USN

Internal Assessment Test II – March 2025

Sub:	Discrete Mathematics and Graph Theory					Sub Code:	MMC102	Branch:	MCA
Date:	18/03/2025	Duration:	90 minutes	Max Marks:	50	Sem / Sec:	I A&B		OBE

Note: Answer FIVE FULL Questions, choosing ONE full question from each part.

		MARKS	CO	RBT
	PART I			
1	Define a tautology. Check whether $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology, contradiction or contingency.	[10]	CO1	L2
	OR			
2	(a) Define the logical connective ‘disjunction’ with its truth table. (b) Write the converse, inverse and contrapositive of “If Gold is a compound then Oxygen is a gas.”	[10]	CO1	L1
	PART II			
3	Write the negation of (i) All Americans eat cheese burgers. (ii) $\exists x, [p(x) \vee q(x)]$ (iii) $\exists x, [\{p(x) \vee q(x)\} \to r(x)]$	[10]	CO1	L3
	OR			
4	Give the direct and indirect proof of “If n is an odd integer then n+9 is an even integer.”	[10]	CO1	L2
5	Check whether the following argument is valid or not. “If today is Tuesday, I have a test in Mathematics or Economics. If my Economics professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics professor is sick. Therefore, I have a test in Mathematics.”	[10]	CO1	L3
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	PART III			
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	OR			

6	Using laws of logic, prove (i) $[(p \vee q) \wedge (p \vee \sim q)] \vee q \Leftrightarrow p \vee q$. (ii) $[p \rightarrow (q \wedge r)] \Leftrightarrow [(p \rightarrow q) \wedge (p \rightarrow r)]$	[10]	CO1	L3
	PART IV			
7	Let A={1, 2, 3, 4, 6} and R be the relation on A defined by 'xRy if and only if x divides y'. Write down R as a set of ordered pairs. Draw the digraph of R. Find in-degree and out-degree of each vertex. Is this relation R symmetric?	[10]	CO3	L2
	OR			
8	Define Simple Digraphs, Symmetric Digraphs, Asymmetric Digraphs, Graph Coloring and Chromatic Number with an example for each.	[10]	CO3	L1
	PART V			
9	(a) Find the Chromatic polynomial and Chromatic number for the cycle C_4 of length 4. (b) Find the Chromatic number of the complete bipartite graph $K_{3,3}$.	[10]	CO4	L3
	OR			
10	(a) Write down the Chromatic polynomial of a Null graph, Path and Complete graph with n vertices. (b) Explain Four Color Problem.	[10]	CO4	L3

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9	(a) Find the Chromatic polynomial and Chromatic number for the cycle C_4 of length 4. (b) Find the Chromatic number of the complete bipartite graph $K_{3,3}$.	[10]	CO4	L3
	OR			
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MMC102 . IAT2

Discrete Maths & Graph Theory

1. A compound proposition which is always false regardless of truth values of it's components is called a Tautology.

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Since all the entries of the last column are 1s, the given compound proposition is a tautology.

2(a) A compound proposition obtained by inserting the word "OR" between two propositions is called a disjunction.

If p and q are two propositions then.

"p or q" is denoted by "$p \vee q$".

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

2(b) Let p : Gold is a compound

q : Oxygen is a gas.

Given $p \rightarrow q$

Converse : $q \rightarrow p$

ie., If oxygen is a gas then Gold is a compound.

Inverse : $\neg p \rightarrow \neg q$

ie., If Gold is not a compound then oxygen is not a gas.

Contrapositive : $\neg q \rightarrow \neg p$

ie., If Oxygen is not a gas then Gold is not a compound.

3 (i) Let the Universe be set of all Americans.

$p(x)$: x eat cheese burgers.

Given $\forall x, p(x)$

Negation : $\neg [\forall x, p(x)]$

$\Leftrightarrow \exists x, \neg p(x)$

ie., Some Americans do not eat cheese burgers.

(ii) Negⁿ $\neg [\exists x, \{p(x) \vee q(x)\}]$

$\Leftrightarrow \forall x, \neg \{p(x) \vee q(x)\}$

$\Leftrightarrow \forall x, \neg p(x) \wedge \neg q(x)$

(iii) Negⁿ $\neg \{\exists x, [\{p(x) \vee q(x)\} \rightarrow r(x)]\}$

$\Leftrightarrow \forall x, \neg [\{p(x) \vee q(x)\} \rightarrow r(x)]$

$\Leftrightarrow \forall x, \{p(x) \vee q(x)\} \wedge \neg r(x)$

4. Let p: n is an odd integer.

q: $n+9$ is an even integer.

