USN

Internal Assessment Test I – Feb 2025

Sub:	Discrete Mathematics and Graph Theory					Sub Code:	MMC102	Branch:	MCA
Date:	05/02/2025	Duration:	90 minutes	Max Marks:	50	Sem / Sec:	I A&B		OBE

<u>Note: Answer FIVE FULL Questions, choosing ONE full question from each part.</u>

		MARKS	CO	RBT
	PART I			
1	Define a set, an empty set and a singleton set with an example for each. Let U={1,2,3,4,5,6,7,8,9}, A={1,2,4,6,8}, B={2,4,5,9}. Compute $(i)\ A\cup B \quad (ii)\ A\cap B \quad (iii)\ A-B \quad (iv)\ \bar{A}$.	[10]	CO1	L1
	OR			
2	For any two sets A and B, state and prove De-Morgan's law.	[10]	CO1	L3
	PART II			
3	A total of 1232 students have taken a course in Java, 879 in C and 114 in C++. Further, 103 have taken courses in both Java and C, 23 in both Java and C++ and 14 in both C and C++. If 2092 students have taken at least one of Java, C and C++, how many students have taken a course in all three subjects?	[10]	CO1	L3
	OR			
4	Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$.	[10]	CO1	L3
	PART III			
5	State Pigeon-hole Principle. ABC is an equilateral triangle whose sides are of length 1m each. If we select 5 points inside the triangle, prove that at least two of these points are such that the distance between them is less than ½ m.	[10]	CO4	L3
	OR			

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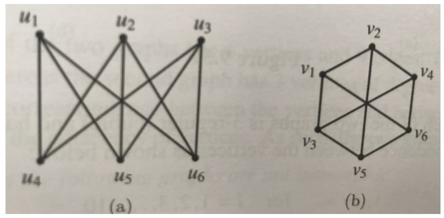
6 Let $f: R \to R$ be defined by $f(x) = \begin{cases} 3x-5 & for\ x > 0 \\ -3x+1 & for\ x \le 0 \end{cases}$.
Determine $f(5/3)$, $f(-5/3)$, $f^{-1}(0)$, $f^{-1}(-6)$, $f^{-1}([-5,5])$. [10] CO1 L2

PART IV

7 Define spanning subgraph with an example. Check whether the following graphs are isomorphic. [10] CO2 L3



OR

8 Define the following with an example for each. (i) Simple graph (ii) Regular graph
(iii) Null graph (iv) Complete graph. [10] CO2 L1

PART V

9 Determine the order $|V|$ of the graph $G = (V, E)$ in the following cases: (i) G is a cubic graph of 9 edges. (ii) G is regular with 15 edges. (iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3. [10] CO2 L3

OR

10 Define Euler Circuit and Write a note on Konigsberg Bridge Problem. [10] CO4 L2

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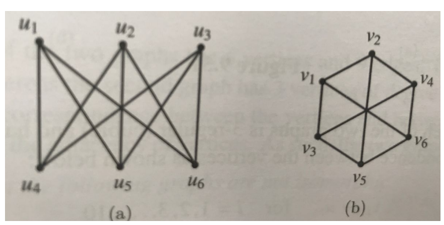

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IAT 1 – Feb 2025

Discrete Mathematics & Graph Theory – MMC102

1. A set is a collection of well defined objects.

Eg: $\{2, 4, 6, \ldots\}$

Empty set: A set that contains no elements is called an empty set.

Eg: Set of all prime numbers from 8 to 10.

Singleton set: A set containing a single element is called a singleton set.

Eg: Set of all multiples of 3 from 7 to 10.

(i) $A \cup B = \{1, 2, 4, 6, 8\} \cup \{2, 4, 5, 9\}$

$= \{1, 2, 4, 5, 6, 8, 9\}$

(ii) $A \cap B = \{1, 2, 4, 6, 8\} \cap \{2, 4, 5, 9\}$

$= \{2, 4\}$

(iii) $A - B = \{1, 2, 4, 6, 8\} - \{2, 4, 5, 9\}$

$= \{1, 6, 8\}$

(iv) $\overline{A} = U - A$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 4, 6, 8\}$

$= \{3, 5, 7, 9\}$

2) <u>De-Mogan's Law:</u> For any two sets A and B,

$$(i)\ \overline{A\cup B} = \overline{A}\cap\overline{B}$$

$$(ii)\ \overline{A\cap B} = \overline{A}\cup\overline{B}$$

<u>Proof:</u> ~~$\overline{A\cup B}$~~ (i) $RHS = \overline{A}\cap\overline{B}$

$= \{x \mid x\in\overline{A} \text{ and } x\in\overline{B}\}$

$= \{x \mid x\notin A \text{ and } x\notin B\}$

$= \{x \mid x\notin A\cup B\}$

$= \overline{A\cup B} = LHS$

(ii) $RHS = \overline{A}\cup\overline{B} = \{x \mid x\in\overline{A} \text{ or } x\in\overline{B}\}$

