CBCS SCHEME

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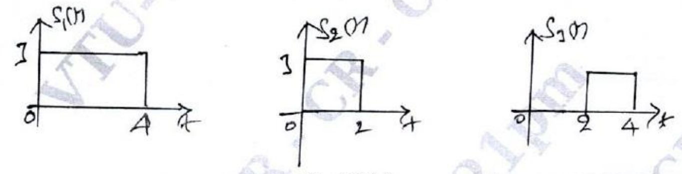
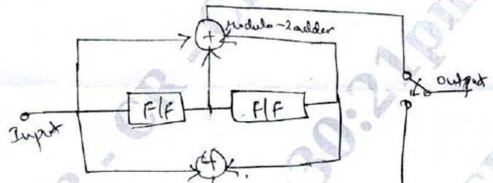
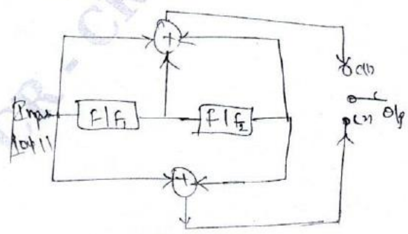
BEC503

Fifth Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Digital Communication

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	**M**	**L**	**C**
Q.1	**a.**	Explain Hilbert transform and its properties.	**6**	**L2**	**CO1**
	b.	Describe the canonical representation of bandpass signal.	**7**	**L2**	**CO1**
	c.	Describe the correlation receiver with neat diagram.	**7**	**L2**	**CO1**
		OR			
Q.2	**a.**	Apply gram Schmidt orthogonalizaiton procedure find the set of orthonormal basis function to represent the signals $S_1(t)$, $S_2(t)$ and $S_3(t)$ as shown in Fig.Q2(a). Also express each of these figures interms of set of basis function.  Fig.Q2(a)	**10**	**L3**	**CO1**
	b.	Derive the equation for converting continuous AWGN channel into a vector channel.	**10**	**L2**	**CO1**
		Module – 2			
Q.3	**a.**	Describe with a neat diagram, the generation and detection of BPSK signal.	8	**L2**	**CO2**
	b.	Define bandwidth efficiency. Tabulate the comment on the bandwidth efficiency of M-ary PSK signal.	**8**	**L2**	**CO2**
	c.	Encode the binary sequence using DPSK 11011011. Assume reference bit as 1.	**4**	**L2**	**CO2**
		OR			
Q.4	**a.**	Derive the expression for probability of error of QPSK signal.	**8**	**L2**	**CO2**
	b.	Discuss the non-coherent detection of BFSK signal.	**8**	**L2**	**CO2**
	c.	Calculate the average power required for a DPSK signal operation gat a data rate of 1000 bit/sec, over a band-pass channel having a bandwidth of 3000 Hz, $\frac{N_0}{2} = 10^{-10}\,\text{w}/\text{H}_z$ probability of error $P_e = 10^{-5}$.	**4**	**L3**	**CO2**
		Module – 3			
Q.5	**a.**	Define entropy and summaries its properties.	**6**	**L2**	**CO3**
	b.	A source has five symbols $S = \{S_1.\ S_2,\ S_3,\ S_4,\ S_5\}$ with probabilities $P = \{0.4,\ 0.2,\ 0.2,\ 0.1,\ 0.1\}$ respectively. compute the source code using Huffman binary coding. Also find the average length and entropy.	**8**	**L3**	**CO3**
	c.	Briefly discuss instantaneous code with an example.	**6**	**L2**	**CO3**
		OR			
Q.6	**a.**	Derive the expression for mutual information and summarize its properties.	**10**	**L2**	**CO3**
	b.	Derive the expression for the channel capacity of binary symmetric channel.	**10**	**L3**	**CO3**

		Module – 4			
Q.7	**a.**	Indicate the advantages and disadvantages of error control coding. Also differentiate between block code and convolution code.	**8**	**L2**	**CO4**
	b.	If 'C' is a valid code vector then show that $CH^T = 0$ where H is parity check matrix of code.	**5**	**L2**	**CO4**
	c.	Design an encoder for the (7, 4) binary cyclic code generated by : $g(x) = 1 + x + x^3$ for the message vector [1001].	**7**	**L3**	**CO4**
		OR			
Q.8	**a.**	Describe the block diagram of generator and parity check matrix with equation. Also write the syndrome equation and list its properties.	**10**	**L2**	**CO4**
	b.	A (7, 4) Linear block code has : $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ i) All possible code vector ii) Determine the Hamming weight of each code word iii) If the received vector is [1100010]. Determine its syndrome correct the codeword.	**10**	**L3**	**CO4**
		Module – 5			
Q.9	**a.**	For a given convolutional encoder shown in Fig.Q9(a), with D = 10011. Compute output sequence using transform domain approach. Also draw the code free diagram.  Fig.Q9(a)	**10**	**L3**	**CO5**
	b.	Describe the recursive systematic convolutional code encoder with an example.	**10**	**L3**	**CO5**
		OR			
Q.10	**a.**	A convolution encoder has two flip-flop with two states, three modulo – 2 adders and an output multiplexer. The generator sequences of the encoder. $g^{(1)} = (1, 0, 1)$, $g^{(2)} = (1, 1, 0)$, $g^{(3)} = (1, 1, 1)$. i) Generator matrix [G] ii) Draw the encoder block diagram iii) Calculate the codeword for the message input vector 11101.	**10**	**L3**	**CO5**
	b.	For a given convolution encoder shown in Fig.Q10(b). Build state table, state transaction table, sketch diagram and describe the Trellis diagram for the input message vector (10111).  Fig.Q10(b)	**10**	**L3**	**CO5**