Given $p \to q$

Direct Proof

Assume p is true.

$\Rightarrow n$ is an odd integer.

$\Rightarrow n = 2k+1 \quad ; k \in \mathbb{Z}$

$\Rightarrow n+9 = 2k+1+9 = 2k+10 = 2(k+5) = 2l$ which is even.

$\Rightarrow q$ is true.

$\therefore p \to q$ is true.

Indirect Proof:

wkt $p \to q \Leftrightarrow \neg q \to \neg p$

Assume $\neg q$ is true.

$\Rightarrow n+9$ is odd

$\Rightarrow n+9 = 2k+1$

$\Rightarrow n = 2k-8 = 2(k-4) = 2l$ which is even.

$\Rightarrow \neg p$ is true.

Hence, $\neg q \to \neg p$ is true.

5. Let p: Today is Tuesday

q: I have a test in Maths

r: I have a test in Economics.

s: My Economics Professor is sick.

Given, $p \to (q \wedge r)$.

$s \to \neg r$

$p \wedge s$

$\therefore q$

$$\begin{array}{l} p \to (q \vee r) \\ s \to \neg r \\ p \\ s \\ \hline \therefore q \end{array} \quad \begin{array}{l} \text{conjunctive} \\ \text{simp}^n \end{array} \quad \Rightarrow \quad \begin{array}{l} q \vee r \\ \neg r \\ \hline \therefore q \end{array} \quad \begin{array}{l} \text{Modus Ponens for I \& III premises} \\ \text{— " — II \& IV} \end{array}$$

$$\Longleftrightarrow \quad \begin{array}{l} \neg q \to r \\ \neg r \\ \hline \therefore q \end{array}$$

This is a valid argument in view of Modus Tollens.

6. (i) LHS $= [(p \vee q) \wedge (p \vee \neg q)] \vee q$

$\Longleftrightarrow [p \vee (q \wedge \neg q)] \vee q$ — Distributive property

$\Longleftrightarrow (p \vee F_0) \vee q$ — Inverse Law

$\Longleftrightarrow p \vee q$ = RHS — Identity law

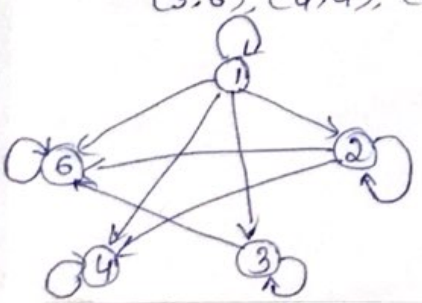
(ii) $[p \to (q \wedge r)]$ — using $p \to q \Longleftrightarrow \neg p \vee q$

$\Longleftrightarrow \neg p \vee (q \wedge r)$

$\Longleftrightarrow (\neg p \vee q) \wedge (\neg p \vee r)$ — Distributive law

$\Longleftrightarrow (p \to q) \wedge (p \to r)$ — using $p \to q \Longleftrightarrow \neg p \vee q$

7. $R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6)\}$



Vertex	In-degree	Out-degree
1	1	5
2	2	3
3	2	2
~~4~~	3	1
6	4	1

$(1,2) \in R$. But $(2,1) \notin R$

If a divides b then b doesn't divide a for $a \neq b$.

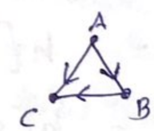
i.e., $(a,b) \in R \Rightarrow (b,a) \notin R$

Hence, R is not symmetric.

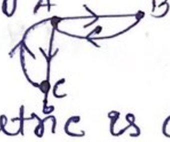
$\downarrow$

(8) **Simple Digraphs:** A digraph that has no self-loop or parallel edges is called a simple digraph.

Asymmetric Digraphs: Digraphs that have at most one directed edge between a pair of vertices, but are allowed to have self loops are called asymmetric. Eg:



Symmetric Digraph: Digraphs in which for every edge (a, b) there is also an edge (b, a) is called symmetric digraph



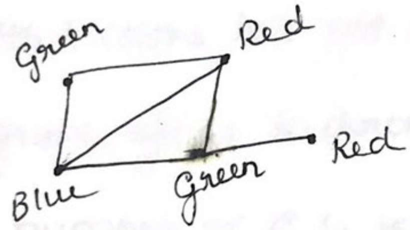
A digraph that is both simple & symmetric is called a

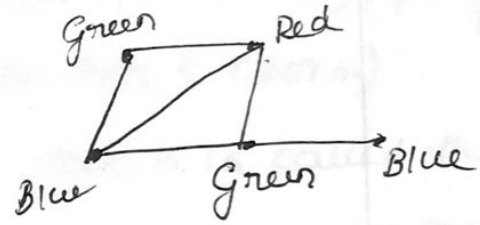
Graph Coloring

Given a planar or non-planar graph G, if we assign colours to its vertices in such a way that no two adjacent vertices have the same color, then we say that the graph G is properly colored.

