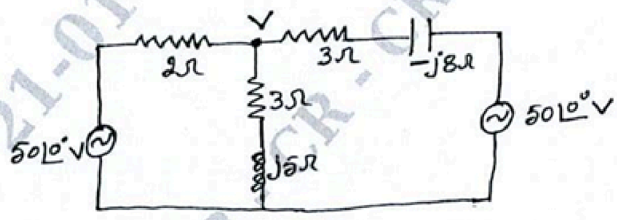
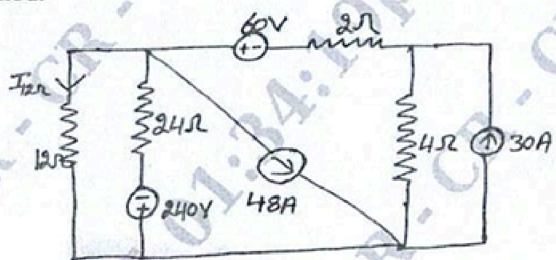
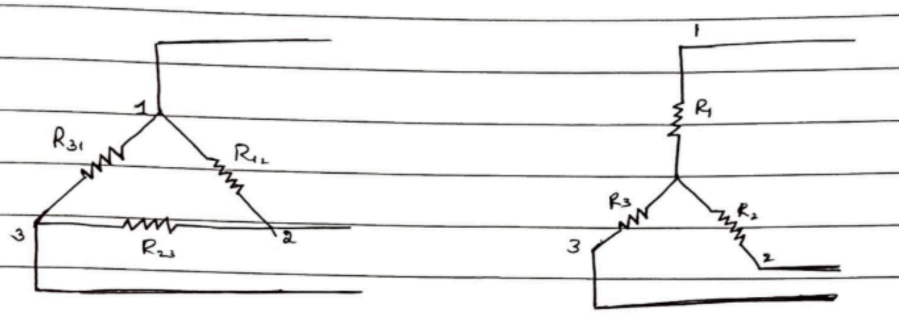
		Module – 1	M	L	C
Q.1	a.	Three impedances are connected in Delta. Obtain the star equivalent of the network.	7	L3	CO1
	b.	For the circuit shown in Fig. Q1(b). Find the voltage 'V' at node by using nodal analysis. Fig. Q1(b) 	6	L3	CO1
	c.	Determine the current in 12Ω resistor shown in Fig. Q1(c) using source transformation method.  Fig. Q1(c)	7	L3	CO1

1a

Star - Delta Transformation

The replacement of delta connection by equivalent star connection is known as delta-star transformation



Delta (Δ) Connection Star (Y) Connection

- Consider the Δ connection, the three corner points 1, 2 and as shown in the figure has electrical resistance R_{12}, R_{23} & R

- Consider the Δ connection, the three corner points 1, 2 and 3 as shown in the figure has electrical resistance R_{12}, R_{23} & R_{31} respectively terminal.
- Three arms of star system R_1, R_2, R_3 are connected with 1, 2, 3 resp.

⇒ These two arrangements (Δ & Y) will be electrically equivalent if the resistance measured b/w any part of terminal is same in both the arrangements.

In delta connection, resistance b/w terminal ① & ② is

$$R_{eq} = R_{12} \parallel (R_{31} + R_{23})$$

$$1/R_{eq} = 1/R_{12} + 1/(R_{31} + R_{23})$$

$$R_{eq} = \frac{R_{12}(R_{31} + R_{23})}{R_{12} + R_{23} + R_{31}}$$

In Y connection resistance between terminal 1 & 2 is $R_1 + R_2$

(Here resistance b/w both the cases should be same)

$$\therefore \quad R_1 + R_2 = \frac{R_{12}(R_{31} + R_{23})}{R_{12} + R_{23} + R_{31}} \longrightarrow eq^n \text{ 1}$$

Similarly

b/w 2 & 3

$$R_2 + R_3 = \frac{R_{23}(R_{31} + R_{12})}{R_{23} + (R_{31} + R_{12})} \longrightarrow eq^n \text{ 2}$$

lly b/w 3 & 1

$$R_3 + R_1 = \frac{R_{31}(R_{12} + R_{23})}{R_{31} + (R_{12} + R_{23})} \longrightarrow eq^n \text{ 3}$$

Adding eq^n 1, 2 & 3

we get $\cancel{2}(R_1 + R_2 + R_3) = \cancel{2}\dfrac{(R_{12}R_{23} + R_{31}R_{12} + R_{23}R_{31})}{R_{12} + R_{23} + R_{31}} \longrightarrow eq^n \text{ 4}$

Subtract eq^n 1, 2, 3 from 4 we get

[Δ to Y transformation]

$$R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{23}R_{12}}{R_{12} + R_{23} + R_{31}} \qquad R_3 = \frac{R_{31}R_{23}}{R_{12} + R_{23} + R_{31}}$$

Again multiply eq^n 1 & 2, eq^n 2 & 3, eq^n 3 & 1

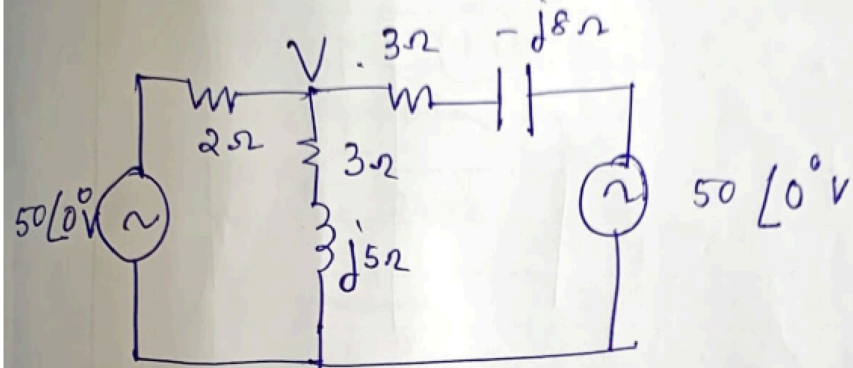
we get [Y to Δ transformation]

$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = R_1 + R_3 + \frac{R_3R_1}{R_2}$$

1b



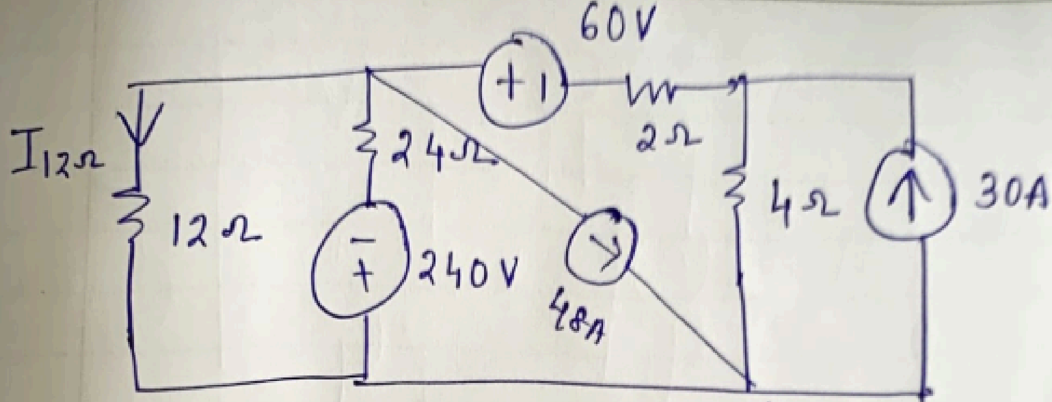
KCL at node V.

$$\cancel{\frac{V - 50\angle 0^\circ}{1/2}} +$$

$$V\left(\frac{1}{2} + \frac{1}{3+j5} + \frac{1}{3-j8}\right) = \frac{50\angle 0^\circ}{2} + \frac{50\angle 0^\circ}{3-j8}$$

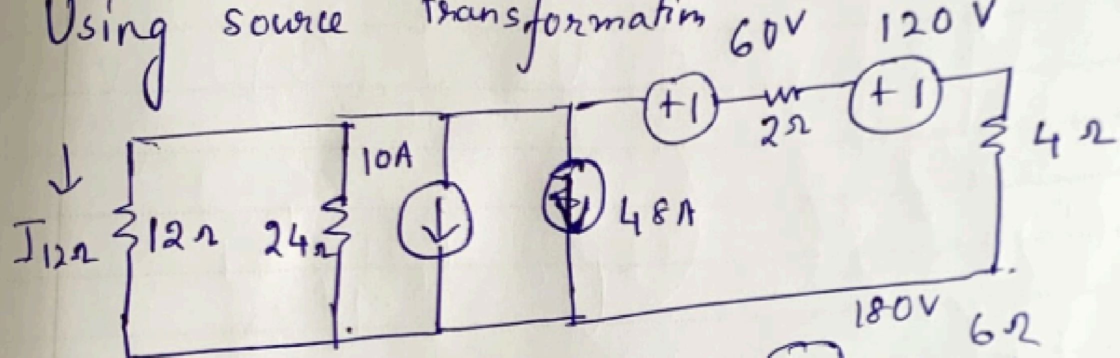
$$\text{Or } V = \frac{50\angle 0^\circ\left[\frac{1}{2} + \frac{1}{3-j8}\right]}{\left[2^{-1} + (3+j5)^{-1} + (3-j8)^{-1}\right]}$$

