CBCS SCHEME

USN

BEC502

Fifth Semester B.E/B.Tech. Degree Examination, Dec.2024/Jan.2025
Digital Signal Processing

Time: 3 hrs. Max. Marks:100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
1	a.	List and discuss different discrete time signals.	7	L2	CO1
	b.	Explain the steps of converting along to digital signal interms of frequencies.	7	L2	CO1
	c.	Discuss the advantages and limitations of Digital Signal Processing (DSP).	6	L2	CO1
		OR			
2	a.	With an example, explain how to verify any signal is periodic or Not.	6	L2	CO1
	b.	Derive the equation for output of a LTI system and list the steps of convolution.	8	L3	CO2
	c.	Write a program to generate : i) Circuit step sequence ii) Sinusoidal sequence.	6	L3	CO2
		Module – 2			
3	a.	Describe the properties of Z – transformation.	7	L3	CO2
	b.	Show that Discrete Fourier Transform (DFT) is a Liner Transformation.	7	L3	CO2
	c.	Compute the A-point DFT of $x(n) = \{1, 1, 0, 0\}$.	6	L3	CO2
		OR			
4	a.	Compute the N-point DFT of, $x(n) = e^{j\omega mn}$.	6	L3	CO2
	b.	State and prove symmetry property of DFT for real valued sequence.	6	L3	CO2
	c.	Compute circular convolution of sequences : $x_1(n) = \{2, 1, 2, 1\}$ and $x_2(n) = \{1, 2, 3, 4\}$.	8	L3	CO2
		Module – 3			
5	a.	State and prove circular item shift property of DFT.	6	L3	CO2
	b.	Compare DFT and FFT with examples.	6	L2	CO3
	c.	Compute Radix – 2 DIT FFT of the following – sequence, $x(n) = n + 1$, for $0 \le n \le 7$.	8	L3	CO3
		OR			
6	a.	State and prove Parseval's theorem for – DFT's.	6	L3	CO2
	b.	Explain overlap – save method used for the convolution of long input sequences.	6	L2	CO3
	c.	Develop an algorithm for Radix – 2 FFT without using built in function.	8	L3	CO3

		Module – 4			
7	a.	Obtain the frequency response expression for the symmetric linear phase FIR filter.	8	L3	CO4
	b.	Compare different widows used to design FIR filters.	6	L2	CO4
	c.	Design an FIR filter using hamming window for N = 7. The desired frequency response is given by $$H_d(\omega) = \begin{cases} e^{-j3\omega} & \lvert\omega\rvert \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} < \lvert\omega\rvert \pi \end{cases}.$$	6	L3	CO4
		OR			
8	a.	Discuss the characteristics of practical frequency selective filters.	6	L3	CO4
	b.	Explain the steps of designing linear phase FIR high pass filter.	8	L2	CO4
	c.	Realize the system function of following FIR filter in cascade form. $H(z) = 1 - 2z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} - \frac{1}{2}z^{-4}$.	6	L3	CO4
		Module – 5			
9	a.	Explain the design procedure of analog Butter worth lowpass prototype – filter?	8	L3	CO5
	b.	Construct the system function in S – domain for N = A.	6	L3	CO5
	c.	Realize direct form – II for the IIR filter represented by $$y(n) - \frac{1}{4}y(n-1) + \frac{1}{8}y(x-2) = x(n) + \frac{1}{2}x(n-2).$$	6	L3	CO5
		OR			
10	a.	Design the digital IIR filter for following details. –3dB gain at 0.5π rads and the stop band automation of 15dB at 0.75π rads. Assume $T_s = 15$.	8	L3	CO5
	b.	Explain the significance of : i) Prewarping ii) Bilinear transformation.	6	L2	CO5
	c.	Obtain the direct form-I realization of following IIR filter : $$H(z) = \frac{1 + 0.4z^{-1}}{1 - 0.5z^{-1} + 0.06z^{-2}}.$$	6	L3	CO5

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1a List and discuss different discrete time signals. (VTU, Jan 2025, 7 Marks)

i) Even and odd signals.

A signal is said to be even signal if

$x(n) = x(-n)$ for all n.

For ex. $x(n) = \cos(n)$

A signal is said to be odd signal if

$x(n) = -x(-n)$ for all n

For ex. $x(n) = \sin(n)$

ii) Energy signals and power signals

A signal is said to be energy signal if its energy (E) is finite. ie,

$$0 < E < \infty$$

where $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

For example, $x(n) = \delta(n)$

$$= \begin{cases} 1 & @ n=0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

A signal is said to be power signal if its power (P) is finite, ie,

$0 < P < \infty$ where

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

For ex. $x(n) = u(n)$

$$= \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

iii) Periodic and aperiodic signals

A signal $x(n)$ is said to be periodic with period 'N' if

$$x(n) = x(n+N) \text{ for all } n,$$

where $0 < N < \infty$. and 'N' is an integer

If such 'N' does not exist, then the signal is said to be aperiodic.

Example of periodic signal

$$x(n) = \sin\left(\frac{2\pi}{3} n\right)$$

Example of aperiodic signal.

$$x(n) = \sin(n)$$

1b Explain the steps of converting analog to digital signal in terms of frequencies (VTU, Jan 2025, 7 Marks)

Sampling, quantizing and encoding are the steps involved in analog to digital conversion.