(Proper coloring of a graph means assigning colors to its vertices such that adjacent vertices have different colours.)

Eg:-





These two are properly colored.

Chromatic Number

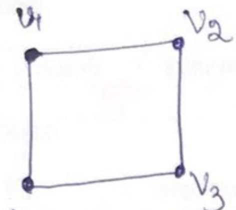
A graph G is said to be k-colorable if we can properly color it with k colors.

A graph G which is k-colorable, but not $(k-1)$-colorable is called a k-chromatic graph.

(A k-chromatic graph is a graph that can be properly colored with k colors but not with less than k colors.)

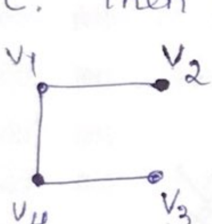
If a graph G is k-chromatic, then k is called the chromatic number of G & is usually denoted by $\chi(G)$.

9(a) Find the chromatic polynomial for the cycle C_4 of length 4.

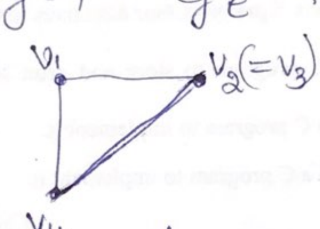


$C_4 = G$

Let us call this graph as G & denote the edge $\{v_2, v_3\}$ as e. Then the graph G_e & G'_e will be as follows.



G_e



G'_e

We note that the graph G_e is a path with 4 vertices.

Therefore, $P(G_e, \lambda) = \lambda(\lambda-1)^3$

Also G_e' is complete graph with 3 vertices.

$\therefore P(G_e', \lambda) = \lambda(\lambda-1)(\lambda-2)$

Using Decomposition theorem,

$$P(C_4, \lambda) = P(G, \lambda) = P(G_e, \lambda) - P(G_e', \lambda)$$

$$= \lambda(\lambda-1)^3 - \lambda(\lambda-1)(\lambda-2)$$

$$= \lambda(\lambda-1)\left[(\lambda-1)^2 - (\lambda-2)\right]$$

$$= \lambda(\lambda-1)\left[\lambda^2 - 2\lambda + 1 - \lambda + 2\right]$$

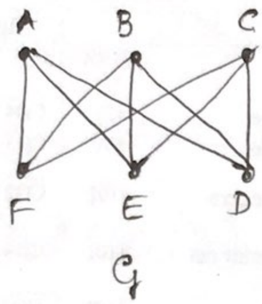
$$= (\lambda^2 - \lambda)(\lambda^2 - 3\lambda + 3)$$

$$= \lambda^4 - 3\lambda^3 + 3\lambda^2 - \lambda^3 + 3\lambda^2 - 3\lambda$$

$$= \lambda^4 - 4\lambda^3 + 6\lambda^2 - 3\lambda$$

This is the chromatic polynomial for the given circle.

9 (b) Find the chromatic number of the complete bipartite graph $K_{3,3}$.



This is $K_{3,3}$ with bipartites

$V_1 = \{A, B, C\}$

$V_2 = \{D, E, F\}$

A − C_1, B − C_1, C − C_1, D − C_2, E − C_2, F − C_2

$\chi(G) = 2$

10(a) (i) $P(N_n, \lambda) = \lambda^n$ where N_n is a null graph with n vertices.

(ii) $P(K_n, \lambda) = 0$ if $\lambda < n$

$= 1$ if $\lambda = n$

$= \lambda(\lambda-1)(\lambda-2) \cdots (\lambda-n+1)$ if $\lambda > n$.

K_n – complete graph with n vertices.

(iii) $P(L_n, \lambda) = \lambda(\lambda-1)^{n-1}$ if $\lambda \geq 2$.

where L_n is a path consisting of n vertices.

(10 b) Four Color ~~Theorem~~ Problem: Having proved that every simple connected planar graph is properly colorable with 5 colors, the question arises: Is it possible to properly color such a graph with just 4 colors? This question is posted in 1852 and was called Four Color Problem. This problem remained unsolved for over a century. This question was eventually settled by two American mathematicians, Kenneth Appel and Wolfgang Haken in 1976.

They proved the the following theorem, now known as the "Four Color Theorem": Every simple, connected planar graph is 4-colorable.

This is equivalent to saying that every map canbe properly colored with just four colors.