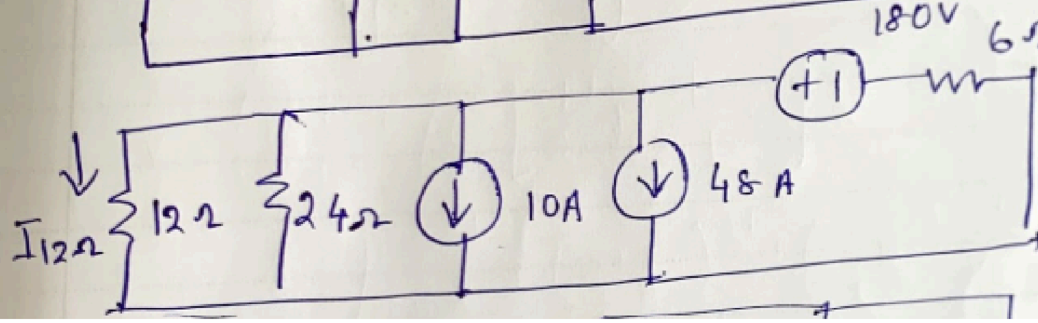
1c

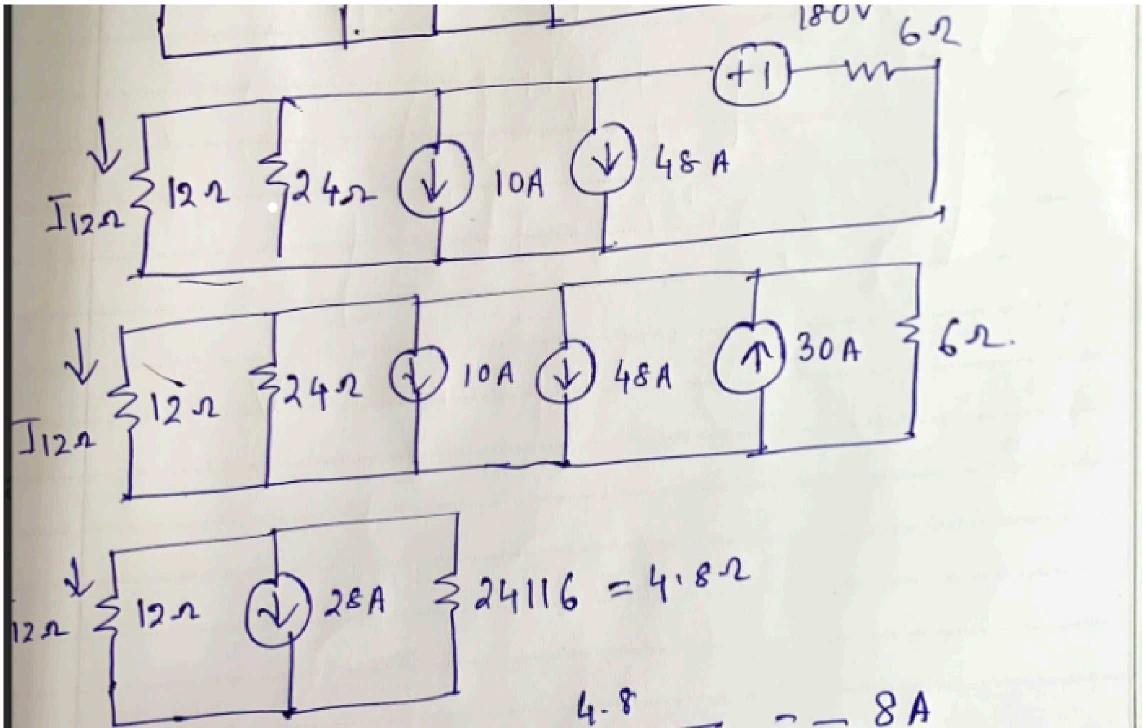


~~Using Current trans source transformatio~~

Using source transformation







$$\therefore\ I_{12\Omega} = -\,28 \times \frac{4.8}{12+4.8} = -\,8A$$

.2 **a.** Find the loop currents I_1, I_2, and I_3 in the circuit shown in Fig. Q2(a).

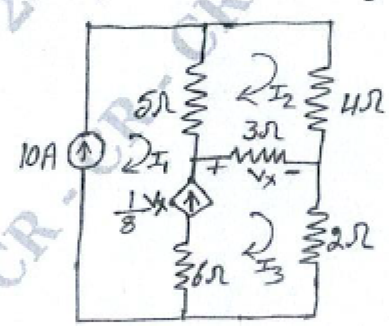
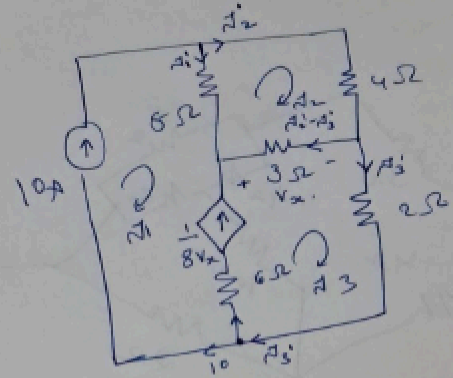


Fig. Q2(a)

2a.



$I_1' + I_2' = 10$ —— ①

L_2, $-4I_2' - 3(I_2' - I_3') + 5I_1' = 0$

$5I_1' - 7I_2' + 3I_3' = 0$ —— ②

L_3, $-(I_3' - 10)6 + 3(I_2' - I_3') - 2I_3' = 0$

$3I_2' - 11I_3' = -60$

$I_1' = 4.06A$ $\qquad I_1 = 10$

$I_2' = 5.93A$ $\qquad I_2 = 5.93A$

$I_3' = 7.07A$ $\qquad I_3 = 7.07A$

4 b. Determine the resistance between the terminals X, Y using star delta transformation in the network shown in Fig. Q2(b).

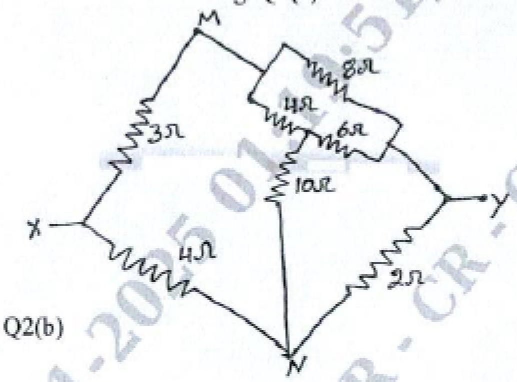
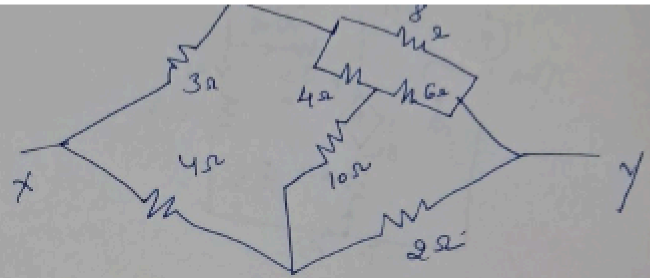
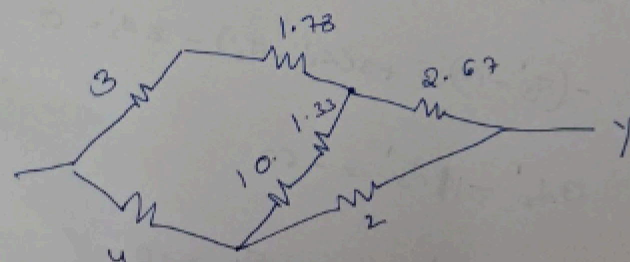
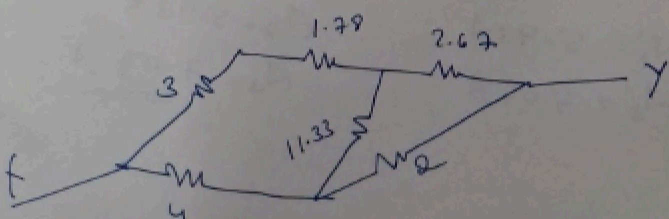


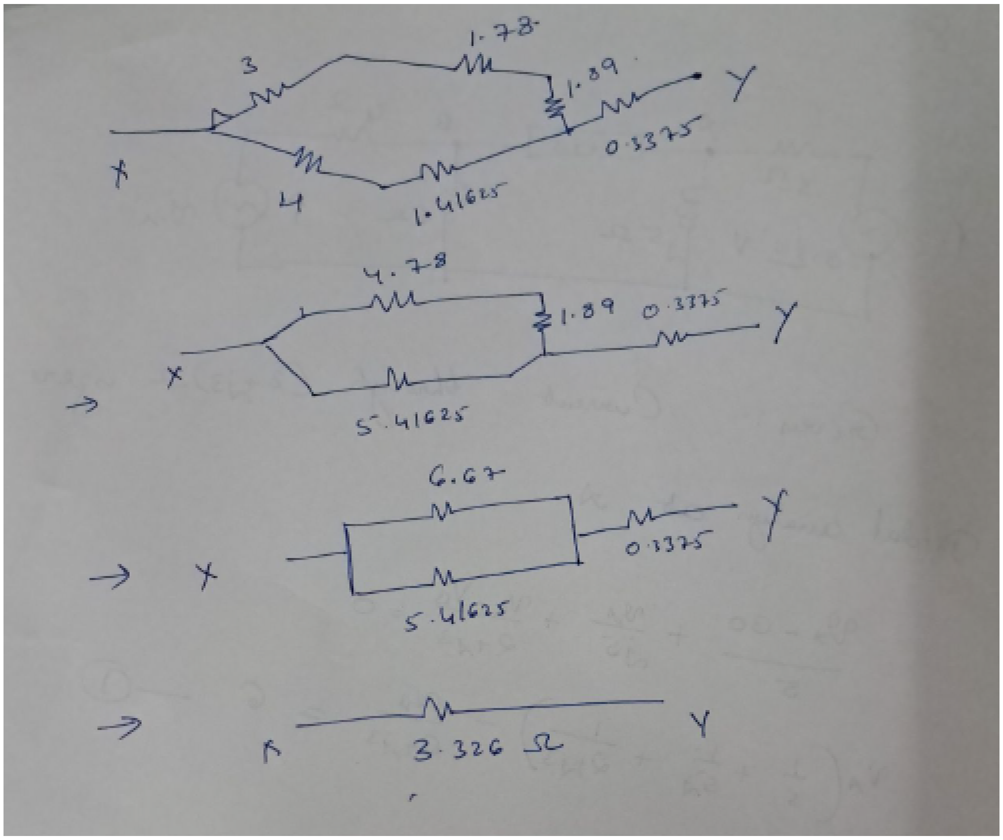
Fig. Q2(b)



$\frac{32}{18}$ $\frac{16}{9}$







2 c. Use the nodal analysis to find the value of V_X and the circuit shown in Fig. Q2(c). Such that the current through (2 + j3) Ω Impedance is zero.

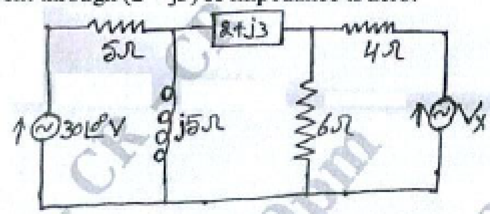
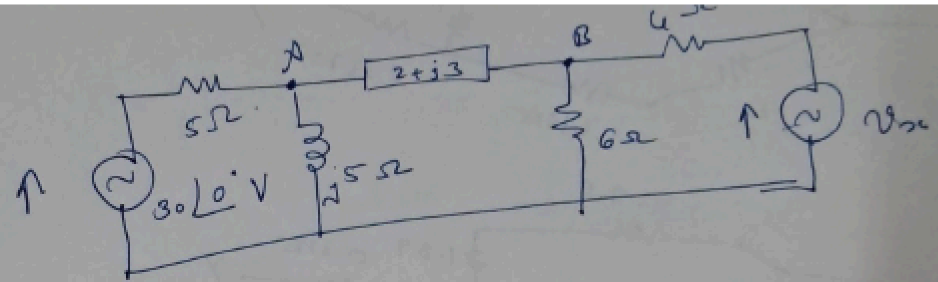


Fig. Q2(c)



Given : Current through $(2+j3)\Omega$ is zero

Nodal analysis at A.

$$\frac{v_A - 30}{5} + \frac{v_A}{j5} + \frac{v_A - V_B}{2+j3} = 0$$

$$v_A\left(\frac{1}{5} + \frac{1}{5j} + \frac{1}{2+j3}\right) - \frac{v_B}{2+j3} = 6 \quad \text{— (1)}$$

Nodal analysis at B.

$$\frac{v_B - v_A}{2+j3} + \frac{v_B}{6} + \frac{v_B - v_x}{4} = 0.$$

$$-v_A\left(\frac{1}{2+j3}\right) + v_B\left(\frac{1}{6} + \frac{1}{4} + \frac{1}{2+j3}\right) - \frac{v_x}{4} = 0 \quad \text{— (2)}$$

current through $(2+j3)\Omega$ is zero

$$\Rightarrow \frac{v_A - V_B}{2+j3} = 0. \qquad \underline{v_A = v_B}$$

from eqn ①

$$V_A \left[\frac{1}{5} + \frac{1}{5j} + \frac{1}{2+j3} - \frac{1}{2+j3} \right] = 6.$$

$$V_A (0.2 - 0.2j) = 6.$$

$$V_A = 15 + 15j = 21.21\angle 45^\circ \text{ V}.$$

Eqn ②

$$\frac{15+15j}{6} + \frac{15+15j}{4} = \frac{V_x}{4}.$$

$$V_x = 25 + 25j$$

$$= 35.35\angle 45^\circ \text{ V}.$$

.3 | a. | Sate and prove Superposition theorem.