$= \{x \mid x\notin A \text{ or } x\notin B\}$

$= \{x \mid x\notin A\cap B\}$

$= \overline{A\cap B} = LHS$

3 Let A, B, C be the set of all students who have taken a course in Java, C, C^{++} respectively.

Given $|A| = 1232$, $|B| = 879$, $|C| = 114$, $|A\cap B| = 103$, $|A\cap C| = 23$,

$|B\cap C| = 14$, $|A\cup B\cup C| = 2092$, $|A\cap B\cap C| = ?$

From Principle of Inclusion - Exclusion,

$|A\cup B\cup C| = |A|+|B|+|C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$

$2092 = 1232 + 879 + 114 - 103 - 23 - 14 + |A\cap B\cap C|$

$\Rightarrow |A \cap B \cap C| = 7$

$\therefore$ 7 students have taken all three courses.

4. Let $A = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$

Characteristic eq^n of A : $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & 2 \\ -1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow -\lambda(3-\lambda) + 2 = 0$

$\Rightarrow \lambda^2 - 3\lambda + 2 = 0$

$\Rightarrow \lambda^2 - 2\lambda - \lambda + 2 = 0$

$\Rightarrow \lambda(\lambda-2) - 1(\lambda-2) = 0$

$\Rightarrow (\lambda-2)(\lambda-1) = 0$

$\Rightarrow \lambda = 2, 1$

To find Eigen vectors, consider

$(A - \lambda I)X = 0$

$$\begin{pmatrix} 3-\lambda & 2 \\ -1 & -\lambda \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{aligned} (3-\lambda)x + 2y &= 0 \\ -x - \lambda y &= 0 \end{aligned}\right\} - ①$$

Case (i)

Put $\lambda = 1$ in ①

$2x + 2y = 0$

$\rightarrow \cancel{2}\ x = -y$

$(1, -1)$ is the eigen vector. corresponding to $\lambda = 1$

Case (ii): Put $\lambda = 2$

$x + 2y = 0$

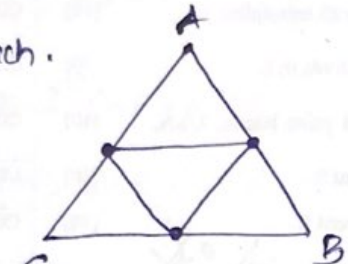
$\Rightarrow x = -2y$

$\Rightarrow \frac{x}{-2} = \frac{y}{1}$

$(-2, 1)$ is the eigen vector corresponding to $\lambda = 2$.

5. Pigeon-hole Principle : "If there are m pigeons and n pigeon-holes with $m > n$, then atleast one pigeon hole contains $p+1$ or more pigeons in it, where $p = \lfloor \frac{m-1}{n} \rfloor$."

Consider a triangle ABC whose sides are of length 1m each.



Bisect each side of the triangle and join them as shown in the diagram.

Now we have 4 sub-triangles with their sides $\frac{1}{2}$m. Let us treat these sub-triangles as pigeonholes and 5 ~~poi~~ points as pigeons. From pigeon hole principle, atleast 2 points must be in the same sub-triangle and obviously the distance b/w them must be less than $\frac{1}{2}$m.

6. $f(x) = \begin{cases} 3x-5 & \text{for } x>0 \\ -3x+1 & \text{for } x \leq 0 \end{cases}$

$f(5/3) = 3(5/3) - 5 = 0$

$f(-5/3) = -3(-5/3) + 1 = 6$

Let $f^{-1}(0) = x$

$\Rightarrow f(x) = 0$

$3x - 5 = 0 \qquad -3x + 1 = 0$

$3x = 5 \qquad -3x = -1$

$x = 5/3 > 0 \qquad x = 1/3 \not> 0$

✓ $\qquad$ ×

$\therefore f^{-1}(0) = \{5/3\}$

Let $f^{-1}(-6) = x$

$\Rightarrow f(x) = -6$

$3x - 5 = -6 \qquad -3x + 1 = -6$

$3x = -1 \qquad -3x = -7$

$x = -1/3 \not> 0 \qquad x = 7/3 \not\leq 0$

× $\qquad$ ×

$\therefore f^{-1}(-6) = \{\ \}$

Let $f^{-1}([-5, 5]) = \{x / f(x) \in [-5, 5]\}$

$= \{x / -5 \leq f(x) \leq 5\}$

$-5 \leq 3x - 5 \leq 5 \qquad -5 \leq (-3x + 1) \leq 5$

$0 \leq 3x \leq 10 \qquad -6 \leq -3x \leq 4$

$\div$ by 3 $\qquad \div$ by -3

$0 \leq x \leq 10/3$

$2 \geq x \geq -4/3$

$-4/3 \leq x \leq 2$

$\therefore f^{-1}([-5, 5]) = [-4/3, 10/3]$

7. A subgraph $G_1(V_1, E_1)$ of $G(V, E)$ is said to be a spanning subgraph of G if $V_1 = V$.

Both the graphs (a) and (b) have 6 vertices.

