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Discrete Mathematics and Graph Theory Sub Code: MMC102 Br					Bra	nch:	MCA	1				
05/02/2025	Duration: 9	0 minutes	Max Marks:	50	Sem / Sec:	I A	А&В		OBE			
Note: Answer	r FIVE FULL			ull qu	estion from	each part.		MAI	RKS	CO	RBT	
		singleton set	t with an exampl				9},	[1	0]	CO1	L1	
For any two sets A and B, state and prove De-Morgan's law. [10]						0]	CO1	L3				
PART II A total of 1232 students have taken a course in Java, 879 in C and 114 in C++. Further, 103 have taken courses in both Java and C, 23 in both Java and C++ and 14 in both C and C++. If 2092 students have taken at least one of Java, C and C++, how many students have taken a course in all three subjects?					CO1	L3						
ind the eigen va	lues and eigen	vectors of th	-	²).				[1	0]	CO1	L3	
elect 5 points in	side the triangl	C is an equil	t at least two of					[1	0]	CO4	L3	
	Note: Answer efine a set, an e ={1,2,4,6,8}, B or any two sets total of 1232 s ken courses in udents have tak ree subjects? and the eigen va cate Pigeon-hole lect 5 points in	Note: Answer FIVE FULL efine a set, an empty set and a ={1,2,4,6,8}, B={2,4,5,9}. Co or any two sets A and B, state a total of 1232 students have ta ken courses in both Java and udents have taken at least one ree subjects? Ind the eigen values and eigen cate Pigeon-hole Principle. AB	Note: Answer FIVE FULL Questions, or \underline{P}_{i} efine a set, an empty set and a singleton set ={1,2,4,6,8}, B={2,4,5,9}. Compute(<i>i</i>) <i>A</i> or any two sets A and B, state and prove De total of 1232 students have taken a course ken courses in both Java and C, 23 in bo udents have taken at least one of Java, C a ree subjects? and the eigen values and eigen vectors of th eate Pigeon-hole Principle. ABC is an equil- lect 5 points inside the triangle, prove tha	Note: Answer FIVE FULL Questions, choosing ONE f PART I efine a set, an empty set and a singleton set with an exampl ={1,2,4,6,8}, B={2,4,5,9}. Compute(i) $A \cup B$ (ii) $A \cap A$ OR or any two sets A and B, state and prove De-Morgan's law. PART II total of 1232 students have taken a course in Java, 879 in ken courses in both Java and C, 23 in both Java and C+ udents have taken at least one of Java, C and C++, how m ree subjects? OR and the eigen values and eigen vectors of the matrix $\begin{pmatrix} 3\\ -1 \end{pmatrix}$ PART III tate Pigeon-hole Principle. ABC is an equilateral triangle w elect 5 points inside the triangle, prove that at least two of	Note: Answer FIVE FULL Questions, choosing ONE full questions, choosing ONE full questions, choosing ONE full questions of the set	Note: Answer FIVE FULL Questions, choosing ONE full question from $(PART I)$ efine a set, an empty set and a singleton set with an example for each. Let U={ ={1,2,4,6,8}, B={2,4,5,9}. Compute(<i>i</i>) $A \cup B$ (<i>ii</i>) $A \cap B$ (<i>iii</i>) $A - B$ (OR or any two sets A and B, state and prove De-Morgan's law. PART II total of 1232 students have taken a course in Java, 879 in C and 114 in C+- ken courses in both Java and C, 23 in both Java and C++ and 14 in both C udents have taken at least one of Java, C and C++, how many students have t ree subjects? OR and the eigen values and eigen vectors of the matrix $\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$. PART III ate Pigeon-hole Principle. ABC is an equilateral triangle whose sides are of le lect 5 points inside the triangle, prove that at least two of these points are su tetween them is less than $\frac{1}{2}$ m.	Note: Answer FIVE FULL Questions, choosing ONE full question from each part. PART I efine a set, an empty set and a singleton set with an example for each. Let $U=\{1,2,3,4,5,6,7,8,5,-1,2,4,6,8\}$, $B=\{2,4,5,9\}$. Compute(i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) \overline{A} . OR or any two sets A and B, state and prove De-Morgan's law. total of 1232 students have taken a course in Java, 879 in C and 114 in C++. Further, 103 I ken courses in both Java and C, 23 in both Java and C++ and 14 in both C and C++. If 2 udents have taken at least one of Java, C and C++, how many students have taken a course in ree subjects? OR and the eigen values and eigen vectors of the matrix $\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$. PART III Eate Pigeon-hole Principle. ABC is an equilateral triangle whose sides are of length 1m each. If eate Pigeon-hole Principle. ABC is an equilateral triangle whose sides are such that the distate the triangle, prove that at least two of these points are such that the distate the matrix is less than $\frac{1}{2}$ m.	Note: Answer FIVE FULL Questions, choosing ONE full question from each part. PART I efine a set, an empty set and a singleton set with an example for each. Let $U=\{1,2,3,4,5,6,7,8,9\}$, $=\{1,2,4,6,8\}, B=\{2,4,5,9\}.$ Compute(<i>i</i>) $A \cup B$ (<i>ii</i>) $A \cap B$ (<i>iii</i>) $A - B$ (<i>iv</i>) \overline{A} . OR or any two sets A and B, state and prove De-Morgan's law. $\frac{PART II}{OR}$ total of 1232 students have taken a course in Java, 879 in C and 114 in C++. Further, 103 have ken courses in both Java and C, 23 in both Java and C++ and 14 in both C and C++. If 2092 udents have taken at least one of Java, C and C++, how many students have taken a course in all ree subjects? OR and the eigen values and eigen vectors of the matrix $\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$. $\frac{PART III}{2}$ The prove that at least two of these points are such that the distance etween them is less than $\frac{1}{2}$ m.	Note: Answer FIVE FULL Questions, choosing ONE full question from each part.MAIPART Iefine a set, an empty set and a singleton set with an example for each. Let $U=\{1,2,3,4,5,6,7,8,9\}$,[1] $=\{1,2,4,6,8\}, B=\{2,4,5,9\}. Compute(i) A \cup B (ii) A \cap B (iii) A - B (iv) \overline{A}.$ [1]ORor any two sets A and B, state and prove De-Morgan's law.[1]total of 1232 students have taken a course in Java, 879 in C and 114 in C++. Further, 103 have ken courses in both Java and C, 23 in both Java and C++ and 14 in both C and C++. 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	CMR INSTITUTE OF TECHNOLOGY, BENGALURU.