The principle of superposition is applicable only for linear systems. The concept of superposition can be explained mathematically by the following response and excitation principle :

$$i_1 \rightarrow v_1$$
$$i_2 \rightarrow v_2$$

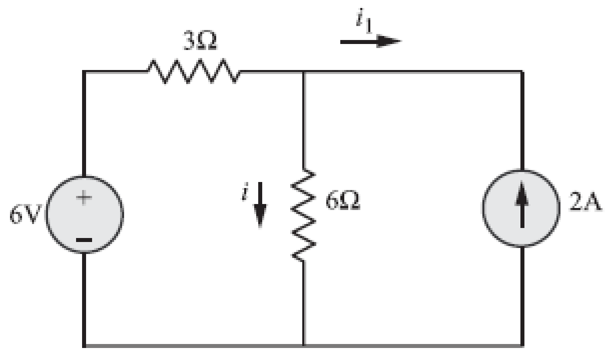
then,
$$i_1 + i_2 \rightarrow v_1 + v_2$$

Superposition theorem states that,

In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as algebraic sum of the individual contributions of each source acting alone.

To prove, take any example and solve.

Find the current in the 6 Ω resistor using the principle of superposition for the circuit



As a first step, set the current source to zero. That is, the current source appears as an open circuit as shown in Fig. 3.2.

$$i_1 = \frac{6}{3+6} = \frac{6}{9}\text{A}$$

As a next step, set the voltage to zero by replacing it with a short circuit as shown in Fig. 3.3.

$$i_2 = \frac{2 \times 3}{3+6} = \frac{6}{9}\text{A}$$

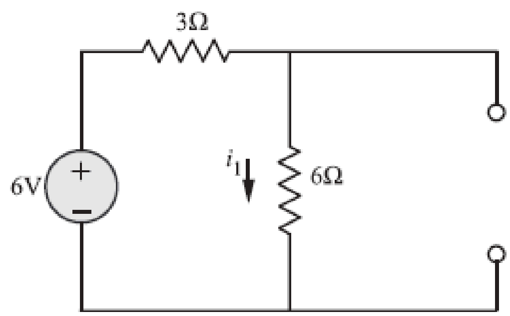


Figure 3.2

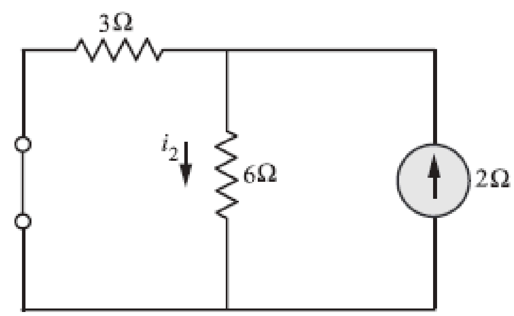


Figure 3.3

The total current i is then the sum of i_1 and i_2

$$i = i_1 + i_2 = \frac{\mathbf{12}}{\mathbf{9}}\mathbf{A}$$

The same answer i sobtained when used KVL on two loops.Thus proved

3 | b. | For the circuit shown in Fig. Q3(b), obtain the Thevenin's equivalent circuit.

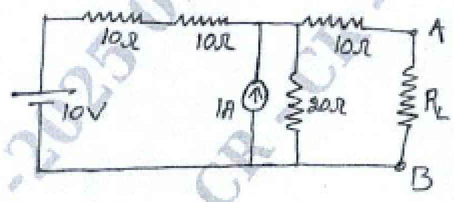
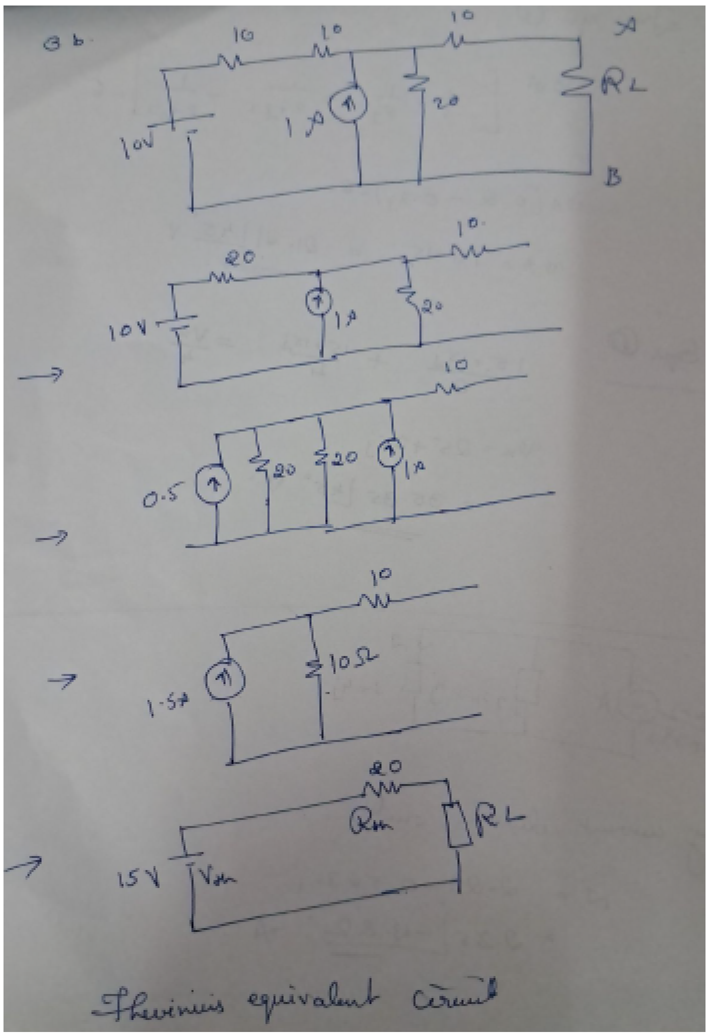


Fig. Q3(b)



3 | c. | Using Millman's theorem, find current flowing through (3 + j4) Ω impedance for the circuit shown in Fig. Q3(c).

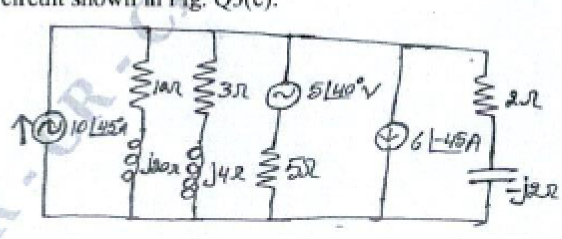
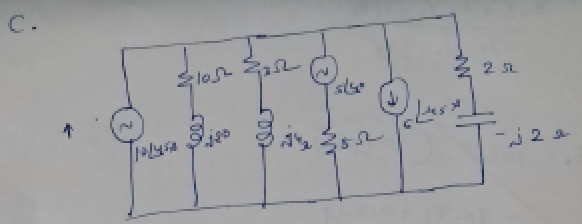
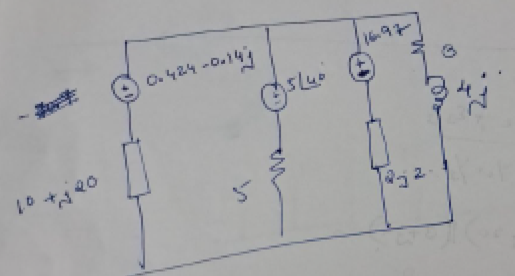


Fig. Q3(c)

C.





$$E = \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3}$$

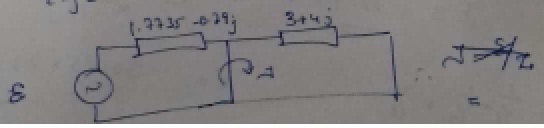
$Y_1 = \frac{1}{10+j20}$ $\qquad E = \frac{5.011 + 4.865j}{0.47 + 0.21j}$

$Y_2 = \frac{1}{5}$

$E = 12.7422 + 4.657$

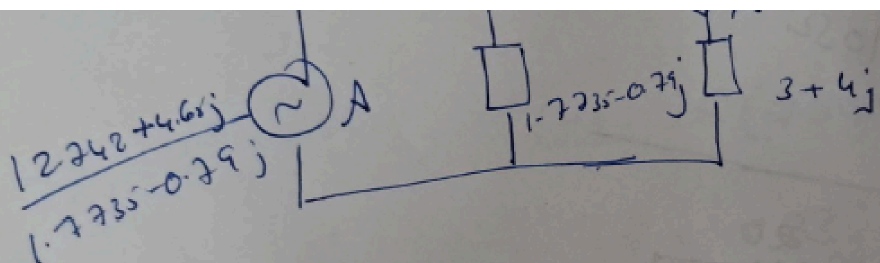
$Y_3 = \frac{1}{2-j2}$

$Z_0 = 1.7735 - 0.7924j$



$\therefore I = \cancel{E/Z_0}$

$=$



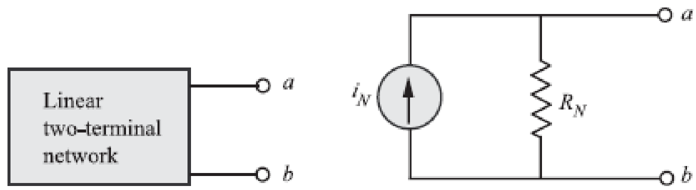
By current division rule,

$I = 2.29 - 0.5635j$

$= 2.35\angle{-13.82}^\circ$ A

Q.4	a.	State and prove Norton's theorem.

Norton's theorem states that a linear two-terminal network can be replaced by an equivalent circuit consisting of a current source i_N in parallel with resistor R_N, where i_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off. If one does not wish to turn off the independent sources, then R_N is the ratio of open circuit voltage to short–circuit current at the terminal pair.