" " have 9 edges.

" " have all vertices of degree 3.

Mapping b/w the vertices:

$u_1 \leftrightarrow v_1$ $\quad$ $u_3 \leftrightarrow v_4$

$u_4 \leftrightarrow v_3$ $\quad$ $u_5 \leftrightarrow v_2$

$u_2 \leftrightarrow v_5$ $\quad$ $u_6 \leftrightarrow v_6$

Mapping b/w the edges:

$\{u_1, u_4\} \leftrightarrow \{v_1, v_3\}$ $\quad$ $\{u_2, u_4\} \leftrightarrow \{v_5, v_3\}$

$\{u_1, u_5\} \leftrightarrow \{v_1, v_2\}$ $\quad$ $\{u_2, u_5\} \leftrightarrow \{v_5, v_2\}$

$\{u_1, u_6\} \leftrightarrow \{v_1, v_6\}$ $\quad$ $\{u_2, u_6\} \leftrightarrow \{v_5, v_6\}$

$\{u_3, u_4\} \leftrightarrow \{v_4, v_3\}$

$\{u_3, u_5\} \leftrightarrow \{v_4, v_2\}$

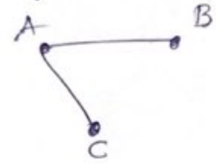
$\{u_3, u_6\} \leftrightarrow \{v_4, v_6\}$

There exists a 1-1 mapping between the vertices and edges of both the graphs. Hence, they are isomorphic.

8. (i) Simple Graph

A graph that has no loops and parallel / multiple edges is called a simple graph.

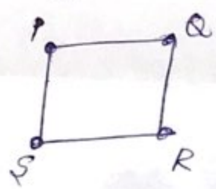
Eg:-



(ii) Regular Graph

A graph in which degree of every vertex is same is called a regular graph.

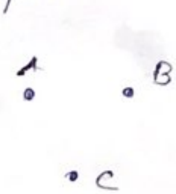
Eg:-



(iii) Null graph

A graph having only vertices, but no edges is called a Null graph.

Eg:-



(iv) Complete Graph: A simple graph of order $n \geq 2$ in which there is an edge between every pair of vertices is called a complete graph. Eg:-

9) Let $|V| = n$

(i) G is a cubic graph.

$\Rightarrow$ Degree of every vertex is 3. & $|E| = 9$

From Hand-shaking property,

$$\sum deg(v_i) = 2|E|$$

$$\Rightarrow \quad 3n = 2(9)$$

$$\Rightarrow \quad n = 6$$

(ii) G is regular graph.

Let the degree of every vertex be k. & $|E| = 15$

From Hand-shaking property,

$$\sum deg(v_i) = 2|E|$$

$$kn = 2(15)$$

$$k = \frac{30}{n}$$

Since k is a +ve integer, possible values of n are:

$$1, 2, 3, 5, 6, 10, 15, 30$$

(iii) $|E| = 10$

From H-S prop, $\sum deg(v_i) = 2|E|$

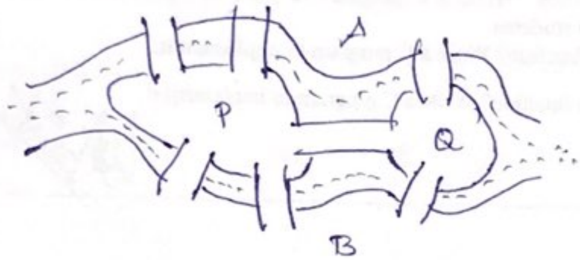
$$2(4) + (n-2)3 = 2(10)$$

$$8 + 3n - 6 = 20$$

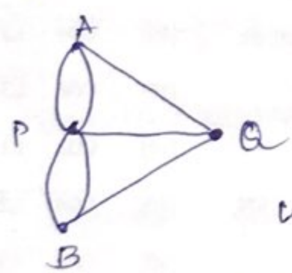
$$3n = 18$$

$$n = 6$$

In 18^{th} century, in a city named Konigsberg, a river flowed named Piegel river which divides city into 4 parts – 2 banks of the river and 2 islands. There 4 land areas were connected to each other by 7 bridges. Citizens of the city posed a problem – "by starting at any of the four land areas, can we return to that area after crossing each bridge exactly once?". This problem is known as Konigsberg bridge problem.



In 1736, Euler analyzed the problem with the help of a graph & gave the solution. Denote the land areas as A, B, P, Q as shown in the diagram. Construct a graph by treating the four land areas as four vertices & 7 bridges as edges connecting the vertices.



We note that in this graph

$\deg(A) = \deg(B) = \deg(Q) = 3$ & $\deg(P) = 5$

which are not even. ∴ the graph doesn't have an Euler circuit. Hence, it is not possible to walk over each of the seven bridges exactly once & return to the starting point.