Sampling is the process of converting continuous time signal into discrete time signal.

Let $x(t)$ be a continuous time signal.

Let f_s be the sampling frequency and T_s be the sampling interval.

$$T_s = \frac{1}{f_s}$$

After sampling $x(t)$, we get

$$x(n) = x(nT_s)$$

For example, if

$$x(t) = \sin(2\pi f t),$$

after sampling, we get

$$x(n) = \sin(2\pi f n T_s)$$

$$= \sin\left(2\pi f n \frac{1}{f_s}\right)$$

$$= \sin\left(2\pi \frac{f}{f_s} n\right)$$

Note that the original signal $x(t)$ had a frequency of 'f' Hz.

After converting $x(t)$ into $x(n)$, the frequency of the resulting signal becom-

-es $\frac{f}{f_s}$.

In otherwords, the original signal $x(t)$ had a frequency of

$$\Omega = 2\pi f \quad \text{rad/s}$$

The converted signal $x(n)$ has a frequen-

-cy of $\omega = 2\pi \frac{f}{f_s}$ rad.

Hence, $f \Rightarrow \frac{f}{f_s}$ or

$$2\pi f \Rightarrow 2\pi \frac{f}{f_s} \quad \text{or}$$

$$\Omega \Rightarrow \frac{\Omega}{f_s}$$

is the mapping from analog frequeny to digital frequency.

The second step in the conversion of analog signal to digital signal is quantiz-ing.

Quantizing is the process of converting continuous range of amplitudes into a discrete set of amplitudes.

The third step is encoding.

Encoding is the process of representing the quantized amplitude interms of 0s and 1s.

1c Discuss the advantages and limitations of digital signal processing.
(VTU, Jan 2025, 6 Marks)

Advantages :

1. Digital signal processing is more flexible than analog signal processing in the

sense that digital systems can be reconfigured just by changing the program.

Reconfiguring analog system requires changing hardware.

2. Digital systems are more accurate than analog systems in doing comput-ations.

3. Digital signals are easy to store in devices such as memory card or hard disk.

4. Digital signal processing can be cheaper than analog signal processing due to flexibility it offers.

Limitations:

1. Most practical signals such as speech music, video are analog signals. processing them in digital domain requires extra hardware such as analog to digital converter and digital to analog converter.

2. Analog signals with high bandwidth require very high sampling rates in the conversion of analog to digital signal; which may be difficult to implement.

2a With an example, explain how to verify any signal is periodic or not.
(VTU, Jan 2025, 6 Marks)

By definition, a discrete time signal $x(n)$ is periodic with period 'N' if and only if

$$x(n+N) = x(n) \quad \text{for all } n$$

where N is an integer and $0 < N < \infty$

Consider a sinusoidal signal with frequency f.

$$x(n) = \sin(2\pi f n).$$

If $x(n)$ has to be periodic with period N, then

$$x(n+N) = x(n), \quad \forall n$$

$$\Rightarrow \sin(2\pi f(n+N)) = \sin(2\pi f n)$$

$$\Rightarrow \sin(2\pi f n + 2\pi f N) = \sin(2\pi f n)$$

This is possible only if

$2\pi f N = 2\pi m$ where m is an integer.

$\Rightarrow f = \frac{m}{N}$. (rational)

$\Rightarrow$ A discrete time sinusoidal is periodic only if its frequency is a rational number.

2b Derive the equation for output of LTI system and list the steps of convolution (VTU, Jan 2025, 8 Marks)

Consider an LTI system with impulse response $h(n)$.

Let $x(n)$ be the input and $y(n)$ be the output.

$$x(n) = \dots + x(-2)\delta(n+2) + x(-1)\delta(n+1) + x(0)\delta(n) + x(1)\delta(n-1) + x(2)\delta(n-2) + \dots$$

$$y(n) = \dots + x(-2)h(n+2) + x(-1)h(n+1) + x(0)h(n) + x(1)h(n-1) + x(2)h(n-2) + \dots$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \quad \ldots (1)$$

This is the equation for output of LTI system.

The sum on the RHS of Eq. (1) is called convolution sum.

Steps involved in linear convolution:

Given $x(n)$ and $h(n)$

Step 1: Plot $h(k)$ and reflect it to obtain $h(-k)$.

Step 2: Shift $h(-k)$ to right or left depending on the value of n.

Step 3: Multiply with $x(k)$ and find the sum.

3a Describe the properties of z-transform. (VTU, Jan 2025, 7 Marks)

1. Linearity property

If $x_1(n) \xleftrightarrow{z} X_1(z)$ and

$x_2(n) \xleftrightarrow{z} X_2(z)$, then

$$a x_1(n) + b x_2(n) \xleftrightarrow{z} a X_1(z) + b X_2(z)$$

where a, b are constants.