Derivation of Norton's theorem:

Let us now assume that the linear circuit described earlier is driven by a voltage source v as shown in Fig. 3.64.

The current flowing into the circuit can be obtained by superposition as

$$i = c_0 v + d_0 \tag{3.11}$$

where $c_0 v$ is the contribution to i due to the external voltage source v and d_0 contains the contributions to i due to all independent sources within the linear circuit. The constants c_0 and d_0 are determined as follows :

(i) When terminals $x - y$ are short-circuited, $v = 0$ and $i = -i_{sc}$. Hence from equation (3.11), we find that $i = d_0 = -i_{sc}$, where i_{sc} is the short-circuit current flowing out of terminal x, which is same as Norton current i_N

Thus, $$d_0 = -i_N$$

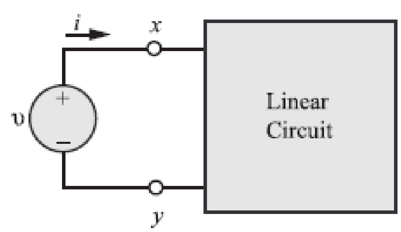


Figure 3.64
Voltage-driven circuit

(ii) Let all the independent sources within the linear network be turned off, that is $d_0 = 0$. Then, equation (3.11) becomes

$$i = c_0 v$$

For dimensional validity, c_0 must have the dimension of conductance. This enforces $c_0 = \frac{1}{R_t}$ where R_t is the equivalent resistance of the linear network as seen from the terminals $x - y$. Thus, equation (3.11) becomes

$$\begin{aligned} i &= \frac{1}{R_t}v - i_{sc} \\ &= \frac{1}{R_t}v - i_N \end{aligned}$$

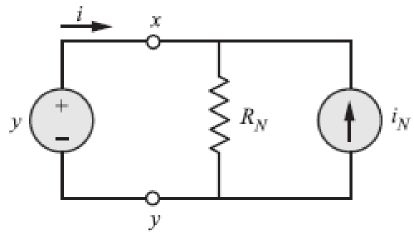
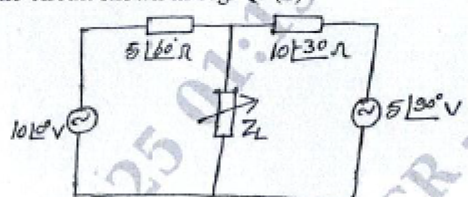


Figure 3.65 Norton's equivalent of voltage driven circuit

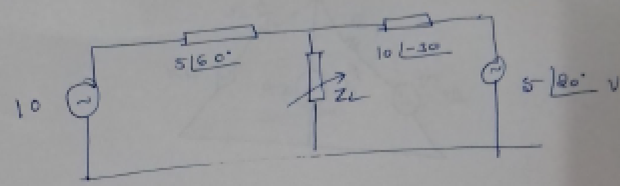
This expresses the voltage-current relationship at the terminals $x - y$ of the circuit in Fig. (3.65), validating that the two circuits of Figs. 3.64 and 3.65 are equivalents.

4 **b.** Find the value of Z_L for Maximum Power transfer and the value of Maximum power for the circuit shown in Fig. Q4(b).

Fig. Q4(b)

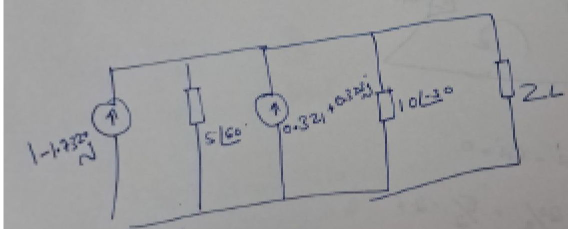


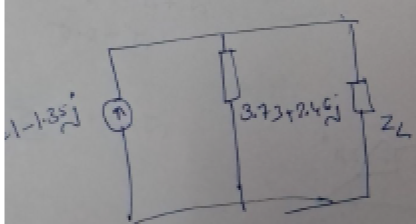
4 b.



For maximum power transfer

$Z_L = Z_{th}^*$



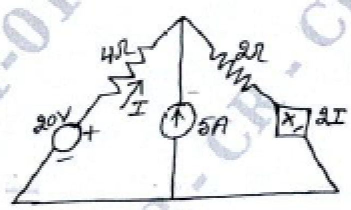


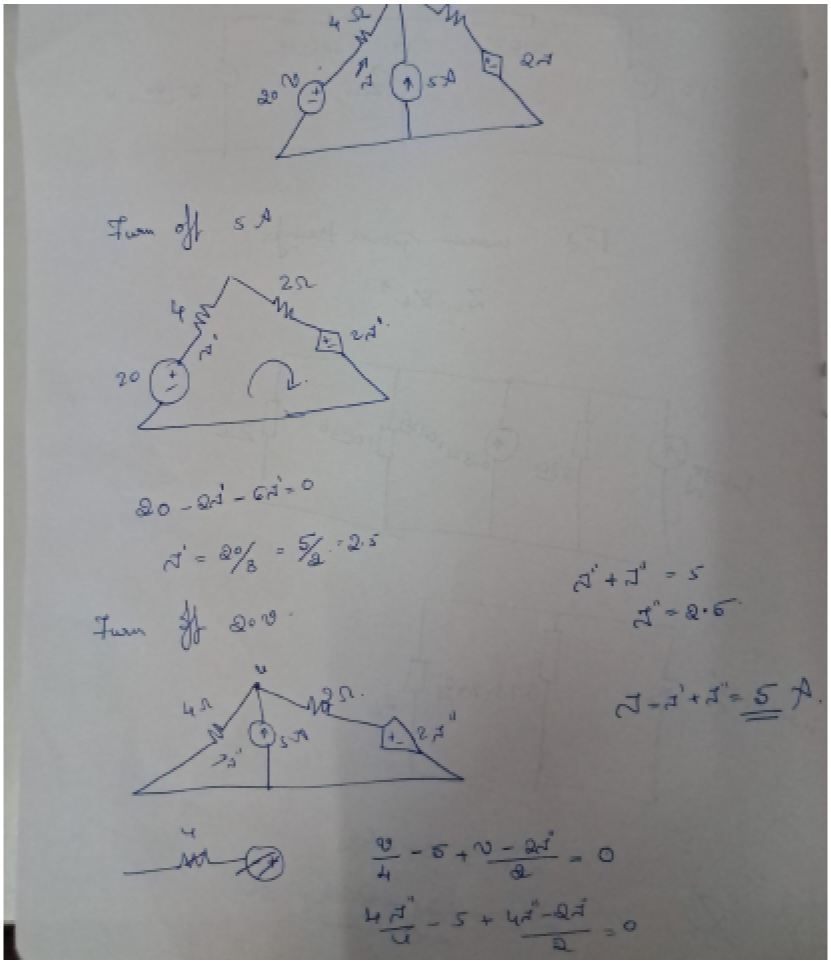
$Z_{th} = 3.73 + 2.46j$

$\therefore Z_L = \underline{3.73 - 2.46j}\ \Omega$

4	c.	Find current 'I' using Super position theorem for the circuit shown in Fig. Q4(c).

Fig. Q4(c)





Turn off 5 A

$20 - 2i' - 6i' = 0$

$i' = 20/8 = 5/2 = 2.5$

$i' + i'' = 5$

$i'' = 2.5$

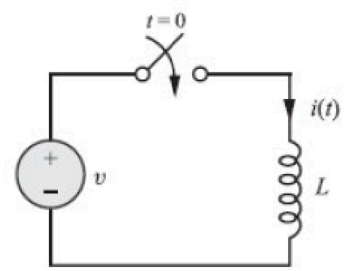
Turn off 20 V

$i = i' + i'' = \underline{\underline{5}}\ A.$

$\frac{v}{4} - 5 + \frac{v - 2i''}{2} = 0$

$\frac{4i''}{4} - 5 + \frac{4i'' - 2i''}{2} = 0$

Q.5	a.	Use the concepts of initial condition to illustrate the voltage behavior in inductor circuit for DC supply.



- The inductor : The switch is closed at $t = 0$

current through inductor is

$i = \frac{1}{L}\int_{-\infty}^{t} v\, d\tau$ — ①

$i_{(t)} = \frac{1}{L}\int_{-\infty}^{0^-} v\, d\tau + \frac{1}{L}\underbrace{\cancel{\int_{0^-}^{0^+} v\, d\tau}}_{0} + \frac{1}{L}\int_{0^+}^{t} v\, d\tau$

$i(t) = i(0^-) + \frac{1}{L}\int_{0^+}^{t} v\, d\tau$ — (A) ⇒ current in an inductor cannot change instantaneously.

⇒ $i(0^+) = i(0^-)$

5

b. In the circuit steady state is reached with switch 'K' open. The switch is closed at t = 0. Compute i , di/dt and d^2i/dt^2 at t = 0^+.

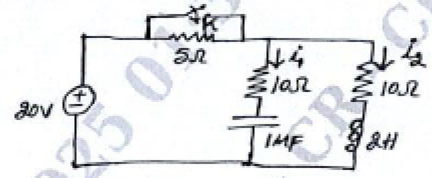


Fig. Q5(b)

5

c. The switch is moved from position (1) to position (2) at t = 0. The steady state has been reached before switching. Computer i , di/dt and d^2i/dt^2 at t = 0^+ for Fig. Q5(c).

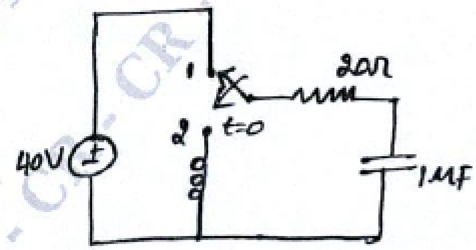
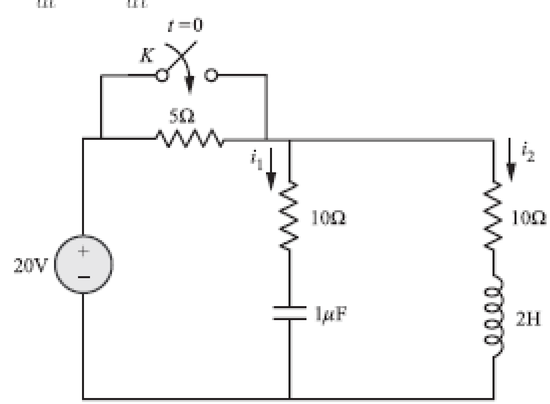


Fig. Q5(c)

In the circuit shown in Fig. 4.34, steady state is reached with switch K open. The switch is closed at $t = 0$.

Determine: i_1, i_2, $\frac{di_1}{dt}$ and $\frac{di_2}{dt}$ at $t = 0^+$



5b.