	Internal Assessment Te	est I – Feb 202	5					
Sub:	Discrete Mathematics and Graph Theory	Sub Code	MMC102	MMC102 Branch: MC		A		
Date:	05/02/2025 Duration: 90 minutes Max Marks: 5	0 Date:	05/0	2/2025	1	OBE		
	Note: Answer FIVE FULL Questions, choosing ONE full	l question fron	i each part.	MA	RKS	CO	RBT	
1	Define a set, an empty set and a singleton set with an example f A={1,2,4,6,8}, B={2,4,5,9}. Compute(<i>i</i>) $A \cup B$ (<i>ii</i>) $A \cap B$ OR				10]	CO1	L1	
2	For any two sets A and B, state and prove De-Morgan's law.			[10]	CO1	L3	
3	PART II A total of 1232 students have taken a course in Java, 879 in C taken courses in both Java and C, 23 in both Java and C++ students have taken at least one of Java, C and C++, how man three subjects?	and 14 in both	C and C++. If	2092 [10]	C01	L3	
4	Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$).		[10]	CO1	L3	
5	PART III State Pigeon-hole Principle. ABC is an equilateral triangle who select 5 points inside the triangle, prove that at least two of th between them is less than ½ m. OR				10]	CO4	L3	

6	Let $f: R \to R$ be defined by $f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$ Determine $f(5/3), f(-5/3), f^{-1}(0), f^{-1}(-6), f^{-1}([-5,5]).$	[10]	CO1	L2
7	Define spanning subgraph with an example. Check whether the following graphs are isomorphic.	[10]	CO2	L3
	OR			
8	Define the following with an example for each. (i) Simple graph (ii) Regular graph (iii) Null graph (iv) Complete graph.	[10]	CO2	L1
9	PART VDetermine the order $ V $ of the graph $G = (V, E)$ in the following cases: (i) G is a cubic graph of 9edges. (ii) G is regular with 15 edges. (iii) G has 10 edges with 2 vertices of degree 4 and allOR	[10]	CO2	L3
10	Define Euler Circuit and Write a note on Konigsberg Bridge Problem.	[10]	CO4	L2

6	Let $f: R \to R$ be defined by $f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$	[10]	CO1	L2
	Determine $f(5/3)$, $f(-5/3)$, $f^{-1}(0)$, $f^{-1}(-6)$, $f^{-1}([-5,5])$.	54.03		
7	PART IV Define spanning subgraph with an example. Check whether the following graphs are isomorphic.	[10]	CO2	L3
	$ \begin{array}{c} $			
	OR			
8	Define the following with an example for each. (i) Simple graph (ii) Regular graph	[10]	CO2	L1
0	(iii) Null graph (iv) Complete graph.	[10]		
	PART V			
9	Determine the order $ V $ of the graph $G = (V, E)$ in the following cases: (i) G is a cubic graph of 9	[10]	CO2	L3
	edges. (ii) G is regular with 15 edges. (iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3. OR			
10	Define Euler Circuit and Write a note on Konigsberg Bridge Problem.	[10]	CO4	L2

$$IAT 1 - Feb 2025$$
Discrete Mathematics & Graph Theory - MMC/02

A set is a collection of well defined objects.