Resulting ROC = ROC1 $\cap$ ROC2

2. Time shift property

If $x(n) \xleftrightarrow{z} X(z)$, then

$x(n-k) \xleftrightarrow{z} z^{-k} X(z)$ with same ROC

3. Multiplication by exponential.

If $x(n) \xleftrightarrow{z} X(z)$ with $R_1 < ROC < R_2$

then $a^n x(n) \xleftrightarrow{z} X\left(\frac{z}{a}\right)$ with new ROC

$$|a| R_1 < ROC < |a| R_2$$

4. Differentiation in z-domain (Multiplication by ramp)

If $x(n) \xleftrightarrow{z} X(z)$, then

$$n x(n) \xleftrightarrow{z} -z \frac{d X(z)}{dz}$$

with same ROC.

5. Convolution in time domain

If $x(n) \xleftrightarrow{z} X(z)$ with ROC1

and $h(n) \xleftrightarrow{z} H(z)$ with ROC2

then $x(n) * h(n) \xleftrightarrow{z} X(z) H(z)$ with

ROC $=$ ROC1 $\cap$ ROC2

6. Time Reversal

If $x(n) \xleftrightarrow{z} X(z)$ with ROC1, then

$x(-n) \xleftrightarrow{z} X\left(\frac{1}{z}\right)$ with ROC $= \frac{1}{\text{ROC1}}$

7. Time scaling

If $x(n) \xleftrightarrow{z} X(z)$ with ROC1, then

$x(n/k) \xleftrightarrow{z} X(z^k)$ with ROC $= (\text{ROC1})^{\frac{1}{m}}$

8. Initial value theorem.

If $x(n)$ is a causal sequence, ie,

$x(n) = 0$ for $n < 0$, then

$$x(0) = \lim_{z \to \infty} X(z)$$

9. Final value theorem

If $x(n) \xleftrightarrow{z} X(z)$ then

$$x(\infty) = \lim_{z \to 1} (z-1) X(z)$$

3b Show that DFT is a linear transformation (VTU, Jan 2025, 7 Marks)

DFT, $$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} x(n) W_N^{kn} \quad \ldots (1)$$

where $W_N = e^{-j\frac{2\pi}{N}}$

Put $K=0$ in Eq. (1).

$$X(0) = \sum_{n=0}^{N-1} x(n) W_N^0$$

$$= x(0) W_N^0 + x(1) W_N^0 + x(2) W_N^0 + \cdots + x(N-1) W_N^0 \quad \ldots (2)$$

Put $k=1$ in Eq (1).

$$X(1) = \sum_{n=0}^{N-1} x(n) W_N^n$$

$$= x(0)W_N^0 + x(1)W_N^1 + x(2)W_N^2 + \dots + x(N-1)W_N^{(N-1)} \quad \dots (3)$$

Put $K=2$ in Eq (1).

$$X(2) = \sum_{n=0}^{N-1} x(n)\, W_N^{2n}$$

$$= x(0)W_N^0 + x(1)W_N^2 + x(2)W_N^4 + \dots + x(N-1)W_N^{2(N-1)} \quad \dots (4)$$

Put $K = N-1$ in Eq (1).

$$X(N-1) = \sum_{n=0}^{N-1} x(n)\, W_N^{(N-1)n}$$

$$= x(0)W_N^0 + x(1)W_N^{(N-1)} + x(2)W_N^{2(N-1)} + \dots + x(N-1)W_N^{(N-1)^2} \quad \dots (5)$$

We can put Eq. (2), (3), (4), (5) into a matrix form and evaluate DFT as follows.

$$X(K) = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & \cdots & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & \cdots & W_N^{(N-1)} \\ W_N^0 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & & & & \\ W_N^0 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)^2} \end{bmatrix}_{N\times N} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{bmatrix}_{N\times 1}$$

This proves that DFT is a linear transformation

3c Compute the 4-point DFT of $x(n) = (1,1,0,0)$ (VTU, Jan 2025, 6 Marks)

$$X(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

4a Compute the N-point DFT of $x(n) = e^{j\omega mn}$ (VTU, Jan 2025, 6 Marks)

$$x(n) = e^{j\omega mn}$$

As per VTU scheme of evaluation, we have to assume

$$\omega = \frac{2\pi}{N}$$

Then, $x(n) = e^{j\frac{2\pi}{N}mn}$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}mn}\, e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k-m)n}$$

This is in the form of $\sum_{n=0}^{N-1} \alpha^n$

where $\alpha = e^{-j\frac{2\pi}{N}(k-m)}$

We know that

$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} N & \text{for } \alpha = 1 \\ \dfrac{1-\alpha^N}{1-\alpha} & \text{for } \alpha \neq 1 \end{cases}$$

$\alpha = 1$ when $k = m$. Then

$$X(k) = N$$

$\alpha \neq 1$ when $k \neq m$. Then

$$X(k) = \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k-m)n}$$

$$= \sum_{n=0}^{N-1} \alpha^{n}$$

$$= \frac{1-\alpha^{N}}{1-\alpha}$$

$$= \frac{1-e^{-j\frac{2\pi}{N}(k-m)N}}{1-e^{-j\frac{2\pi}{N}(k-m)}}$$

$$= \frac{1-e^{-j2\pi(k-m)}}{1-e^{-j\frac{2\pi}{N}(k-m)}}$$

$$= \frac{1-1}{1-e^{-j\frac{2\pi}{N}(k-m)}}$$

$$= 0$$

Hence, $X(K) = \begin{cases} N & @\, k=m \\ 0 & \text{otherwise} \end{cases}$

$$= N\,\delta(k-m)$$

4b State and prove symmetry properties of DFT for real valued sequence.
(VTU, Jan 2025, 6 Marks)

N-point DFT, $X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$

$$= \sum_{n=0}^{N-1} [x_r(n) + j x_i(n)] \left[\cos\left(\tfrac{2\pi}{N}kn\right) - j\sin\left(\tfrac{2\pi}{N}kn\right)\right]$$

If $x(n)$ is real, $x_i(n) = 0$.