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Q1a) Solution:

When the phase angles of positive frequency components of a signal $x(t)$ are shifted by -90° and phase angles of negative frequency components are shifted by 90°, the resulting function of time is called Hilbert Transform of the signal denoted as $\hat{x}(t)$.
However, the amplitude of all frequency components are unaffeted by this operation.
Hilbert transformer is an LTI system with the following frequency response.

$$H(f) = \begin{cases} e^{-j\frac{\pi}{2}}, & \text{for } f>0 \\ 0, & \text{for } f=0 \\ e^{j\frac{\pi}{2}}, & \text{for } f<0 \end{cases} \quad \ldots (1)$$

But $e^{\pm j\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) \pm j\sin\left(\frac{\pi}{2}\right)$

$$= \pm j \quad \ldots (2)$$

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$\therefore$ $H(f)$ can be written as,

$$H(f) = \begin{cases} -j, & \text{for } f>0 \\ 0, & \text{for } f=0 \\ j, & \text{for } f<0 \end{cases} \quad \ldots (3)$$

Alternatively, $H(f)$ can be written as,

$$H(f) = -j\,\mathrm{sgn}(f) \quad \ldots (4)$$

where,

$$\mathrm{sgn}(f) = \begin{cases} 1, & \text{for } f>0 \\ 0, & \text{for } f=0 \\ -1, & \text{for } f<0 \end{cases} \quad \ldots (5)$$

We know that, the frequency resonse of the system is the Fourier transform of the impulse response.

Hilbert Transform has the following 3 properties :

Property 1: A signal $x(t)$ and its Hilbert transform $\hat{x}(t)$ have the same magnitude spectrum.

Proof:

Let $h(t)$ be the impulse response of Hilbert transformer.

$$h(t) = \frac{1}{\pi t} \quad \ldots (1)$$

Frequency response of Hilbert transformer,

$$H(f) = \begin{cases} -j & \text{for } f > 0 \\ 0 & \text{for } f = 0 \\ j & \text{for } f < 0 \end{cases} \quad \ldots (2)$$

$$\therefore |H(f)| = 1 \quad \forall f \quad \text{except } f = 0$$

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$$\hat{x}(t) = x(t) * h(t) \quad \ldots (3)$$

We know that convolution of two functions in time domain is transformed into multiplication of their Fourier transforms.

$$\therefore \hat{X}(f) = X(f)\, H(f)$$

$$\therefore |\hat{X}(f)| = |X(f)|\,|H(f)|$$

$$= |X(f)| \quad \ldots (4)$$

Property 2: If $\hat{x}(t)$ is the Hilbert transform of $x(t)$, then the Hilbert transform of $\hat{x}(t)$ is $-x(t)$.

Proof:

$$\hat{x}(t) = x(t) * h(t) \quad \ldots (1)$$

$$\therefore \hat{X}(f) = X(f)\, H(f)$$

$$= -j\, \mathrm{sgn}(f)\, X(f) \quad \ldots (2)$$

Hilbert transform of $\hat{x}(t)$,

$$\hat{\hat{x}}(t) = \hat{x}(t) * h(t) \quad \ldots (3)$$

$\therefore \quad \hat{\hat{X}}(f) = \hat{X}(f)\, H(f)$

$$= \left[-j\, \mathrm{sgn}(f)\, X(f)\right]\left[-j\, \mathrm{sgn}(f)\right] \quad \ldots (4)$$

(Using (2))

But, $j^2 = -1$ and

$$\mathrm{sgn}(f)\, \mathrm{sgn}(f) = 1$$

$\therefore$ (1) can be written as,

$$\hat{\hat{X}}(f) = -X(f) \quad \ldots (5)$$

Taking Inverse FT, we get

$$\hat{\hat{x}}(t) = -x(t) \quad \ldots (6)$$

Property 3: A signal $x(t)$ and its Hilbert transform $\hat{x}(t)$ are orthogonal over the entire time interval $(-\infty, \infty)$.

Proof:

To prove that $x(t)$ and $\hat{x}(t)$ are orthogonal, we have to prove that

$$\int_{-\infty}^{\infty} x(t)\, \hat{x}(t)\, dt = 0 \quad \ldots (1)$$

(12)

But,

$$\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = \int_{-\infty}^{\infty} X(f)\,\hat{X}^{*}(f)\,df \quad \cdots (2)$$

We know that,

$$\hat{x}(t) = x(t) * h(t) \quad \cdots (3)$$

$$\therefore \hat{X}(f) = X(f)\,H(f)$$

$$= X(f)\left[-j\,\mathrm{sgn}(f)\right] \quad \cdots (4)$$

$$\therefore \hat{X}^{*}(f) = j\,\mathrm{sgn}(f)\,X^{*}(f) \quad \cdots (5)$$

Using (5), (2) can be written as,

$$\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = \int_{-\infty}^{\infty} X(f)\left[j\,\mathrm{sgn}(f)\,X^{*}(f)\right] df$$

$$= j\int_{-\infty}^{\infty} \mathrm{sgn}(f)\,|X(f)|^{2}\,df$$

$$= 0 \quad \cdots (6)$$

$\because$ sgn(f) is an odd function.

$|X(f)|^2$ is an even function

sgn(f) $|X(f)|^2$ is an odd function.

Hence, $\int_{-\infty}^{\infty} \mathrm{sgn}(f)\,|X(f)|^2\,df = 0$

(6) Proves the orthogonality property.

