_2''y = d_2'' \; ; \; c_3''z = d_3''$$

Problems

Solve the foll. system of eq^n by gauss jordan method:

1. $$x + y + z = 9$$
$$2x + y - z = 0$$
$$2x + 5y + 7z = 52$$

$$[A \,;\, B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & 1 & -1 & 0 \\ 2 & 5 & 7 & 52 \end{array}\right] \qquad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\begin{array}{r} 52 \\ -18 \\ \hline 34 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 3 & 5 & 34 \end{array}\right] \qquad \begin{array}{l} R_1 \to R_1 + R_2 \\ R_3 \to R_3 + 3R_2 \end{array}$$

$34 - 54$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -2 & -9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array}\right] \qquad R_3 \to \frac{R_3}{-4}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & -9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & 1 & 5 \end{array}\right] \qquad \begin{array}{l} R_1 \to R_1 + 2R_3 \\ R_2 \to R_2 + 3R_3 \end{array}$$

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$$[A ; B] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & +1 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$x = 1 \quad ; \quad -y = -3 \quad ; \quad z = 5$

$x = 1 \quad ; \quad y = 3 \quad ; \quad z = 5$

→ Investigate the values of λ & μ such that of Equation or for what values of λ & μ it satisfies the following conditions:-

1 Imp

$x+y+z=6$
$x+2y+3z=10$
$x+2y+\lambda z=\mu$

1. unique solutions or consistent
2. Infinite solution
3. No solution Or inconsistent

Soln:- $[A;B] = \begin{bmatrix} 1 & 1 & 1 & ; 6 \\ 1 & 2 & 3 & ; 10 \\ 1 & 2 & \lambda & ; \mu \end{bmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$[A;B] = \begin{bmatrix} 1 & 1 & 1 & ; 6 \\ 0 & 1 & 2 & ; 4 \\ 0 & 1 & \lambda-1 & ; \mu-6 \end{bmatrix}$

$R_3 \to R_3 - R_2$

$[A;B] = \begin{bmatrix} 1 & 1 & 1 & ; 6 \\ 0 & 1 & 2 & ; 4 \\ 0 & 0 & \lambda-3 & ; \mu-10 \end{bmatrix}$

1. for unique solutions :- $\rho[A;B] = \rho[A] =$ No. of. unknowns
$\lambda \neq 3 \cdot \mu \neq 10$

(2) for infinite solutions $\rho[A;B] = \rho[A] <$ No. of unknowns

$\lambda = 3$, $\mu = 10$

(3) for No solutions $\rho[A;B] \neq \rho[A]$

$\lambda = 3$; $\mu \neq 10$.

$$AX^{(0)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4 \end{bmatrix} = 6\begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \lambda^{(1)} X^{(1)}$$

$$AX^{(1)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 7.34 \\ -2.67 \\ 4.01 \end{bmatrix} = 7.34\begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \lambda^{(2)} X^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \begin{bmatrix} 7.82 \\ -3.63 \\ 4.01 \end{bmatrix} = 7.82\begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \lambda^{(3)} X^{(3)}$$

$$AX^{(3)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \begin{bmatrix} 7.94 \\ -3.89 \\ 3.99 \end{bmatrix} = 7.94\begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \lambda^{(4)} X^{(4)}$$

$$AX^{(4)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 7.98 \\ -3.97 \\ 3.99 \end{bmatrix} = 7.98\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \lambda^{(5)} X^{(5)}$$

$$AX^{(5)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 4 \end{bmatrix} = 8\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \lambda^{(6)} X^{(6)}$$

Thus the dominant eigen value is 8 and the corresponding eigen vector is **[1, –0.5, 0.5]′ or [2, –1, 1]′ equivalently.**

10. a

$$A = \begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 92 & 93 & 94 & 95 & 96 \\ 93 & 94 & 95 & 96 & 97 \\ 94 & 95 & 96 & 97 & 98 \\ 95 & 96 & 97 & 98 & 99 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, $R_4 \to R_4 - R_1$

$R_5 \to R_5 - R_1$

$$A = \begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix}$$

$R_2 \to \frac{R_2}{2}$, $R_3 \to \frac{R_3}{3}$, $R_4 \to \frac{R_4}{4}$

$$A = \begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$\to 91R_2 - R_1$

$R_2 \to 91R_2 - R_1$, $R_3 \to 91R_3 - R_1$, $R_4 \to 91R_4 - R_1$

$R_5 \to 91R_5 - R_1$

$$A = \begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -1 & -2 & -3 & -4 \end{bmatrix}$$

$R_5 \rightarrow R_5 - R_4$

$$A = \begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_4 \rightarrow R_4 - R_3$

$$A = \begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$A = \begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \rightarrow \dfrac{R_1}{91}, \quad R_2 \rightarrow R_2 \times -1$

$$A = \begin{bmatrix} 1 & 92/91 & 93/91 & 94/91 & 95/91 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho[A] = 2$

$\Rightarrow \quad x = \frac{1}{27}[85 - 6y + z]$

$y = \frac{1}{15}[72 - 6x - 2z]$

$z = \frac{1}{54}[110 - x - y]$

I :

$x = 0, \quad y = 0, \quad z = 0$

$x^{(1)} = \frac{1}{27}[85 - 6(0) + 0]$

$x^{(1)} = 3.148$

$y^{(1)} = \frac{1}{15}(72 - 6(3.148) - 2(0)]$

$y^{(1)} = 3.5408$

$z^{(1)} = \frac{1}{54}[110 - 3.148 - 3.5408]$

$z^{(1)} = 1.9131$

II

$x^{(1)} = 3.148, \quad y^{(1)} = 3.5408 \quad z^{(1)} = 1.9131$

$x^{(2)} = \frac{1}{27}[85 - 6(3.5408) + 1.9131]$

$x^{(2)} = 2.4321$

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$$y^{(2)} = \frac{1}{15}\left[72 - (6 \times 2.4321) - (2 \times 1.9137)\right]$$

$$y^{(2)} = 3.5720.$$

$$z^{(2)} = \frac{1}{54}\left[110 - 2.4321 - 3.5720\right]$$

$$z^{(2)} = 1.9259$$

III : $x^{(2)} = 2.4321 \quad y^{(2)} = 3.5720 \; , \; z^{(2)} = 1.9259$

$$x^{(3)} = \frac{1}{27}\left[85 - 6(3.5720) + 1.9259\right]$$

$$x^{(3)} = 2.4257$$

$$y^{(3)} = \frac{1}{15}\left[72 - 6(2.4257) - (2 \times 1.9259)\right]$$

$$y^{(3)} = 3.5729.$$

$$z^{(3)} = \frac{1}{54}\left[110 - 2.4257 - 3.5729\right]$$

$$z^{(3)} = 1.9260$$

$\therefore \; x = 2.426 \; , \; y = 3.573 \; ; \; z = 1.926$

```
import numpy as np
I = np.array([[1,1,3],[1,5,1],[3,1,1]])
print("\n given matrix :\n", I)
w,v = np.linalg.eig(I)
print(" \n eigen values :\n", w)
print(" \n eigen vectors : \n", v)
print(" Eigen value : \n", w[0])
print(" \n Corresponding Eigen vector: ", v[:,0])
```