Then $X(K) = \sum_{n=0}^{N-1} x_r(n) \left[\cos\left(\tfrac{2\pi}{N}kn\right) - j\sin\left(\tfrac{2\pi}{N}kn\right)\right]$

$$X_R(K) = \sum_{n=0}^{N-1} x_r(n) \cos\left(\tfrac{2\pi}{N}kn\right)$$

$$X_I(K) = -\sum_{n=0}^{N-1} x_r(n) \sin\left(\tfrac{2\pi}{N}kn\right)$$

$\cos\left(\frac{2\pi}{N}kn\right)$ is circularly even

$\sin\left(\frac{2\pi}{N}kn\right)$ is circularly odd.

If $x(n)$ is circularly even, then

$$X_I(K) = 0$$

$$X_R(K) = \sum_{n=0}^{N-1} x_r(n) \cos\left(\frac{2\pi}{N} kn\right)$$

Hence, DFT will be real and circularly even.

If $x(n)$ is circularly odd, then

$$X_R(K) = 0$$

$$X_I(K) = -\sum_{n=0}^{N-1} x_r(n) \sin\left(\frac{2\pi}{N} kn\right)$$

Hence, DFT will be imaginary and circularly odd.

4c Compute the circular convolution of the sequences $x_1(n) = (2,1,2,1)$ and $x_2(n) = (1,2,3,4)$. (VTU, Jan 2025, 8 Marks)

$$y(n) = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

5a State and prove circular time shift property of DFT.
(Jan 2025, 6Marks)

Statement :

If $x(n) \xleftrightarrow{DFT} X(K)$, then

$$x(n-l)_N \xleftrightarrow{DFT} e^{-j\frac{2\pi}{N}Kl} X(K)$$

Proof:

N-point DFT of $x(n)$, $X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$

$\therefore$ N-point DFT of $x(n-l)_N$, $X'(K) = \sum_{n=0}^{N-1} x(n-l)_N e^{-j\frac{2\pi}{N}kn}$

Put $n-l = m$

$$\therefore X'(K) = \sum_{m=-l}^{-l+N-1} x(m) e^{-j\frac{2\pi}{N}K(m+l)}$$

$$= e^{-j\frac{2\pi}{N}Kl} \sum_{m=0}^{N-1} x(m) e^{-j\frac{2\pi}{N}km}$$

$$= e^{-j\frac{2\pi}{N}Kl} X(K)$$

5b Compare DFT and FFT with examples.
(VTU, Jan 2025, 6 Marks)

Suppose $N = 128$

	DFT	FFT
No. of complex multiplications.	N^2 (16384)	$\frac{N}{2} \log_2 N$ (448)
No. of complex additions	$N(N-1)$ (16256)	$N \log_2 N$ (896)
No. of real additions.	$4N^2 - 2N$ (65280)	$2N\log_2 N + N\log_2 N$ (2688)
No. of real multiplications.	$4N^2$ (65536)	$2N \log_2 N$ (1792)