Eq: $[2:4,4,6,...]$

Empty set : A set that contains no elements is called an empty

set.

Eq: Set of all prime numbers from 8 to 10.

Singleton set : A set containing a single element is called a

singleton set :

Eq: Set of all onulliples of 3 from T to 10.

(i) AUB = $\{1, 2, 4, 6, 8\} \cup \{2, 4, 5, 9\}$

= $\{1, 2, 4, 6, 8\} \cap \{2, 4, 5, 9\}$

(ii) AOB = $\{1, 2, 4, 6, 8\} \cap \{2, 4, 5, 9\}$

= $\{1, 2, 4, 6, 8\}$

(iii) A-B = $\{1, 2, 4, 6, 8\} - \{2, 4, 5, 9\}$

= $\{1, 2, 4, 6, 8\}$

(iv) $\overline{A} = U - A$

= $\{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 4, 6, 8\}$

= $\{3, 5, 7, 9\}$

2) Be-Mogan's Law For any two sets A and B,
(i)
$$\overline{AUB} = \overline{AOB}$$

(ii) $\overline{AOB} = \overline{AUB}$
[iii) $\overline{AOB} = \overline{AUB}$
[ive and \overline{AOB}]
 $= \{x \mid x \in \overline{A} \text{ and } x \in \overline{B}\}$
 $= \{x \mid x \notin A \text{ and } x \notin B\}$
 $= \{x \mid x \notin A \text{ and } x \notin B\}$
 $= \overline{AUB} = LHS$
(ii) $RHS = \overline{AUB} = \{x \mid x \in \overline{A} \text{ or } x \in \overline{B}\}$
 $= \{x \mid x \notin A \text{ or } x \notin B\}$
 $= \{x \mid x \notin A \text{ or } x \notin B\}$
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 $= \{x \mid x \notin A \text{ or } x \notin B\}$

3 Let A, B, C be the set of all students who have taken a course in Java, C, C^{++} respectively. Given |A| = 12.32, |B| = 879, |C| = 114, $|A \cap B| = 103$, $|A \cap d| = 28$ $|B \cap C| = 14$, $|A \cup B \cup C| = 2092$, $|A \cap B \cap C| = 7$. From Principle of Inclusion - Exclusion, $|A \cup B \cup C| = |A + 1B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$ $2092 = 1232 + 879 + 114 - 103 - 23 - 14 + |A \cap B \cap C|$

=)
$$|A \cap B \cap C| = 7$$

 $\therefore 7$ students have taken all three courses.
4. Let $A = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$
Characteristic eqⁿ of $A : |A - \pi T| = 0$
 $\begin{vmatrix} 3 - \pi & 2 \\ -1 & -\pi \end{vmatrix} = 0$
 $\Rightarrow -\pi(3 - \pi) + \pi = 0$
 $\Rightarrow -\pi(3 - \pi) + \pi = 0$
 $\Rightarrow \pi^2 - 3\pi + \pi = 0$
 $\Rightarrow \pi^2 - 3\pi + \pi = 0$
 $\Rightarrow \pi(\pi - \pi) - 1(\pi - \pi) = 0$
 $\Rightarrow \pi(\pi - \pi) - 1(\pi - \pi) = 0$
 $\Rightarrow \pi = 2, 1$

Tofond Eigen vectors, consider

$$(A - \pi I) \times = 0$$

$$\begin{pmatrix} (3 - \pi 2) & (\pi) \\ (-1 - \pi) & (y) \\ (y) & = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} (3 - \pi) \chi + 2y = 0 \\ -\pi - \pi \chi = 0 \end{pmatrix} = 0$$

$$\frac{(3 - \pi) \chi + 2y = 0}{-\pi - \pi \chi = 0} = 0$$

$$\frac{(3 - \pi) \chi + 2y = 0}{-\pi - \pi \chi = 0}$$

$$\frac{(3 - \pi) \chi + 2y = 0}{-\pi - \pi \chi = 0}$$

$$\frac{2\chi + 2\chi = 0}{-\pi - \chi}$$

$$\frac{(1, -1)}{\pi} \text{ is the eigen vector.}$$

$$\frac{(3 - \pi) \chi}{(2 - \pi)^{2}} = 0$$

Case (ii): Put
$$\lambda = 2$$

 $2(+2y=0)$
 $=> \chi = -2y$
 $=> \frac{\chi}{-2} = \frac{y}{1}$
(-2,1) is the eigen vector
corresponding to $\lambda = 2$.