Q1b) Solution :

Consider a real valued bandpass signal $x(t)$.

$\hat{x}(t)$ – Hilbert transform of $x(t)$

$X(f)$ – Fourier transform of $x(t)$

$x_+(t)$ – Pre-envelope of $x(t)$ with +ve frequencies

$\tilde{x}(t)$ – Complex envelope of $x(t)$.

We know that,

$$x_+(t) = x(t) + j\,\hat{x}(t) \quad \ldots (1)$$

and

$$\tilde{x}(t) = x_+(t)\, e^{-j2\pi f_c t} \quad \ldots (2)$$

and

$$x_+(t) = \tilde{x}(t)\, e^{j2\pi f_c t} \quad \ldots (3)$$

Using (1), (2) can be written as

$$\tilde{x}(t) = [x(t) + j\,\hat{x}(t)]\, e^{-j2\pi f_c t}$$

$$= \left[x(t) + j\hat{x}(t)\right]\left[\cos(2\pi f_c t) - j\sin(2\pi f_c t)\right]$$

$$= \left[x(t)\cos(2\pi f_c t) - j\sin(2\pi f_c t)\, x(t) + j\hat{x}(t)\cos(2\pi f_c t) + \hat{x}(t)\sin(2\pi f_c t)\right]$$

$$= x(t)\cos(2\pi f_c t) + \hat{x}(t)\sin(2\pi f_c t) + j\left[\hat{x}(t)\cos(2\pi f_c t) - x(t)\sin(2\pi f_c t)\right]$$

$$= x_I(t) + j\, x_Q(t) \quad \cdots (4)$$

where $x_I(t) = x(t)\cos(2\pi f_c t) + \hat{x}(t)\sin(2\pi f_c t) \quad \cdots (5)$

and $x_Q(t) = \hat{x}(t)\cos(2\pi f_c t) - x(t)\sin(2\pi f_c t) \quad \cdots (6)$

Using (4), (3) can be written as,

$$x_+(t) = \left[x_I(t) + j\, x_Q(t)\right]\left[\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right]$$

$$= x_I(t)\cos(2\pi f_c t) + j\, x_I(t)\sin(2\pi f_c t) + j\, x_Q(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$$

$$= x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t) + j\left[x_I(t)\sin(2\pi f_c t) + x_Q(t)\cos(2\pi f_c t)\right] \quad \ldots (7)$$

But, (1) suggests that,

$$x(t) = \text{Real part of } x_+(t) \quad \ldots (8)$$

$\therefore$ Using (7) we can write,

$$x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t) \quad \ldots (9)$$

(9) gives the canonical representation of bandpass signal $x(t)$.

$x_I(t)$ is called in-phase component of $x(t)$ and $x_Q(t)$ is called quadrature component of $x(t)$.

Note that both $x_I(t)$ and $x_Q(t)$ are low-pass signals.

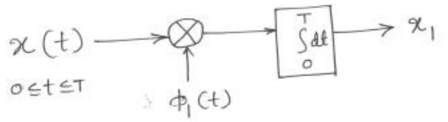
Q1c) Solution :

Correlation receiver consists of multiple correlators which involve multipliers and integrators.

Analog multipliers are hard to build.

Matched filter is an alternative to correlator which avoids the use of multipliers. 53

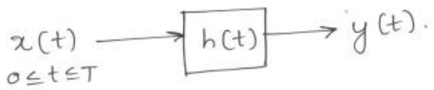
Consider the following correlator.



Output of the correlator,

$$x_1 = \int_0^T x(t)\,\phi_1(t)\,dt \quad \ldots (1)$$

Consider the following LTI system with impulse response $h(t)$.



$$y(t) = x(t) * h(t)$$

$$= \int_0^T x(\tau)\, h(t-\tau)\, d\tau \quad \ldots (2)$$

Sampling $y(t)$ @ $t=T$, we get

$$y(T) = \int_0^T x(\tau)\, h(T-\tau)\, d\tau \quad \ldots (3)$$

(1) may also be written as, 54

$$x_1 = \int_0^T x(\tau)\, \phi_1(\tau)\, d\tau \quad \ldots (4)$$

Comparing (3) and (4), we may state that for $y(T)$ to be equal to x_1, $h(T-\tau)$ should be equal to $\phi_1(\tau)$.

ie, $h(T-\tau) = \phi_1(\tau) \quad \ldots (5)$

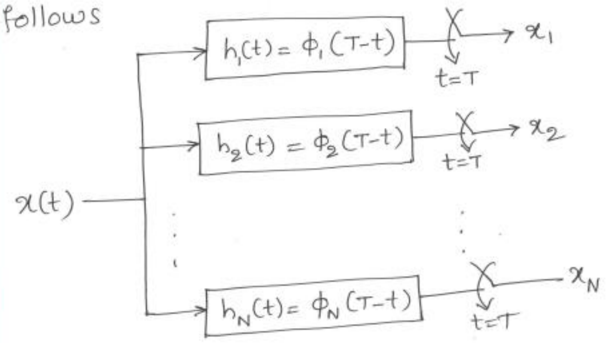
put $T-\tau = t$. We get,

$$h(t) = \phi_1(T-t) \quad \ldots (6)$$

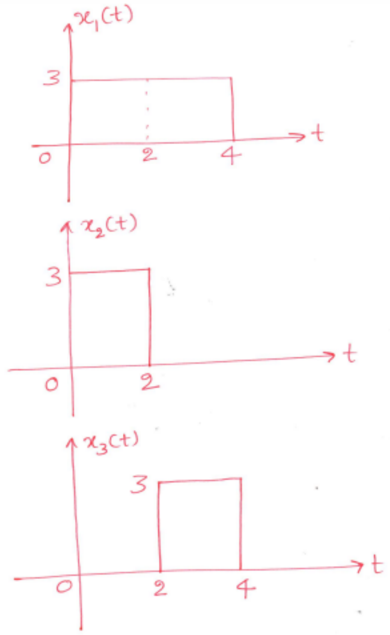
This is the impulse response of the filter matched to $\phi_1(t)$.

matched to $\phi_1(t)$.