Figure 4.34

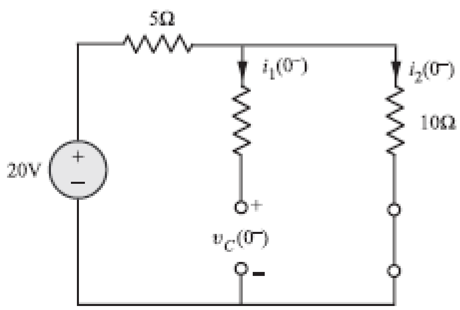
At $t = 0^-$, switch K is open and at $t = 0^+$, it is closed. At $t = 0^-$, the circuit is in steady state and appears as shown in Fig.4.35(a).

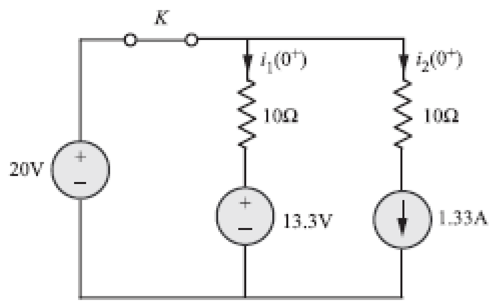
$$i_2(0^-) = \frac{20}{10+5} = 1.33\text{A}$$

Hence, $$v_C(0^-) = 10 i_2(0^-) = 10 \times 1.33 = 13.3\text{V}$$

Since current through an inductor cannot change instantaneously, $i_2(0^+) = i_2(0^-) = \mathbf{1.33\ A}$.
Also, $v_C(0^+) = v_C(0^-) = 13.3$V.
The equivalent circuit at $t = 0^+$ is as shown in Fig.4.35(b).

$$i_1(0^+) = \frac{20 - 13.3}{10} = \frac{6.7}{10} = \mathbf{0.67A}$$





For $t \geq 0^+$, the circuit is as shown in Fig.4.35(c).

Writing KVL clockwise for the left–mesh, we get

$$10 i_1 + \frac{1}{C}\int_{0^+}^{t} i_1(\tau)d\tau = 20$$

Differentiating with respect to t, we get

$$10\frac{di_1}{dt} + \frac{1}{C} i_1 = 0$$

Putting $t = 0^+$, we get

$$10\frac{di_1(0^+)}{dt} + \frac{1}{C} i_1(0^+) = 0$$

$$\Rightarrow \quad \frac{di_1(0^+)}{dt} = \frac{-1}{10 \times 1 \times 10^{-6}} i_1(0^+) = \mathbf{-0.67 \times 10^5 A/\,sec}$$

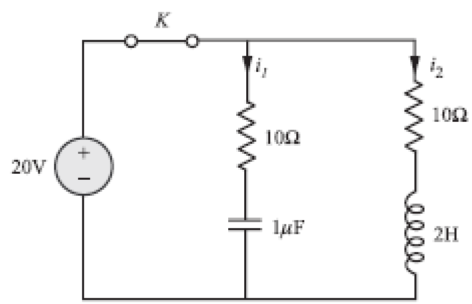


Figure 4.35(c)

Writing KVL equation to the path made of 20V $\rightarrow K \rightarrow$ 10Ω $\rightarrow$ 2H, we get

$$10 i_2 + \frac{2 di_2}{dt} = 20$$

At $t = 0^+$, the above equation becomes

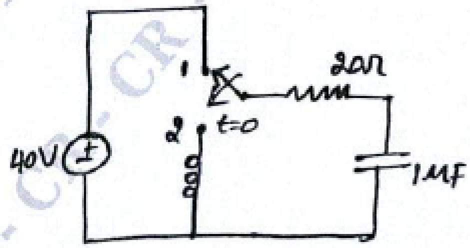
$$10 i_2(0^+) + \frac{2 di_2(0^+)}{dt} = 20$$

$$\Rightarrow \quad 10 \times 1.33 + \frac{2 di_2(0^+)}{dt} = 20$$

$$\Rightarrow \quad \frac{di_2(0^+)}{dt} = \mathbf{3.35A/\,sec}$$

5.c. The switch is moved from position (1) to position (2) at $t = 0$. The steady state has been reached before switching. Computer i , di/dt and d^2i/dt^2 at $t = 0^+$ for Fig. Q5(c).

Fig. Q5(c)



SOLUTION

The symbol for switch K implies that it is in position 1 at $t = 0^-$ and in position 2 at $t = 0^+$. Under steady-state condition, a capacitor acts as an open circuit. Hence at $t = 0^-$, the circuit diagram is as shown in Fig. 4.18(a).

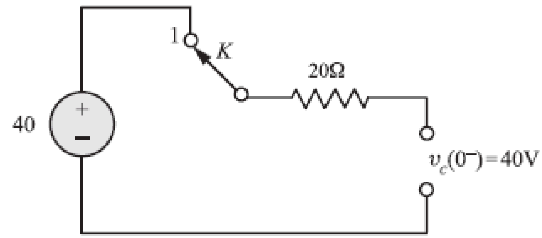


Figure 4.18(a)

We know that the voltage across a capacitor cannot change instantaneously. This means that $v_C\left(0^+\right) = v_C\left(0^-\right) = 40$ V.

At $t = 0^-$, inductor is not energized. This means that $i\left(0^-\right) = 0$. Since current in an inductor cannot change instantaneously, $i\left(0^+\right) = i\left(0^-\right) = \mathbf{0}$. Hence, the circuit diagram at $t = 0^+$ is as shown in Fig. 4.18(b).

The circuit diagram for $t \geq 0^+$ is as shown in Fig.4.18(c).

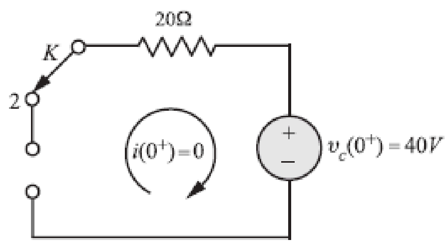


Figure 4.18(b)

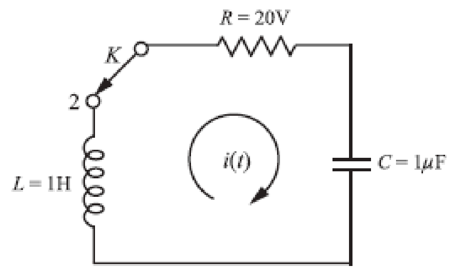


Figure 4.18(c)

Applying KVL clockwise, we get

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{0+}^{t} i(\tau)d\tau = 0$$

$$\Rightarrow \qquad Ri + L\frac{di}{dt} + v_C(t) = 0$$

At $t = 0^+$, we get

$$Ri(0^+) + L\frac{di(0^+)}{dt} + v_C(0^+) = 0$$

$$\Rightarrow \qquad 20 \times 0 + 1\frac{di(0^+)}{dt} + 40 = 0$$

$$\Rightarrow \qquad \frac{di(0^+)}{dt} = \mathbf{-40A/\sec}$$

Diferentiating equation (4.4) with respect to t, we get

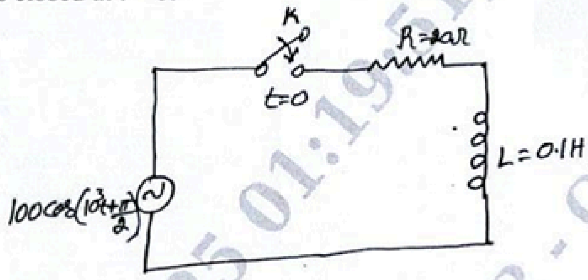
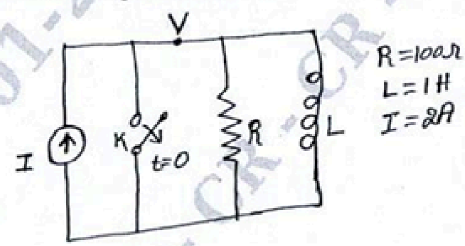
$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

Putting $t = 0^+$ in the above equation, we get

$$R\frac{di(0^+)}{dt} + L\frac{d^2i(0^+)}{dt^2} + \frac{i(0^+)}{C} = 0$$

$$\Rightarrow \qquad R \times (-40) + L\frac{d^2i(0^+)}{dt^2} + \frac{0}{C} = 0$$

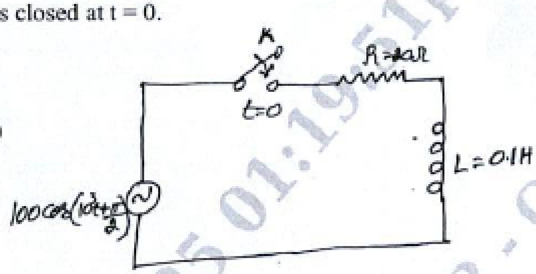
Hence $$\frac{d^2i(0^+)}{dt^2} = \mathbf{800A/\sec^2}$$

Q.6	**a.**	In the circuit shown in Fig. Q6(a), determine complete solution for current when switch 'K' is closed at t = 0. Fig. Q6(a) 	**10**	**L3**	**CO3**
	b.	Compute v , dv/dt , $d^2v/^{dt2}$ at t = 0^+ for the circuit shown in below Fig. Q6(b), when the switch K is opened at t = 0. Fig. Q6(b) 	**10**	**L4**	**CO3**

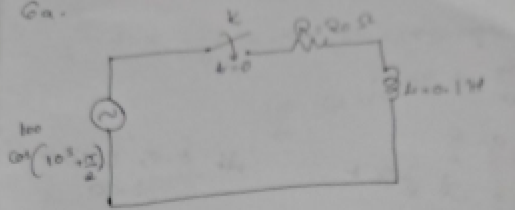
6a

In the circuit shown in Fig. Q6(a), determine complete solution for current when switch 'K' is closed at t = 0.

Fig. Q6(a)



6a.



To obtain complete solution

i.e. i

At $t = 0^-$, $i(0^-) = 0 \Rightarrow i(0^+) = 0$.

At $t = 0$, switch is closed

Applying KVL to the circuit

$$80i + 0.1\frac{di}{dt} = 100\cos\left(10^3 t + \frac{\pi}{8}\right)$$

$$Ri + L\frac{di}{dt} = E \quad \text{———} \quad (1)$$

Solution for $i \rightarrow$

$i = i_f + i_N$,

eqn (1) can be rewritten as

$$\frac{di}{dt} + \frac{R}{L}i = \frac{v}{L} \quad \text{———} \quad (2)$$

Standard eqn $\rightarrow$

$$\frac{dy}{dt} + ay = x$$

$$a = \frac{R}{L}$$

$$i_n = A \cdot e^{-R/L\, t}.$$

At $t = 0^-$, $i(0^-) = 0$

$A = 0$

i_f is due to forcing function v/L.