Pigeon-hole Principle: "If there are on pigeons and 5. n pigeon-holes with m>n, then at least one pigeon hole contains p+1 or more pigeons in it, where $p = \lfloor \frac{m-1}{n} \rfloor$. Consider a triangle ABC whose sides are of length Ion each . Bisect each side of the triangle and join them as shown in the diagram. Now we have 4 sub-triangles with their sides 1/2 m. Let us treat these subtriangles as pigeonholes and 5 per points as pigeons. From pigeon hole principle, atleast 2 points must be in the same sub-triangle and obviously the distance blue them must be less than 2m.

$$f(x) = \begin{cases} 3x - 5 & for x > 0 \\ -3x + 1 & for x \le 0 \end{cases}$$
$$f(5/3) = 3(5/3) - 5 = 0$$
$$f(-5/3) = -3(-5/3) + 1 = 6$$

Let
$$f^{-1}(0) = x$$

 $\rightarrow P(x) = 0$

6

$$3x - 5 = 0 \qquad -3z + 1 = 0$$

$$3x = 5 \qquad -3z = -1$$

$$x = 5/3 > 0 \qquad x = \frac{1}{3} \neq 0$$

$$- \qquad D \qquad \times$$

$$\therefore f^{-1}(0) = \frac{5}{3}\frac{5}{3}\frac{1}{3}$$

$$kut f^{-1}(-6) = x$$

$$= 3x - 5 = -6 \qquad -3x + 1 = -6$$

$$3x = -1 \qquad -3x = -7$$

$$x = -\frac{1}{3} \neq 0 \qquad x = \frac{7}{3} \neq 0$$

$$x = \frac{7}{3} \neq 0$$

$$x = \frac{7}{3} \neq 0$$

$$x = \frac{7}{3} \neq 0$$

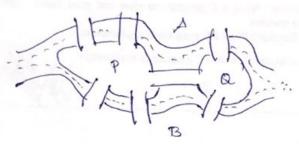
 $-5 \le 3x - 5 \le 5 \qquad -5 \le (-3x + 1) \le 5$ $0 \le 3x \le 10 \qquad -6 \le -3x \le 4$ $+ by 3 \qquad -6 \le -3x \le 4$ $+ by -3 \qquad -5 \le (-3x + 1) \le 5$ $-6 \le -3x \le 4$ $+ by -3 \qquad 2 \ge 2 \ge -\frac{1}{3}$ $2 \ge 2 \ge -\frac{1}{3}$ $-\frac{1}{3} \le x \le 2$ $-\frac{1}{3} \le x \le 2$

$\alpha_{i} \leftrightarrow \nu_{i}$	$U_3 \leftrightarrow V_4$
$U_4 \leftrightarrow V_3$	U5 G> V2
uar v5	$u_6 \iff v_6$

8. O(i) Simple Graph A graph that has no loops and parallel / multiple edges is called a simple graph. AB Eg:-(ii) Regular Graph A graph in which degree of every vertex is same os called a regular graph. Eg:- 19 R Citi) Null graph A graph hoving only vertices, but no edges is called a Null graph. Eg:- * .B · c semple (iv) Complete Graph: A graph of order n≥2 in which there is an edge between every pair of vertices is called a complete graph. Eg:-Ø

9) Let [V]=n (i) G is a cubic graph. =) Degree of every vertex is 3. & IEI=9 From Hand-shaking property, Edeg (vi) = 2/El \Rightarrow 3n = 2(9)=> n=6 (II) G is negular graph. Let the degree of every vertex be k. & IEI= 15 From Hand-shaking property, Edeg (vi) = 21El kn = 2(15) $k = \frac{30}{n}$ Since k is a the integer, possible values of n are: 1,2,3,5,6,10,15,30

(m) |E| = 10From H-S prop, $E \deg(v_i) = 2|E|$ 2(4) + (n-2) = 2(10) 8 + 3n - 6 = 20 3n = 18n = 6 In 18th century, in a city named kongsberg, a niver flowed named Piegel niver which divides city into 4 parts - 2 banks of the niver and 2 islands. There 4 land areas were connected to each other by 7 bridges. Citizens of the city posed a problem - "by starting at any of the four land areas, can we networn to that area after coossing each bridge exactly once?". This problem is known as konigsberg bridge problem.



In 1736, Ewler analyzed the problem with the help of a graph & gove the solution. Denote the land areas as A, B, P, Q as shown in the diagram. Construct a graph by treating the four land areas as four vortices & 7 bridges as edges connecting the vertices.

We note that in this graph deg(A) = deg(B) = deg(O) = 3 & deg(P) = 5 which are not even. :. the graph doesn't have an Euler circuit. Hence, it is not possible to walk over each of the seven bridges exactly once & networn to the starting point.