Correspondingly, the correlation receiver detector part may be implemented as follows



Q2a) Solution :



Note that $x_1(t) = x_2(t) + x_3(t)$.

$\therefore$ The given signals are linearly dependent.

Now, consider $x_2(t)$ and $x_3(t)$.

They are linearly independent.

$\therefore$ There must be two basis functions.

Further, $\int_0^4 x_2(t)\, x_3(t)\, dt = 0$.

$\therefore$ $x_2(t)$ and $x_3(t)$ are orthogonal to each other from 0 to 4.

Hence, an appropriate set of basis functions may be found as follows.

step i) Energy of $x_2(t)$,

$$E_2 = \int_0^2 3^2 \, dt$$

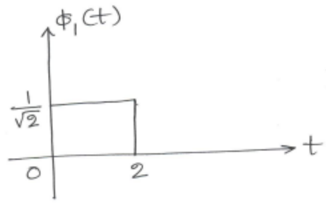
$$= 9t \Big|_0^2$$

$$= 9[2-0]$$

$$= 18$$

step ii) Basis function, $\phi_1(t) = \frac{x_2(t)}{\sqrt{18}}$

$$= \frac{x_2(t)}{3\sqrt{2}}$$



step iii) Energy of $x_3(t)$,

$$E_3 = \int_2^4 3^2\, dt$$

$$= 9\, t \Big|_2^4$$

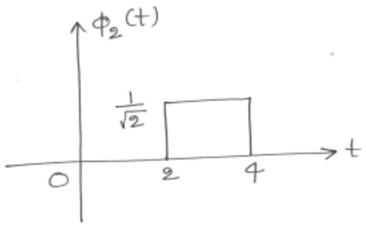
$$= 9\,[4-2]$$

$$= 18$$

step iv) Basis function, $\phi_2(t) = \dfrac{x_3(t)}{\sqrt{E_3}}$

$$= \frac{x_3(t)}{\sqrt{18}}$$

$$= \frac{x_3(t)}{3\sqrt{2}}$$



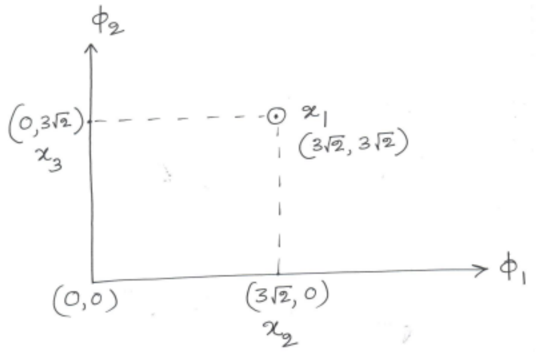
Expressing the signals as a linear combination of basis functions.

$$x_1(t) = 3\sqrt{2}\ \phi_1(t) + 3\sqrt{2}\ \phi_2(t)$$

$$x_2(t) = 3\sqrt{2}\ \phi_1(t) + 0\ \phi_2(t)$$

$$x_3(t) = 0\ \phi_1(t) + 3\sqrt{2}\ \phi_2(t)$$

Constellation diagram
(signal-space diagram)



Q2b) Solution :

Let $x(t), 0 \le t \le T$ be the received symbol where T is the symbol duration.

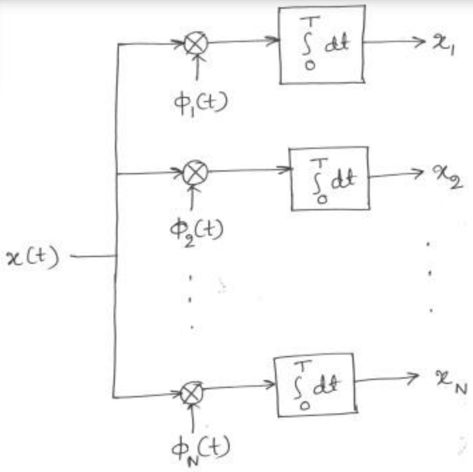
$$x(t) = s_i(t) + w(t), \quad 0 \le t \le T \quad \text{...(1)}$$

where $s_i(t)$ is the transmitted symbol, $i = 1, 2, \dots M$, and $w(t)$ represents zero mean, additive, white, Gaussian noise with power spectral density $\frac{N_0}{2}$.

The received symbol $x(t), 0 \le t \le T$ is applied to a bank of N correlators, where N is the dimensionality of the transmitted signal space.

Let x_j, $j = 1, 2, \dots N$ be the output of j^{th} correlator and the vector,

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$ be the observation vector.



$\phi_j(t)$, $0 \le t \le T$, $j = 1, 2, \dots N$ are the basis functions for the symbols $s_i(t)$, $i = 1, 2, \dots M$

Output of j^{th} correlator,

$$x_j = \int_0^T x(t)\,\phi_j(t)\,dt$$

$$= \int_0^T [s_i(t) + w(t)]\,\phi_j(t)\,dt$$

$$= \int_0^T s_i(t)\,\phi_j(t)\,dt + \int_0^T w(t)\,\phi_j(t)\,dt$$

$$= s_{ij} + w_j \quad \cdots \quad (2)$$

Here, s_{ij} is the projection of $s_i(t)$ over $\phi_j(t)$ ie, j^{th} coordinate of $s_i(t)$. and w_j is the projection of $w(t)$ over $\phi_j(t)$.