From standard set of formulae

$$i_f = D\cos\omega t + E\sin\omega t$$

Considering in eqn (2)

$$\frac{di_f}{dt} + \frac{R}{L} i_f = v/L$$

$$\frac{d}{dt}(D\cos\omega t + E\sin\omega t) + \frac{R}{L}(D\cos\omega t + E\sin\omega t) =$$

$$\frac{100}{L}\cos(10^3 t + \pi/2).$$

$$-D\omega\sin\omega t + E\omega\cos\omega t + \frac{R}{L}D\cos\omega t + \frac{RE}{L}\sin\omega t =$$

$$\frac{100}{L}\cos(10^3 t + \pi/2).$$

$$\left(-D\omega + \frac{RE}{L}\right)\sin\omega t + \left(E\omega + \frac{RD}{L}\right)\cos\omega t = \frac{100}{20}\cos 10^3 t$$

$\omega = 10^3 t$

Equating coefficients

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\left.\begin{aligned}\left(-D\omega + \frac{RE}{L}\right) &= 0 \\ E\omega + \frac{RD}{L} &= \frac{100}{20}\end{aligned}\right] \text{— (4)}$$

In the question $\omega = 10^3$, $R = 20\Omega$, $L = 0.1H$

Eqn (4) $\Rightarrow$

$$-D10^3 + 200E = 0$$
$$10^3 E + 200D = 1000$$

$D = 0.1923$

$E = 0.9615$

$$i = \underbrace{0.1923\cos\omega t + 0.9615\sin\omega t}_{i_p} + \underbrace{Ae^{-t/2}}_{i_{tr}}$$

$$i = 0.1923\cos\omega t + 0.9615\sin\omega t$$

6b

SOLUTION

The switch is opened at $t = 0$. This means that at $t = 0^-$, it is closed and at $t = 0^+$, it is open. Since $i_L(0^-) = 0$, we get $i_L(0^+) = 0$. The circuit at $t = 0^+$ is as shown in Fig. 4.23(a).

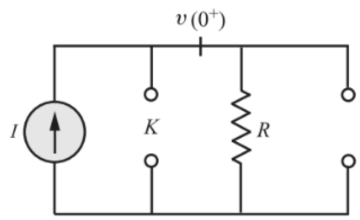


Figure 4.23(a)

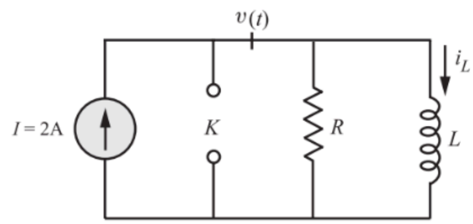


Figure 4.23(b)

$$\begin{aligned} v(0^+) &= IR \\ &= 2 \times 200 \\ &= \mathbf{400\ Volts} \end{aligned}$$

Refer to the circuit shown in Fig. 4.23(b).

For $t \geq 0^+$, the *KCL* at node $v(t)$ gives

$$I = \frac{v(t)}{R} + \frac{1}{L}\int_{0^+}^{t} v(\tau)d\tau \tag{4.8}$$

Differentiating both sides of equation (4.8) with respect to t, we get

$$0 = \frac{1}{R}\frac{dv(t)}{dt} + \frac{1}{L}v(t) \tag{4.8a}$$

At $t = 0^+$, we get

$$\frac{1}{R}\frac{dv(0^+)}{dt} + \frac{1}{L}v(0^+) = 0$$

$$\Rightarrow \quad \frac{1}{200}\frac{dv(0^+)}{dt} + \frac{1}{1} \times 400 = 0$$

$$\Rightarrow \quad \frac{dv(0^+)}{dt} = \mathbf{-8 \times 10^4\ V/\sec}$$

Again differentiating equation (4.8a), we get

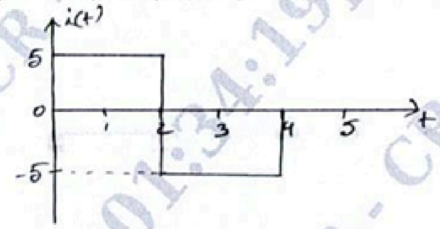
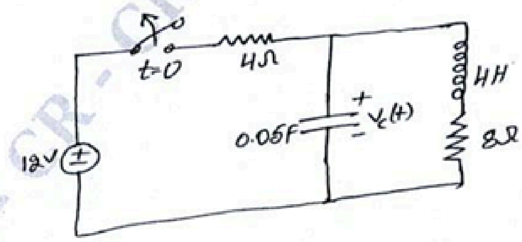
$$\frac{1}{R}\frac{d^2v(t)}{dt^2} + \frac{1}{L}\frac{dv(t)}{dt} = 0$$

At $t = 0^+$, we get

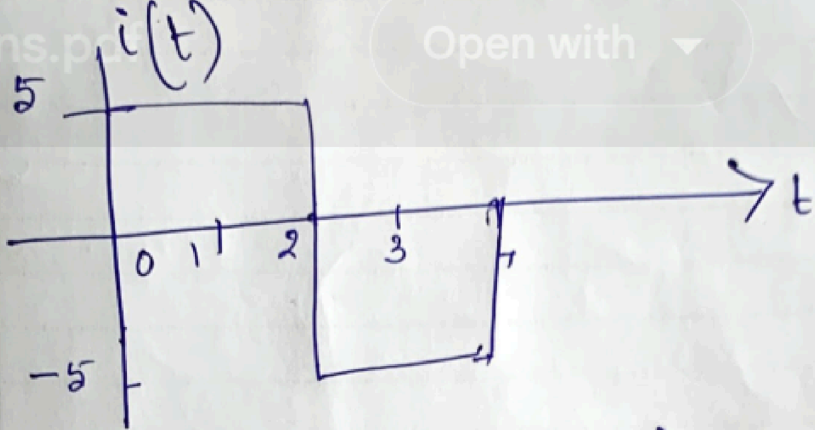
$$\frac{1}{200}\frac{d^2v(0^+)}{dt^2} + \frac{1}{1}\frac{dv(0^+)}{dt} = 0$$

$$\begin{aligned} \Rightarrow \quad \frac{d^2v(0^+)}{dt^2} &= 200 \times 8 \times 10^4 \\ &= \mathbf{16 \times 10^6\ V/\sec^2} \end{aligned}$$

Module – 4

			Marks	Level	CO
Q.7	a.	Using waveform synthesis method to express the voltage pulse interms of unit step. Find i) L{i(t)} ii) L{∫ i(t).dt}.  Fig. Q7(a)	8	L3	CO4
	b.	State and prove initial value and final value theorem for Laplace transform.	6	L2	CO4
	c.	Obtain the Laplace transform of step and ramp function with relevant expressions.	6	L3	CO4
		OR			
Q.8	a.	Determine $i_L(t)$ for $t \geq 0$ using Laplace transform for circuit shown in Fig. Q8(a).  Fig. Q8(a)	10	L3	CO4

7a



$$i(t) = 5u(t) - 5u(t-2) - 5u(t-2) - [-5u(t-4)].$$

$$i(t) = 5u(t) - 10u(t-2) + 5u(t-4).$$

i) $\mathcal{L}\{i(t)\} = \frac{5}{s} - \frac{10e^{-2s}}{s} + \frac{5e^{-4s}}{s}.$

ii) $\mathcal{L}\{\int i(t)\,dt =$

ii) $\mathcal{L}\left\{\int i(t)\,dt =\right.$

Ans If $\mathcal{L}\{x(t)\} = X(s).$

$$y(t) = \int_0^t x(\tau)\,d\tau,$$

$$\mathcal{L}\{y(t)\} = Y(s) = \frac{X(s)}{s}$$

$$\therefore \mathcal{L}\left\{\int i(t)\,dt = \frac{I(s)}{s} = \frac{5}{s^2} - \frac{10e^{-2s}}{s^2} + \frac{5e^{-4s}}{s^2}\right.$$

7b

5.5.9 Initial-value theorem

The initial-value theorem allows us to find the initial value $x(0)$ directly from its Laplace transform $X(s)$.

If $x(t)$ is a causal signal,

then, $$x(0) = \lim_{s\to\infty} sX(s)$$

***Proof*:**

To prove this theorem, we use the time differentiation property.

$$\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = sX(s) - x(0) = \int_0^\infty \frac{dx}{dt}e^{-st}dt$$

If we let $s \to \infty$, then the integral on the right side of equation (5.10) vanishes due to damping factor, e^{-st}.

Thus, $$\lim_{s\to\infty}[sX(s) - x(0)] = 0$$

$$\Rightarrow \quad x(0) = \lim_{s\to\infty} sX(s)$$

5.5.10 Final-value theorem

The final-value theorem allows us to find the final value $x(\infty)$ directly from its Laplace transform $X(s)$.

If $x(t)$ is a causal signal,

then $$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

***Proof*:**

The Laplace transform of $\dfrac{dx(t)}{dt}$ is given by

$$sX(s) - x(0) = \int_0^{\infty} \frac{dx(t)}{dt} e^{-st} dt$$

Taking the limit $s \to 0$ on both the sides, we get

$$\begin{aligned}
\lim_{s\to 0}[sX(s) - x(0)] &= \lim_{s\to 0} \int_0^{\infty} \frac{dx(t)}{dt} e^{-st} dt \\
&= \int_0^{\infty} \frac{dx(t)}{dt} \left[\lim_{s\to 0} e^{-st}\right] dt \\
&= \int_0^{\infty} \frac{dx(t)}{dt} dt \\
&= x(t)\big|_0^{\infty} \\
&= x(\infty) - x(0)
\end{aligned}$$

Since, $$\lim_{s\to 0}[sX(s) - x(0)] = \lim_{s\to 0}[sX(s)] - x(0)$$

we get, $$x(\infty) - x(0) = \lim_{s\to 0}[sX(s) - x(0)]$$

Hence, $$x(\infty) = \lim_{s\to 0}[sX(s)]$$

This proves the final value theorem.

7c

Laplace Transform of unit step function:

$$u(t) = \begin{cases} 1 & t \geq 0+ \\ 0 & t \leq 0- \end{cases}$$

$$\mathcal{L}\{u(t)\} = F(s) = \int_0^\infty 1 \cdot e^{-st}\,dt = \left[-\frac{1}{s}e^{-st}\right]_0^\infty$$

$$= \frac{1}{s}.$$

Laplace transform of ramp function.