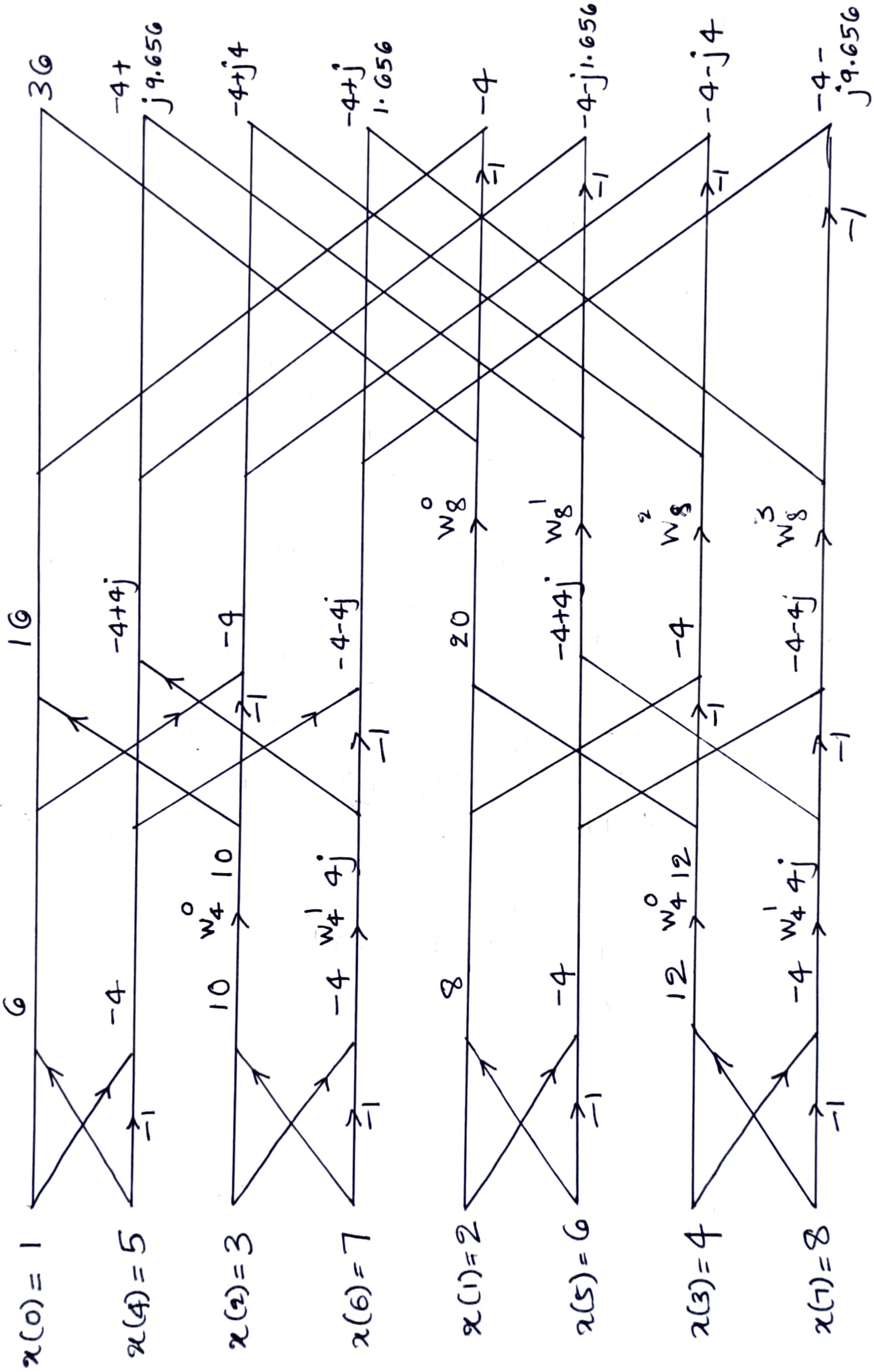
5c Compute DIT-FFT of $x(n) = n+1,\ 0 \le n \le 7$

(VTU, Jan 2025, 8 Marks)

$x(n) = n+1, \quad 0 \le n \le 7$

$= (1, 2, 3, 4, 5, 6, 7, 8)$

$\quad\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7$



6a State and prove Parseval's theorem for DFTs. (VTU, Jan 2025, 6 Marks)

$$E = \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{K=0}^{N-1} |X(K)|^2$$

Proof:

$$E = \sum_{n=0}^{N-1} |x(n)|^2$$

$$= \sum_{n=0}^{N-1} x(n)\, x^*(n)$$

$$= \sum_{n=0}^{N-1} x(n) \frac{1}{N} \sum_{K=0}^{N-1} X^*(K)\, e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X^*(K) \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X^*(K)\, X(K)$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} |X(K)|^2$$

6b Explain overlap-save method for the convolution of long sequences.
(VTU, Jan 2025, 6 Marks).

N - length of circular convolution

lh - length of h(n)

lx - length of x(n)

Step 1: divide x(n) into smaller blocks

$$x_1(n) = [\underbrace{0, 0, \ldots 0}_{lh-1}, x(0), x(1), \ldots]$$

$$x_2(n) = [\underbrace{x(\,), x(\,)}_{lh-1 \text{ previous samples}}, \ldots x(\,), x(\,)]$$

⋮

step 2: perform N-point circular conv.

$$y_1(n) = x_1(n) \; Ⓝ \; h(n)$$

$$y_2(n) = x_2(n) \; Ⓝ \; h(n)$$

⋮

step 3: drop first (lh-1) samples from $y_1(n), y_2(n) \ldots$ and write remaining samples.

7a Obtain the frequency response expression for symmetric linear phase FIR filter. (VTU, Jan 2025, 8 Marks)

Type I filter – h(n) is symmetric & N is odd.

$$H(z) = z^{-\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n) \left\{ z^{\frac{N-1-2n}{2}} + z^{-\frac{N-1-2n}{2}} \right\} \right]$$

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + 2 \sum_{n=0}^{\frac{N-3}{2}} h(n) \cos\left(\omega\left(\frac{N-1-2n}{2}\right)\right) \right]$$

Type II filter – h(n) symmetric & N is even

$$H(z) = z^{-\left(\frac{N-1}{2}\right)} \left[\sum_{n=0}^{\frac{N-2}{2}} h(n) \left\{ z^{\left(\frac{N-1-2n}{2}\right)} + z^{-\left(\frac{N-1-2n}{2}\right)} \right\} \right]$$

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \, 2 \sum_{n=0}^{\frac{N-2}{2}} h(n) \cos\left(\omega\left(\frac{N-1-2n}{2}\right)\right)$$

7b Compare different windows used to design FIR filters.