Mean of output of j^{th} correlator,

$$m_j = E[s_{ij} + w_j]$$

$$= E[s_{ij}] + E[w_j]$$

$$= s_{ij} \quad \cdots \quad (3)$$

Variance of output of j^{th} correlator,

$$\sigma_j^2 = E\left[(x_j - m_j)^2\right]$$

$$= E\left[(s_{ij} + w_j - s_{ij})^2\right]$$

$$= E[w_j^2]$$

$$= E\left[\int_0^T w(t)\,\phi_j(t)\,dt \int_0^T w(u)\,\phi_j(u)\,du\right]$$

$$= \int_0^T \int_0^T \phi_j(t)\, \phi_j(u)\, E[w(t)w(u)]\, dt\, du \quad --- (4)$$

But $E[w(t)w(u)] = R_W(t,u)$ where $R_W(t,u)$ is the autocorrelation function of noise process $W(t)$.

Since $W(t)$ is stationary, $R_W(t,u)$ is a function of $(t-u)$.

$\therefore R_W(t,u) = R_W(t-u)$

= Inverse Fourier Transform of Power Spectral Density

= IFT of $\frac{N_0}{2}$

$= \frac{N_0}{2} \delta(t-u) \quad \ldots (5)$

Using (5), (4) can be written as,

$$\sigma_j^2 = \int_0^T \int_0^T \phi_j(t)\, \phi_j(u)\, \frac{N_0}{2} \delta(t-u)\, dt\, du$$

$$= \frac{N_0}{2} \int_0^T \phi_j^2(t)\, dt$$

$$= \frac{N_0}{2} \quad \ldots (6)$$

$$\because \int_0^T \phi_j^2(t)\, dt = 1$$

which represents energy of basis function $\phi_j(t)$.

$$\text{cov}[x_j x_k] = E\left[(x_j - m_j)(x_k - m_k)\right]$$

$$= E[w_j w_k]$$

$$= E\left[\int_0^T w(t)\phi_j(t)\,dt \int_0^T w(u)\phi_k(u)\,du\right]$$

$$= \int_0^T \int_0^T \phi_j(t)\,\phi_k(u)\, E\left[w(t)\,w(u)\right] dt\, du$$

$$= \int_0^T \int_0^T \phi_j(t)\,\phi_k(u)\,\frac{N_0}{2}\,\delta(t-u)\, dt\, du$$

$$= \int_0^T \phi_j(t)\,\phi_k(t)\,\frac{N_0}{2}\, dt$$

$$= 0 \quad \text{for } j \neq k \quad \ldots (7)$$

$\because$ $\phi_j(t)$ and $\phi_k(t)$ are orthogonal for

$j \neq k$. (46)

Since $\operatorname{cov}(X_j X_K) = 0$, the random variables X_j and X_K are uncorrelated.

Since X_j and X_K are Gaussian, they are also statistically independent.

The conditional PDF of the output of the j^{th} correlator when symbol $s_i(t)$ is transmitted or the message m_i is transmitted is given by.

$$f_{X_j}\left(x_j/m_i\right) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x_j-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi \frac{N_0}{2}}}\, e^{-\frac{(x_j-s_{ij})^2}{2\frac{N_0}{2}}}$$

$$= \frac{1}{\sqrt{\pi N_0}}\, e^{-\frac{(x_j-s_{ij})^2}{N_0}} \quad \ldots (8)$$

Since the output of the correlator are statistically independent, we may express the PDF of the observation vector X

when message m_i was transmitted, (47)

as

$$f_X\left(x/m_i\right) = \prod_{j=1}^{N} f_{X_j}\left(x_j/m_i\right), \quad i = 1, 2, \cdots M$$

$i = 1, 2, \ldots M$.

$$= \left[\frac{1}{\sqrt{\pi N_0}}\right]^N \exp\left[-\frac{1}{N_0}\sum_{j=1}^{N}(x_j - s_{ij})^2\right] \quad \ldots (9)$$

$f_X(x/m_i)$ are called likelihood functions of AWGN channel.

Q3a) Solution :

In binary phase shift keying (BPSK) bit '1' and bit '0' are represented by the following symbols.

Bit 1 :

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad , \quad 0 \le t \le T_b$$

$f_c = \frac{n}{T_b}$

n - nonzero integer

T_b - bit duration

Bit 0 :

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) \quad , \quad 0 \le t \le T_b$$

$$= -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad ; \quad 0 \le t \le T_b$$

To find basis function.