Ramp function is defined as $x(t) = tu(t)$

$$\mathcal{L}\{x(t)\} = \mathcal{L}\{tu(t)\} = \int_0^\infty tu(t)e^{-st}\,dt$$

$$= \int_0^\infty te^{-st}\,dt$$

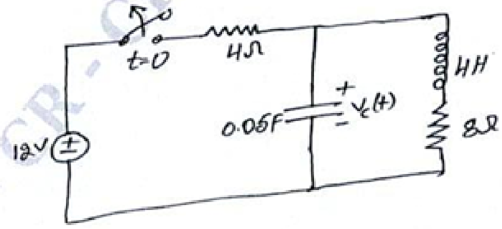
$$= \left[\frac{te^{-st}}{s}\right]_0^\infty - \int_0^\infty (1)\frac{e^{-st}}{-s}\,dt$$

$$= 0 - \left[\frac{e^{-st}}{s^2}\right]_0^\infty = \left(0 - \cancel{\frac{1}{s^2}}\right) = \frac{1}{s^2}$$

OR

Q.8	a.	Determine $i_L(t)$ for $t \geq 0$ using Laplace transform for circuit shown in Fig. Q8(a).	10	L3	CO4

Fig. Q8(a)



8a

SOLUTION

At $t = 0^-$, switch is closed and at $t = 0^+$, it is open. Let us assume that at $t = 0^-$, the circuit is in steady state. In steady state, capacitor is open and inductor is short. The equivalent circuit at $t = 0^-$ is as shown in Fig. 5.25(a).

$$i_L(0^-) = \frac{12}{8+4} = 1\text{A}$$

$$v_C(0^-) = 1 \times 8 = 8\text{V}$$

Therefore, $$i_L(0) = i_L(0^+) = i_L(0^-) = 1\text{A}$$

$$v_C(0) = v_C(0^+) = v_C(0^-) = 8\text{V}$$

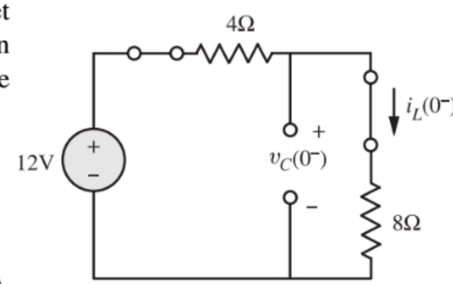


Figure 5.25(a)

For $t \geq 0^+$, the circuit in frequency domain is as shown in Fig. 5.25(b). We will use *KVL* to find $i_L(t)$. Hence, we use series circuits to represent both the capacitor and inductor in the frequency domain. These series circuits contain voltage sources rather than current sources. It is easier to account for voltage sources than current sources when writing mesh equations. This justifies the selection of series representation for both the capacitor and inductor.

Applying *KVL* clockwise to the right mesh, we get

$$\frac{-8}{s}+\frac{20}{s}I_L(s)+4sI_L(s)-4+8I_L(s)=0$$

$$\Rightarrow \quad \frac{8}{s}+4=\left[\frac{20}{s}+8+4s\right]I_L(s)$$

$$\Rightarrow \quad I_L(s)=\frac{2+s}{s^2+2s+5}=\frac{(s+1)+1}{(s+1)^2+4}$$

$$\Rightarrow \quad I_L(s)=\frac{s+1}{(s+1)^2+2^2}+\frac{1}{2}\left[\frac{2}{(s+1)^2\times 2^2}\right]$$

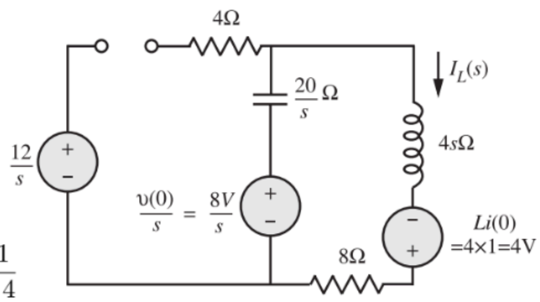


Figure 5.25(b)

We know the Laplace transform pairs:

$$\mathcal{L}\left\{e^{-at}\cos bt\right\}=\frac{s+a}{(s+a)^2+b^2}$$

and

$$\mathcal{L}\left\{e^{-at}\sin bt\right\}=\frac{b}{(s+a)^2+b^2}$$

Hence,

$$\boldsymbol{i_L(t)=\left[e^{-t}\cos 2t+\frac{1}{2}e^{-t}\sin 2t\right]u(t)\,\mathrm{A}}$$

b.	Find the Laplace transform of the periodic signal x(t) as shown in Fig. Q8(b).	10	L3	CO

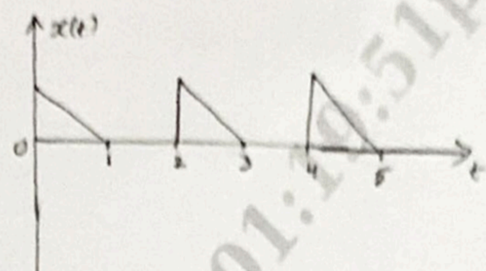


Fig. Q8(b)

8b

SOLUTION

From Fig. 5.17, we find that $T = 2$ Seconds.

The signal $x(t)$ considered over one period is donoted as $x_1(t)$ and shown in Fig. 5.18(a).

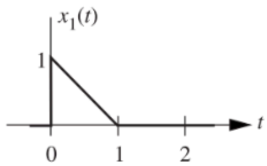


Figure 5.18(a)

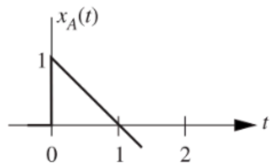


Figure 5.18(b)

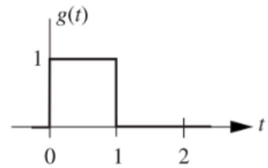


Figure 5.18(c)

The signal $x_1(t)$ may be viewed as the multiplication of $x_A(t)$ and $g(t)$.

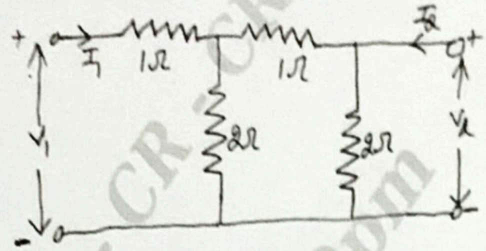
$$
\begin{aligned}
\text{That is,}\quad x_1(t) &= x_A(t)g(t) \\
&= [-t+1][u(t)-u(t-1)] \\
\Rightarrow \quad x_1(t) &= -tu(t) + tu(t-1) + u(t) - u(t-1) \\
&= -tu(t) + (t-1+1)u(t-1) + u(t) - u(t-1) \\
&= -tu(t) + (t-1)u(t-1) + u(t-1) + u(t) - u(t-1) \\
&= u(t) - tu(t) + (t-1)u(t-1) \\
&= u(t) - r(t) + r(t-1)
\end{aligned}
$$

Taking Laplace Transform, we get

$$
\begin{aligned}
X_1(s) &= \frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^2}e^{-s} \\
&= \frac{s-1+e^{-s}}{s^2}
\end{aligned}
$$

Hence,

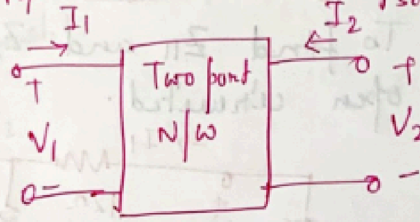
$$\boldsymbol{X(s) = \frac{X_1(s)}{1-e^{-sT}} = \frac{(s-1+e^{-s})}{s^2(1-e^{-2s})}}$$

		Module – 5			
Q.9	a.	Define Z – parameters. Determine Y parameters interms if Z – parameters.	6	L3	CO5
	b.	Show that resonant frequency is geometric mean of cut off frequency in series R – L – C circuit.	7	L3	CO5
	c.	Apply the two – port network analysis technique to determine ABCD – parameters of the network shown in Fig. Q9(c).  Fig. Q9(c)	7	L3	CO5

9a

$\cdots 22 = \quad V_2 \big|_{V_1=0}$

Impedance Parameter! (Linear N/ω, contains no independent sources)



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

In matrix form $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

Z parameters are called <u>open circuit impedance parameters</u>

$Z_{11} = \dfrac{V_1}{I_1}\Big|_{I_2=0}$ ⇒ open circuit I/P impedance

$Z_{22} = \dfrac{V_2}{I_2}\Big|_{I_1=0}$ ⇒ open circuit o/p impedance

$Z_{12} = \dfrac{V_1}{I_2}\Big|_{I_1=0}$, $Z_{21} = \dfrac{V_2}{I_1}\Big|_{I_2=0}$ ⇒ open circuit transfer impedance

If $Z_{12} = Z_{21}$ ⇒ Reciprocal N/ω

If all the Z parameter are identical → symmetrical N/ω.

a) Y parameters in terms of Z parameters

Y parameters equations are:

$$I_1 = y_{11}V_1 + y_{12}V_2 \quad \text{--- } (1)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \quad \text{--- } (2)$$

Z parameters eqns are:

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{--- } (3)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{--- } (4)$$

$$\Delta Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} = Z_{11}Z_{22} - Z_{12}Z_{21}$$

Using ~~Cramer's rule~~:

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\Delta Z} = \frac{V_1 Z_{22} - Z_{12}V_2}{\Delta Z}$$

$$I_1 = \frac{Z_{22}V_1}{\Delta Z} - \frac{Z_{12}V_2}{\Delta Z} \quad \text{--- } (5)$$

$$I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\Delta Z}$$

$$I_2 = \frac{Z_{11}V_2 - Z_{21}V_1}{\Delta Z} = -\frac{Z_{21}}{\Delta Z}(V_1) + \frac{Z_{11}}{\Delta Z}V_2 \quad \text{--- } (6)$$

Compare eqn (1) with (5) and (2) with (6).

$$y = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix} \mho$$

9b

Ex1 Show $f_0 = \sqrt{f_1 f_2}$ and $B.W = \frac{R}{2\pi L}$

Current in the series RLC circuit is given by,

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \quad -(1)$$

Also at ~~resonance~~ cut off frequency.

$$I = \frac{I_0}{\sqrt{2}}$$, where I_0 is maximum current

$$I = \frac{V}{R\sqrt{2}} \quad \text{--- (2)}$$

Equating (1) and (2)

$$\frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{R\sqrt{2}}$$

$$\Rightarrow 2R^2 = R^2 + (X_L - X_C)^2$$

$$\Rightarrow R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\text{or } \omega L - \frac{1}{\omega C} = \pm R. \quad \text{--- (3)}$$

At upper cut off frequency, f_2

$$R = \omega_2 L - \frac{1}{\omega_2 C} \quad \text{--- (4)}$$

At lower cut off frequency

$$-R = \omega_1 L - \frac{1}{\omega_1 C} \quad \text{--- (5)}$$

Adding (4) and (5)

$$\omega_2 L - \frac{1}{\omega_2 C} + \omega_1 L - \frac{1}{\omega_1 C} = R - R$$

$$\Rightarrow L(\omega_2 + \omega_1) - \frac{1}{C}\left(\frac{1}{\omega_2} + \frac{1}{\omega_1}\right) = 0$$

$$\Rightarrow L(\omega_2 + \omega_1) = \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)$$

$$L = \frac{1}{C} \times \frac{1}{\omega_1 \omega_2}$$

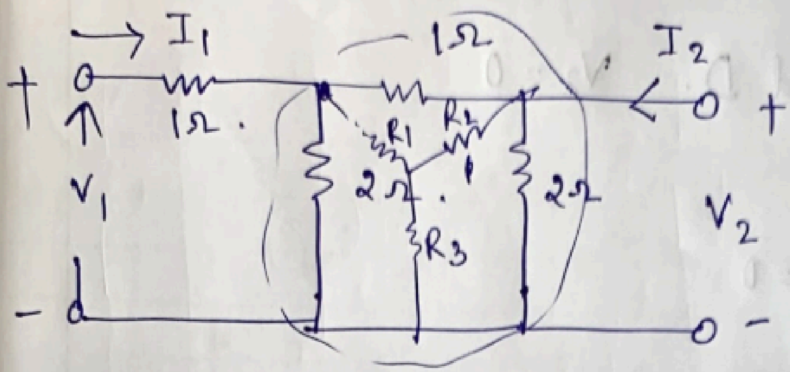
(4)

$$\Rightarrow \omega_1 \omega_2 = \frac{1}{LC}$$

$$\Rightarrow \omega_1 \omega_2 = \omega_0^2 \quad \left[\text{as } \omega_0^2 = \frac{1}{LC}\right].$$

$$\Rightarrow \boxed{f_1 f_2 = f_0^2} \Rightarrow$$ In series resonance circuit resonant frequency f_0 is geometrical mean of f_1 & f_2.