(VTU, Jan 2025, 6 Marks)

Window	Transition Width ($\Delta \omega$)	Stopband attenuation (dB)
Rectangular	$\frac{1.8\pi}{N}$	21
Bartlett	$\frac{6.1\pi}{N}$	26
Hanning	$\frac{6.2\pi}{N}$	44
Hamming	$\frac{6.6\pi}{N}$	53
Blackman	$\frac{11\pi}{N}$	74

7c Design an FIR filter using Hamming window for N=7. The desired frequency response is given by

$$H_d(\omega) = \begin{cases} e^{-j3\omega}, & |\omega| \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} < |\omega| < \pi \end{cases}$$

(VTU, Jan 2025, 6 Marks)

$\omega_c = \frac{3\pi}{4}$ rad.

$N = 7$

$\alpha = \frac{N-1}{2} = 3.$

$$h(n) = \begin{cases} \dfrac{\sin(\omega_c(n-\alpha))}{\pi(n-\alpha)} , & n \neq \alpha \\ \dfrac{\omega_c}{\pi} , & n = \alpha \end{cases}$$

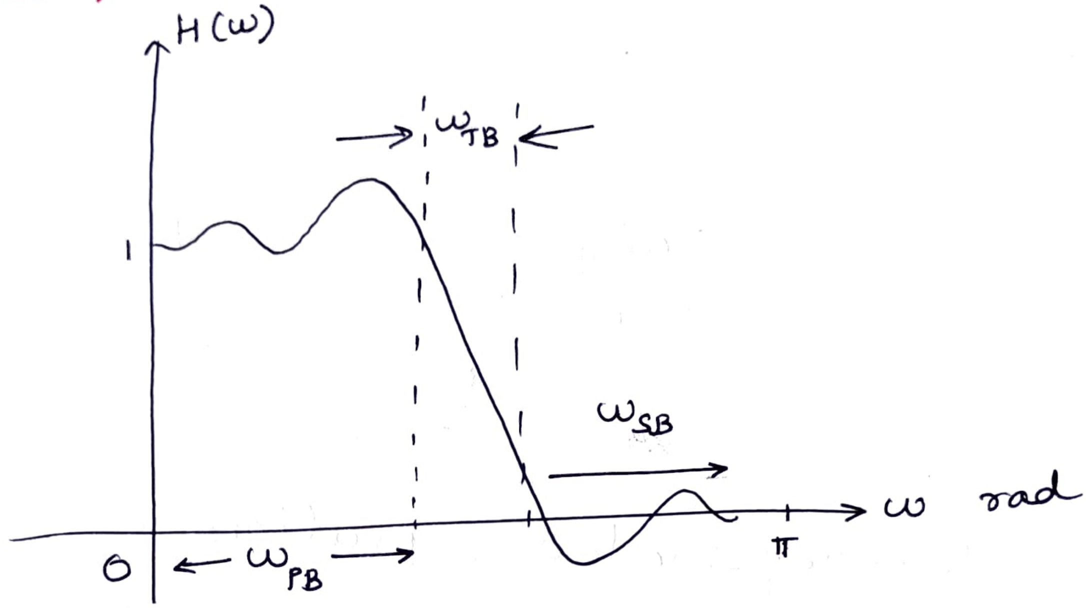
$$= \begin{cases} \dfrac{\sin\left(\frac{3\pi}{4}(n-3)\right)}{\pi(n-3)} , & n \neq 3 \\ \dfrac{3}{4} , & n = 3 \end{cases}$$

Hamming window equation

$$\omega(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) , \quad 0 \leq n \leq N-1$$

n	$h(n)$	$\omega(n)$	$h(n)\,\omega(n)$
0	0.075	0.08	0.006
1	-0.1592	0.31	-0.0493
2	0.2251	0.77	0.1733
3	0.75	1	0.75
4	0.2251	0.77	0.1733
5	-0.1592	0.31	-0.0493
6	0.075	0.08	0.006

8a Discuss the characteristics of practical frequency selective filters. (VTU, Jan 2025, 6 Marks)



ω_{PB} - passband

ω_{SB} - stopband

ω_{TB} - transition band.

1. There are only 2 bands in the frequency response of ideal filters - passband and stopband.
But, practically there is transition band also.

2. Ideally, frequency response is constant in passband.
Practically, there are ripples in passband.

8b Explain the steps of designing linear phase, FIR highpass filter.

(VTU, Jan 2025, 8 Marks)

Given the desired frequency response $H_d(\omega)$.