Energy of $s_1(t) = \int_0^{T_b} |s_1(t)|^2 \, dt$

$$= \int_0^{T_b} \frac{2E_b}{T_b} \cos^2(2\pi f_c t) \, dt$$

$$= \frac{2E_b}{T_b} \int_0^{T_b} \frac{1 + \cos(4\pi f_c t)}{2} \, dt$$

$$= \frac{E_b}{T_b} \int_0^{T_b} 1 \, dt$$

$$= E_b$$

$\therefore$ Basis function, $\phi_1(t) = \dfrac{s_1(t)}{\sqrt{E_b}}$

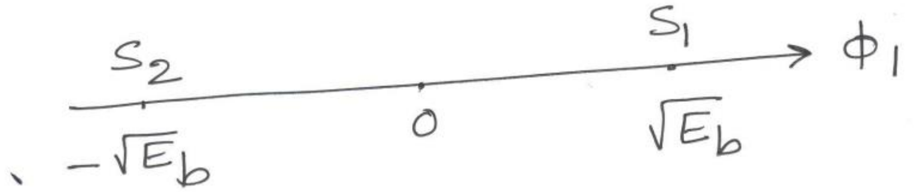
$$= \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

$$0 \leq t \leq T_b$$

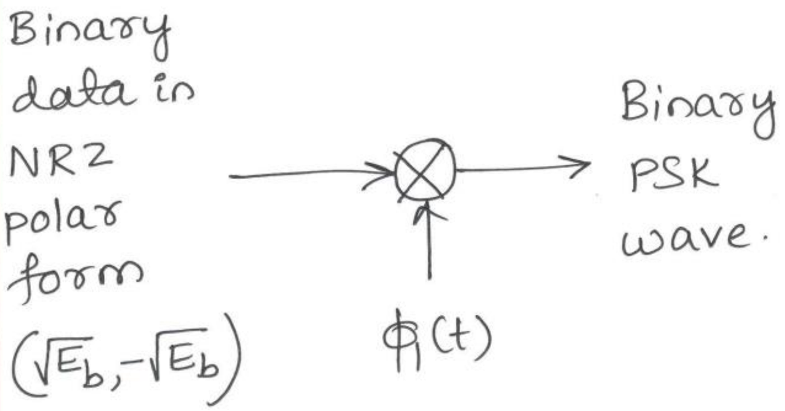
$\therefore$ $s_1(t) = \sqrt{E_b}\, \phi_1(t), \quad 0 \leq t \leq T_b$

$s_2(t) = -\sqrt{E_b}\, \phi_1(t), \quad 0 \leq t \leq T_b$

Signal-space diagram

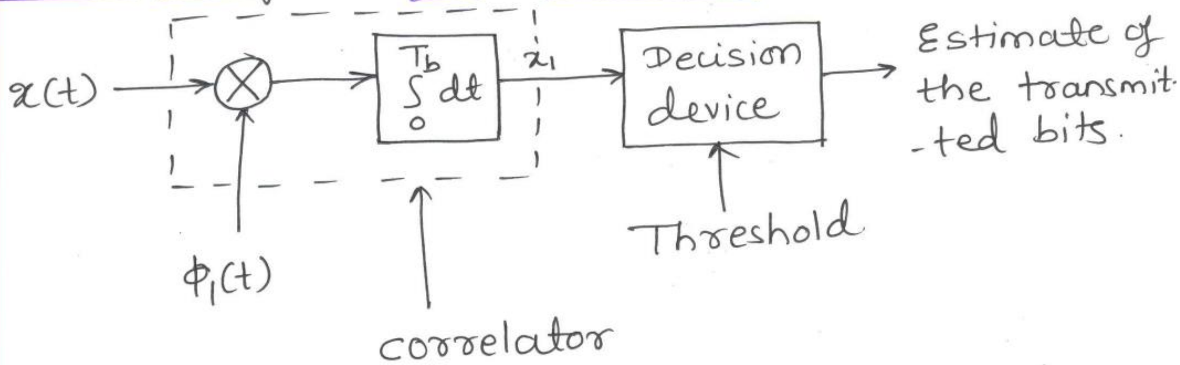


Block diagram of transmitter.



(13)

Block diagram of receiver



Let $x(t)$, $0 \le t \le T_b$ be the received signal.

$$x(t) = S_i(t) + w(t), \quad 0 \le t \le T_b \qquad \ldots (1)$$
$$i = 1, 2$$

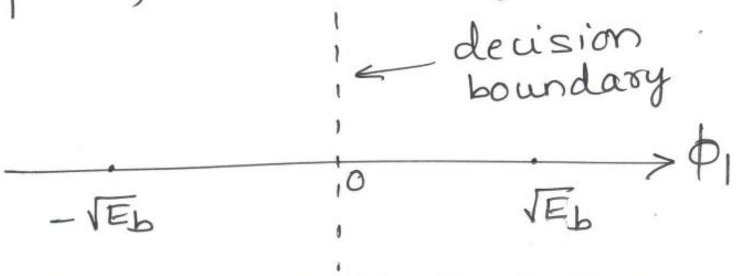
where $w(t)$ represents additive, white Gaussian noise with zero mean and PSD $\frac{N_0}{2}$ ie. variance $\frac{N_0}{2}$.

Decision logic

Let x_1 be the output of the correlator

If $x_1 > 0$, decide in favor of bit '1'

If $x_1 < 0$, decide in favor of bit '0'



Q3b) Solution:

Bandwidth efficiency (ρ) is the ratio of bitrate (R_b) to the required channel bandwidth (B).

$$\rho = \frac{R_b}{B} \quad bits/sec/Hz \quad \ldots (1)$$

But for M-ary PSK, channel bandwidth,

$$B = \frac{2}{T} \qquad (T - \text{symbol duration})$$

$$= \frac{2R_b}{\log_2 M} \quad \ldots \quad (2)$$

Using (2), we may rewrite (1) as follows.

$$\rho = \frac{R_b}{\left(\frac{2R_b}{\log_2 M}\right)}$$

$$= \frac{\log_2 M}{2}$$

2

M	2	4	8	16	32	64
ρ (bits/sec/Hz)	0.5	1	1.5	2	2.5	3

Note that as M increases, the bandwidth efficiency increases. 56

But, along with that, probability of error also increases.

Correspondingly, to keep probability of error within acceptable limit, we have to increase E_b/N_0.