9c



$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$A = \dfrac{V_1}{V_2}\Big|_{I_2=0}$

$C = \dfrac{I_1}{V_2}\Big|_{I_2=0}$

$\Delta \to Y$ Conver

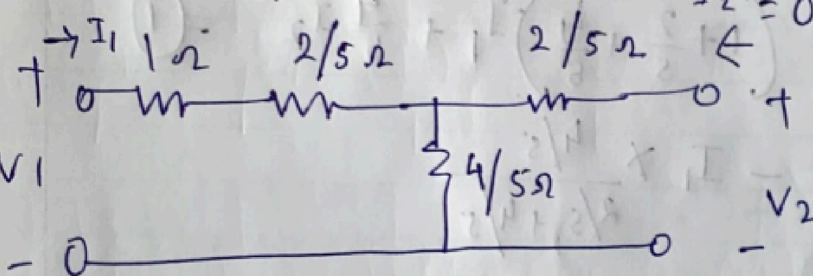
$R_1 = \dfrac{2\times1}{2+1+2} = \dfrac{2}{5}\,\Omega$

$R_2 = \dfrac{2\times1}{2+1+2} = \dfrac{2}{5}$

$R_3 = \dfrac{2\times2}{5} = \dfrac{4}{5}$

$\therefore$ Given network

$I_2 = 0$



To get A and C.

$$V_1 = \left(1 + \frac{2}{5} + \frac{4}{5}\right) I_1$$

$$V_2 = \frac{4}{5} I_1$$

$$A \doteq \frac{V_1}{V_2}\bigg|_{I_2=0} = \frac{7/5}{4/5} = \frac{7}{4}$$

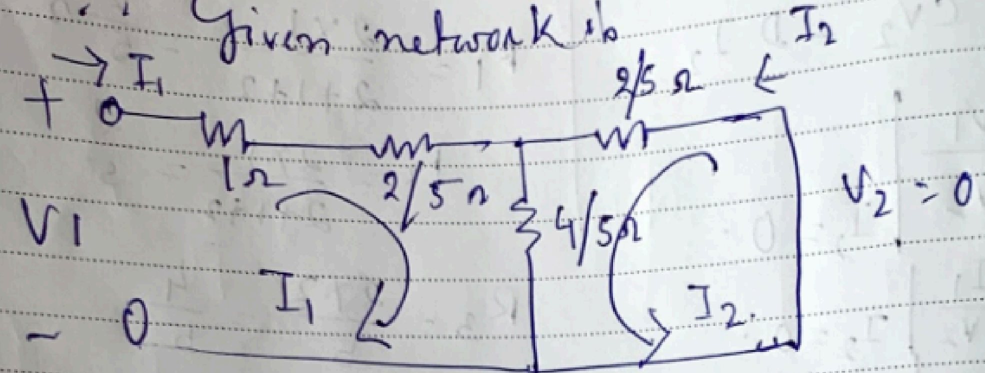
$$C = \frac{I_1}{V_2} = \frac{5}{4}\ \mho$$

To get B and D. $V_2 = 0$

$$B = \frac{V_1}{-I_2}\Bigg|_{V_2=0}$$

$$D = \frac{I_1}{-I_2}\Bigg|_{V_2=0}$$

∴ Given network is



$$V_1 = \left(1 + \frac{2}{5} + \frac{4}{5}\right) I_1 + \frac{4}{5} I_2 \quad \text{—(1)}$$

$$I_2 = -I_1 \times \frac{4/5}{2/5 + 4/5}$$

$$= -I_1 \times \frac{\cancel{4}\,2}{\cancel{6}\,3}$$

$$D: \frac{I_1}{-I_2} = \frac{3}{2}$$

from (1).

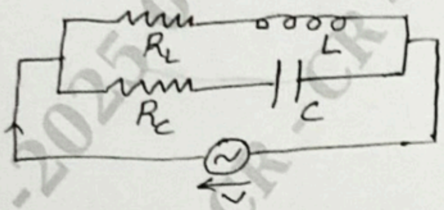
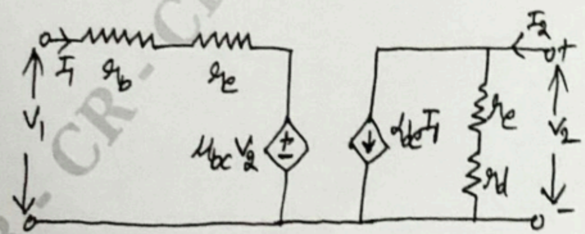
$$V_1 = \frac{11}{5}\left(-I_2 \times \frac{3}{2}\right) + \frac{4}{5} I_2$$

$$= \left(\frac{-33+8}{10}\right) I_2 = -\frac{\overset{5}{\cancel{25}}}{\underset{2}{\cancel{10}}} I_2$$

$$= -\frac{5}{2} I_2$$

$$\therefore D = \frac{V_1}{-I_2} = \frac{5}{2}\ \Omega.$$

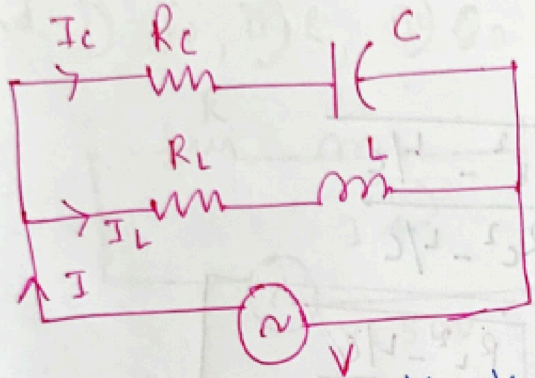
OR

Q.10	**a.**	Derive the expression for the resonant frequency of the circuit shown in Fig. Q10(a). Also show that the circuit resonate at all frequency if $R_L = R_C = \sqrt{\frac{L}{C}}$. Fig. Q10(a) 	**10**	**L3**	**CO5**
	b.	The model of a transistor in the CE mode is shown in Fig. Q10(b). Determine the h – parameters. Fig. Q10(b) 	**10**	**L3**	**CO5**

* * * * *

10a

Generalised Practical Parallel Resonance Circuit



Total admittance $Y = Y_L + Y_C$

$$Y_L = \frac{1}{Z_L} = \frac{1}{R_L + jX_L} = \frac{R_L - jX_L}{R_L^2 + \omega^2 L^2} \quad \text{--- (1)}$$

Also $Y_C = \frac{1}{Z_C} = \frac{1}{R_C - jX_C} =$

$$Y_C = \frac{R_C + j\left(\frac{1}{\omega C}\right)}{R_C^2 + \frac{1}{\omega^2 C^2}} \quad \text{--- (2)}$$

$$\therefore Y = \frac{R_L}{R_L^2 + \omega^2 L^2} - \frac{j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}} + \frac{j/\omega C}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

$$\Rightarrow Y = \left[\frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}}\right] + j\left[\frac{1/\omega C}{R_C^2 + \frac{1}{\omega^2 C^2}} - \frac{\omega L}{R_L^2 + \omega^2 L^2}\right]$$

To find resonant frequency,

$$\frac{1/\omega_0 C}{R_C^2 + \frac{1}{\omega_0^2 C^2}} - \frac{\omega_0 L}{R_L^2 + \omega_0^2 L^2} = 0$$

Solving

$$\omega_0 = \sqrt{\frac{1}{LC}}\,\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

or $$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}}$$

Note! if $R_L^2 = R_C^2 = L/C$.

then the circuit will resonate for all the frequencies or

$$R_L = R_C = \sqrt{L/C}.$$

The model of a transistor in the CE mode is shown in Fig. Q10(b). Determine the h – parameters.

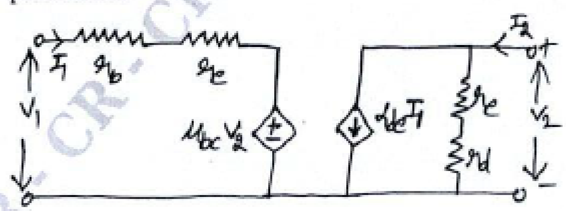
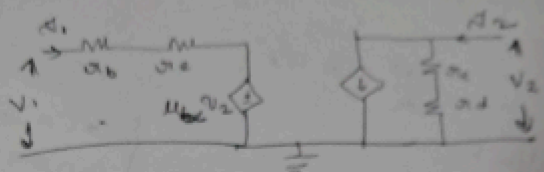


Fig. Q10(b)

10b

10b.



To find h-parameters.

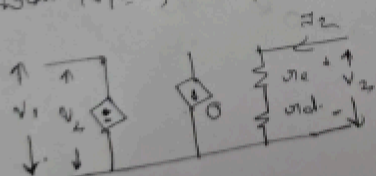
$v_1 = h_{11} i_1 + h_{12} v_2$

$i_2 = h_{21} i_1 + h_{22} v_2$

while $i_1 = 0$, $h_{12} = v_1 / v_2$; $h_{22} = \frac{i_2}{v_2}$

with $v_2 = 0$, $h_{11} = v_1 / i_1$; $h_{21} = \frac{i_2}{i_1}$

with $i_1 = 0$, $v_1 = v_2 \cdot \mu_{bc}$



$-v_2 + i_2(r_e + r_d) = 0$

$v_2 = i_2(r_e + r_d)$

$\boxed{h_{11} = \mu_{bc}}$

$\boxed{h_{22} = \frac{i_2}{V_2} = \frac{1}{r_e + r_d}}$

10b.

With $V_2 = 0$

$$I_2 = \alpha_{dc} I_1$$

$$\boxed{\frac{I_2}{I_1} = h_{21} = \alpha_{dc}}$$

$$v_1 - I_1(r_b + r_e) = 0$$

$$v_1 = I_1(r_b + r_e)$$

$$\boxed{h_{11} = \frac{v_1}{I_1} = r_b + r_e}$$