Step 1: Obtain impulse response

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} \, d\omega.$$

Step 2: Shift $h(n)$ by α to get $h(n-\alpha)$, where $\alpha = \frac{N-1}{2}$

Step 3: Multiply by window function to get final impulse response

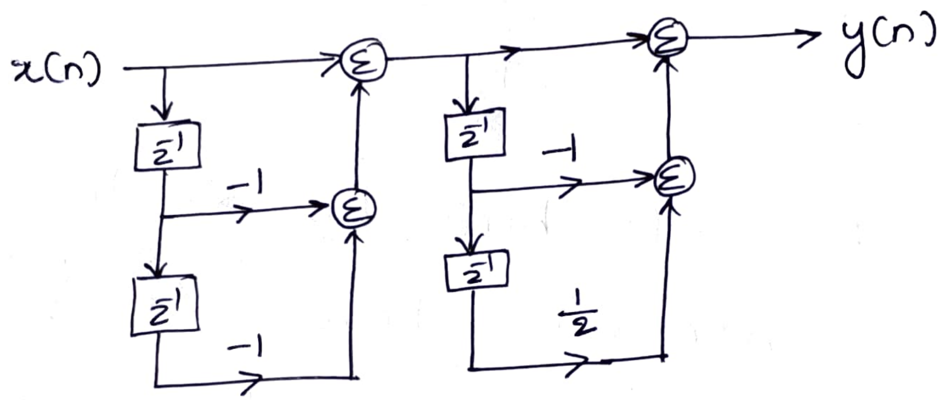
$$h'(n) = h(n-\alpha)\, \omega(n)$$

8c Realize the system function in cascade form (VTU, Jan 2025, 6 Marks)

$$H(z) = 1 - 2z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} - \frac{1}{2}z^{-4}$$

$$= \left(1 - z^{-1} - z^{-2}\right)\left(1 - z^{-1} + \frac{1}{2}z^{-2}\right)$$

Cascade realization



9a Explain the design procedure of analog Butterworth lowpass prototype filter.

(VTU, Jan 2025, 8 Marks)

Step 1: Find the order of the filter.

$$N = \frac{\log_{10}\left(\dfrac{10^{0.1 A_{PB}} - 1}{10^{0.1 A_{SB}} - 1}\right)}{2 \log_{10}\left(\dfrac{\Omega_{PB}}{\Omega_{SB}}\right)}$$

Step 2: Take cut-off frequency

$\Omega_c = 1$ rad/s

as it is a prototype filter

Step 3: Find location of poles.

$$S_k = -\Omega_c \sin\left(\frac{(2k+1)\pi}{2N}\right) + j\Omega_c \cos\left(\frac{(2k+1)\pi}{2N}\right)$$

$k = 0, 1, 2, \ldots, N-1$

Step 4: Find the transfer function $H(s)$

$$H(s) = \frac{\Omega_c^N}{\prod_{k=0}^{N-1} (s - s_k)}$$

9b Construct the system function in s-domain for $N=4$.

(VTU, Jan 2025, 6 Marks)

$N = 4 \qquad \Omega_c = 1$ rad/s

Poles:

$$S_k = -\Omega_c \sin\left(\frac{(2k+1)\pi}{2N}\right) + j\Omega_c \cos\left(\frac{(2k+1)\pi}{2N}\right)$$

$k = 0, 1, 2, 3$

$$S_0 = -\sin\left(\frac{\pi}{8}\right) + j\cos\left(\frac{\pi}{8}\right)$$

$$= -0.3827 + j0.9238$$

$$S_1 = -\sin\left(\frac{3\pi}{8}\right) + j\cos\left(\frac{3\pi}{8}\right)$$

$$= -0.9238 + j0.3827$$

$$S_2 = S_1^* = -0.9238 - j0.3827$$

$$S_3 = S_0^* = -0.3827 - j0.9238$$

System function

$$H(s) = \frac{\Omega_c^N}{(s-s_0)(s-s_1)(s-s_2)(s-s_3)}$$

Consider

$$(s-s_0)(s-s_3) = (s+0.3827+j0.9238)(s+0.3827-j0.9238)$$

$$(a+jb)(a-jb) = a^2+b^2$$

$$\therefore (s-s_0)(s-s_3) = (s+0.3827)^2 + 0.9238^2$$

$$= s^2+0.7654s+0.1465+0.8534$$

$$= s^2+0.7654s+1$$

$$(s-s_1)(s-s_2) = (s+0.9238-j0.3827)(s+0.9238+j0.3827)$$

$$= (s+0.9238)^2 + 0.3827^2$$

$$= s^2+1.8476s + 0.9238^2+0.3827^2$$

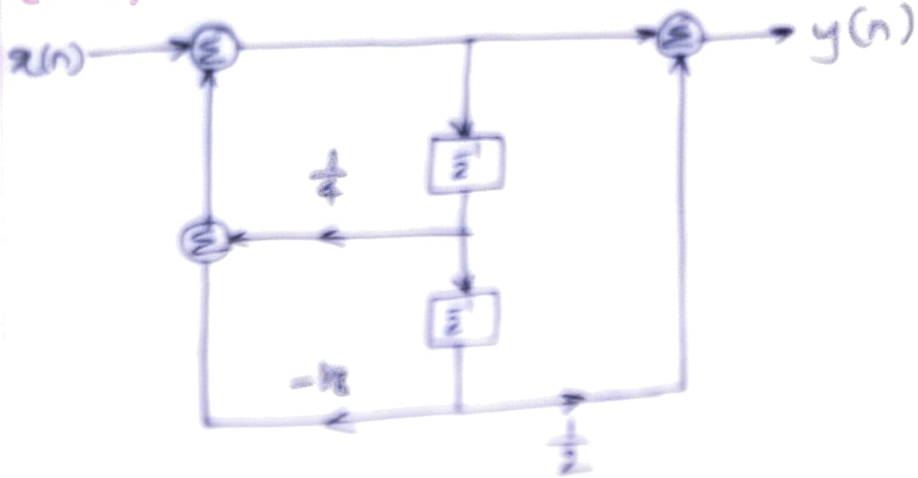
$$= s^2+1.8476s+1$$

$$\therefore H(s) = \frac{1}{(s^2+0.7654s+1)(s^2+1.8476s+1)}$$

9c Realize direct form II for the IIR filter

$$y(n) - \frac{1}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{2}x(n-2).$$