Q3c) Solution :

DPSK= Data XOR ($DPSK_{-1}$)

Data		1	1	0	1	1	0	1	1
DPSK	1	0	1	1	0	1	1	0	1
	Start Bit								

Q4a) Solution :

Probability of error.

Suppose that $s_4(t)$ was transmitted.

From the block diagram of receiver,

we have

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$$x_1 = \int_0^T x(t)\,\phi_1(t)\,dt$$

$$= \int_0^T \left[s_4(t) + w(t) \right] \phi_1(t)\,dt$$

$$= \int_0^T s_4(t)\,\phi_1(t)\,dt + \int_0^T w(t)\,\phi_1(t)\,dt$$

$$= \sqrt{\frac{E}{2}} + w_1 \quad \cdots \quad (1)$$

coordinate of $s_4(t)$ with respect to $\phi_1(t)$

Mean of X_1 when $s_4(t)$ was transmitted.

ie, 11 was transmitted

$$\mu_1 = E[X_1]$$

$$= \sqrt{\frac{E}{2}} \quad \ldots (2)$$

Variance of X_1 when $s_4(t)$ was transmi-
-tted,

$$\sigma_1^2 = \frac{N_0}{2} \quad \ldots (3)$$

$$x_2 = \int_0^T x(t)\,\phi_2(t)\,dt$$

$$= \int_0^T [s_4(t) + w(t)]\,\phi_2(t)\,dt$$

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$$= \int_0^T s_4(t)\,\phi_2(t)\,dt + \int_0^T w(t)\,\phi_2(t)\,dt$$

$$= \sqrt{\frac{E}{2}} + w_2 \quad \ldots (4)$$

↑ coordinate of $s_4(t)$ with respect to $\phi_2(t)$

Mean of X_2 when $s_4(t)$ is transmitted,

$$\mu_2 = \sqrt{\frac{E}{2}} \quad \ldots (5)$$

Variance of x_2 when $s_4(t)$ is transmitted,

$$\sigma_2^2 = \frac{N_0}{2} \quad \ldots (6)$$

$\therefore$ PDF of x_1 when $s_4(t)$ was transmitted,

$$f_{x_1}\left(x_1/11\right) = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(x_1-\mu_1)^2}{2\sigma_1^2}}$$

$$= \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(x_1-\sqrt{\frac{E}{2}}\right)^2}{N_0}} \quad \ldots (7)$$

PDF of x_2 when $s_4(t)$ was transmitted,

$$f_{x_2}\left(x_2/11\right) = \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{\left(x_2-\sqrt{\frac{E}{2}}\right)^2}{2\sigma_2^2}}$$

$$= \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(x_2-\sqrt{\frac{E}{2}}\right)^2}{N_0}} \quad \ldots (8)$$

When $s_4(t)$ is transmitted, corret decision is made when $x_1 > 0$ and $x_2 > 0$. (36)

so that the received signal point lies in the first quadrant of the signal-space diagram.

$\therefore$ Probability of corret decision when $s_4(t)$ is transmitted,

$$P_c(11) = P(X_1 > 0)\, P(X_2 > 0) \quad \ldots (9)$$

$$P(X_1 > 0) = \int_0^\infty f_{X_1}\left(x_1/11\right) dx_1$$

$$= \int_0^\infty \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(x_1 - \sqrt{\frac{E}{2}}\right)^2}{N_0}} dx_1 \quad \ldots (10)$$

put $$\frac{\left(x_1 - \sqrt{\frac{E}{2}}\right)^2}{N_0} = \frac{z^2}{2} \quad \ldots (11)$$

$$\therefore \quad \frac{\left(x_1 - \sqrt{\frac{E}{2}}\right)}{\sqrt{N_0}} = \frac{z}{\sqrt{2}}$$

$$\frac{dx_1}{\sqrt{N_0}} = \frac{dz}{\sqrt{2}}$$

$$\therefore \quad dx_1 = \sqrt{\frac{N_0}{2}}\, dz \quad \cdots \quad (12)$$

when $x_1 = 0, \quad z = -\sqrt{\frac{E}{N_0}} \quad \cdots \quad (13)$

when $x_1 = \infty, \quad z = \infty \quad \cdots \quad (14)$

Using (11), (12), (13), (14), we may write

(10) as,

$$P(x_1 > 0) = \frac{1}{\sqrt{\pi N_0}} \int_{-\sqrt{\frac{E}{N_0}}}^{\infty} e^{-\frac{z^2}{2}} \sqrt{\frac{N_0}{2}}\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\sqrt{\frac{E}{N_0}}}^{\infty} e^{-\frac{z^2}{2}}\, dz$$

$$= Q\left(-\sqrt{\frac{E}{N_0}}\right)$$

$$= 1 - Q\left(\sqrt{\frac{E}{N_0}}\right) \quad \cdots (15)$$

Similarly, we can prove that, $P(X_2 > 0)$ when $s_4(t)$ is transmitted,

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$$P(X_2 > 0) = 1 - Q\left(\sqrt{\frac{E}{N_0}}\right) \quad \cdots (16)$$

Using (15) and (16), we may write (9) as

$$P_c(11) = \left[1 - Q\left(\sqrt{\frac{E}{N_0}}\right)\right]^2$$

$$= 1 + \left[Q\left(\sqrt{\frac{E}{N_0}}\right)\right]^2 - 2Q\left(\sqrt{\frac{E}{N_0}}\right)$$

$$\simeq 1 - 2Q\left(\sqrt{\frac{E}{N_0}}\right) \quad \cdots (17)$$

$\because \left[Q\left(\sqrt{\frac{E}{N_0}}\right)\right]^2$ is negligible.