(VTU, Jan 2015, 6 Marks)



10a Design the digital IIR filter for the following details. −3 dB gain at 0.5π rad, stopband attenuation of 15 dB at 0.75π rad. Assume $T_s = 1$ s. (VTU, Jan 2025, 8 Marks)

$A_{PB} = 3$ dB

$A_{SB} = 15$ dB

$\omega_{PB} = 0.5\pi$ rad

$\omega_{SB} = 0.75\pi$ rad

$T_s = 1$ sec.

step 1: $\Omega_{PB}^{new} = \frac{2}{T_s} \tan\left(\frac{\omega_{PB}}{2}\right)$

$= 2 \tan\left(\frac{\pi}{4}\right)$

$= 2$ rad/s

step 2: $\Omega_{SB}^{new} = \frac{2}{T_s} \tan\left(\frac{\omega_{SB}}{2}\right)$

$= 2 \tan\left(\frac{3\pi}{8}\right)$

$= 4.8284$ rad/s

Step 3: Order of the filter

$$N = \frac{\log_{10}\left(\frac{10^{0.1 A_{PB}} - 1}{10^{0.1 A_{SB}} - 1}\right)}{2\log_{10}\left(\frac{\Omega_{PB}^{new}}{\Omega_{SB}^{new}}\right)}$$

$$= 1.94$$

$$\simeq 2$$ (Take the next nearest integer)

Step 4: Cut-off frequency

$$\Omega_c = \frac{\Omega_{PB}^{new}}{\left(10^{0.1 A_{PB}} - 1\right)^{\frac{1}{2N}}}$$

$$= 2$$

Step 5: Pole location

$$S_K = \Omega_c \left[-\sin\left(\frac{(2K+1)\pi}{2N}\right) + j\cos\left(\frac{(2K+1)\pi}{2N}\right)\right]$$

$$S_0 = 2\left[-\sin\left(\frac{\pi}{4}\right) + j\cos\left(\frac{\pi}{4}\right)\right]$$

$$= -1.4142 + j\,1.4142$$

$$S_1 = S_0^* = -1.4142 - j\,1.4142$$

Step 6: Analog filter transfer function

$$H(s) = \frac{\Omega_c^N}{(s-s_0)(s-s_1)}$$

$$= \frac{2^2}{(s+1.4142-j\,1.4142)(s+1.4142+j\,1.4142)}$$

$$= \frac{4}{(s+1.4142)^2 + 1.4142^2}$$

$$= \frac{4}{s^2 + 2.8284s + 4}$$

Step 7: Conversion of $H(s)$ into $H(z)$

put $$s = \frac{2}{T_s}\,\frac{z-1}{z+1}$$

$$= 2\left(\frac{z-1}{z+1}\right)$$

$$\therefore\; H(z) = \frac{4}{4\left(\frac{z-1}{z+1}\right)^2 + 2.8284\left(\frac{z-1}{z+1}\right) + 4}$$

$$= \frac{4(z+1)^2}{4(z-1)^2 + 2.8284(z-1)(z+1) + 4(z+1)^2}$$

$$= \frac{4(z+1)^2}{4z^2 - 8z + 4 + 2.8284z^2 - 2.8284 + 4z^2 + 8z + 4}$$

$$= \frac{4(z+1)^2}{10.8284z^2 + 5.1716}$$

10b Explain the significance of

i) Bilinear transformation

ii) Prewarping

(VTU, Jan 2025, 6 Marks).

i) Bilinear transformation

To design a digital IIR filter, we first design an analog IIR filter and convert the transfer function of analog filter (H(s)) into transfer function of digital filter (H(z)).

Bilinear transformation is used for this purpose.

Bilinear transformation ensures that a stable analog filter is converted into a stable digital filter.

ii) Prewarping.

In the design of digital IIR filters. we need to first convert digital specifications into analog specifi-cations and design an analog filter.

Prewarping is used for this purpose.

$$\Omega_{PB}^{new} = \frac{2}{T_s} \tan\left(\frac{\omega_{PB}}{2}\right)$$

$$\Omega_{SB}^{new} = \frac{2}{T_s} \tan\left(\frac{\omega_{SB}}{2}\right)$$

Design the analog filter for these specifications and convert $H(s)$ into $H(z)$ using bilinear transformation

10c Obtain the direct form I realization of the following filter.

(VTU, Jan 2025, 6 Marks)

$$H(z) = \frac{1+0.4z^{-1}}{1-0.5z^{-1}+0.06z^{-2}}$$