This is the probability of correct decision when '11' is transmitted

$\therefore$ Probability of error when '11' is transmitted is given by

$$P_e(11) = 1 - P_c(11)$$

$$= 2\,Q\left(\sqrt{\frac{E}{N_0}}\right) \quad \ldots (18)$$

similarly, we may prove that, 39

$$P_e(00) = P_e(01)$$

$$= P_e(10)$$

$$= 2\,Q\left(\sqrt{\frac{E}{N_0}}\right) \quad \ldots (19)$$

Assuming equiprobable dibits, we get, average probability of symbol error,

$$P_e^{symbol} = 2\,Q\left(\sqrt{\frac{E}{N_0}}\right) \quad \ldots (20)$$

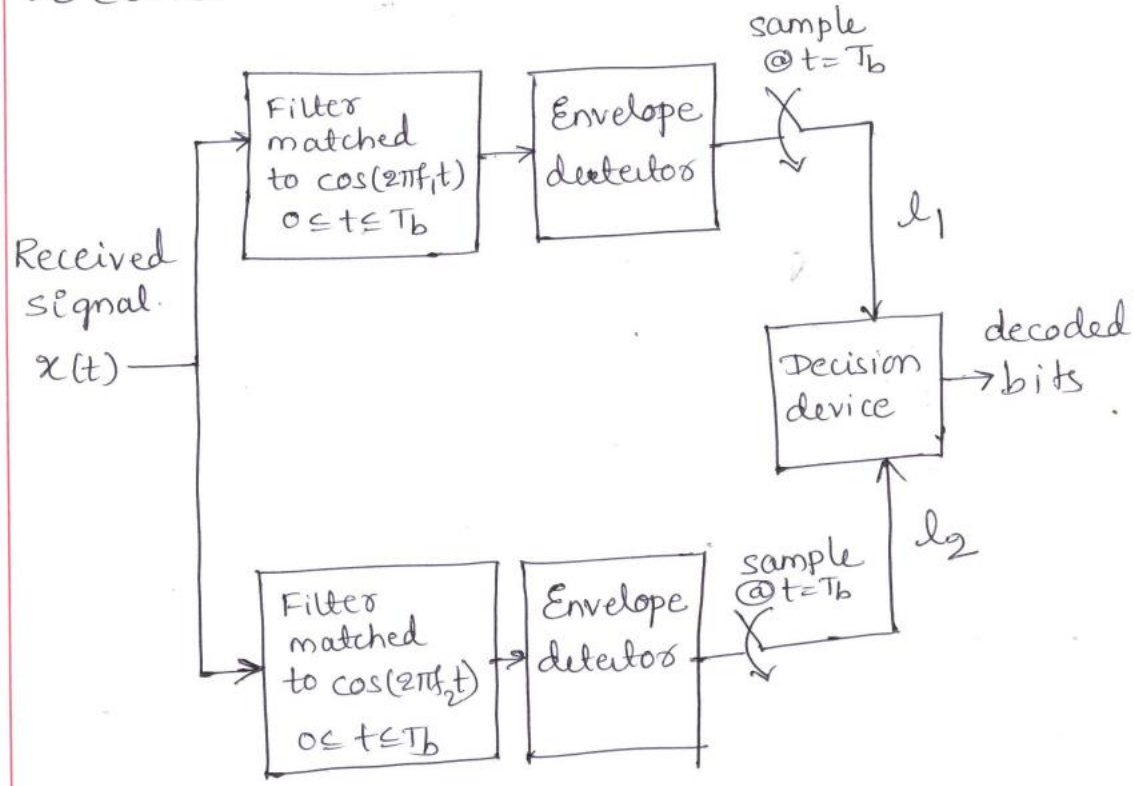
But, each symbol represents 2 bits. Hence, average probability of bit error,

$$P_e^{bit} = \frac{1}{2}\,2\,Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \ldots (21)$$

Q4b) Solution :

Block diagram of non-coherent FSK receiver.



The non-coherent receiver consists of a pair of matched filters followed by envelope detectors.
One of the filters is matched to $\cos(2\pi f_1 t),\ 0 \le t \le T_b$ and the other

one is matched to $\cos(2\pi f_2 t),\ 0 \le t \le T_b$ 51

When bit '1' is transmitted, ie $\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_1 t)$ $0 \le t \le T_b$ is transmitted, l_1 will be greater than l_2 in the absence of noise.

When bit '0' is transmitted, ie $\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_2 t),\ 0 \le t \le T_b$ is transmitted, l_2 will be greater than l_1 in the absence of noise.
Hence, by comparing l_1 and l_2, a decision will be made.

Decision rule:
Decide in favor of '1' if $l_1 > l_2$
Decide in favor of '0' if $l_1 < l_2$.

Note: Probability of error of BFSK with non-coherent detection is given by.

$$P_e = \frac{1}{2} e^{-\left(\frac{E_b}{2N_0}\right)}$$

Q4c) Solution :

Rb=10^3 bps

No/2=10^-10 Watt/Hz

Pe= 10^-5

For DPSK,

Pe=(1/2) *e^(-Eb/No)

(10^-5) = (1/2) * e^(-Eb/No)

e^(-Eb/No)= 2*(10^-5)

Taking ln on both LHS and RHS,

ln[e^(-Eb/No)]= ln[2*(10^-5)]

-Eb/No= ln(2) + ln(10^-5)

-Eb/No= -10.8197

Eb= No*10.8197= 2*(10^-10) * 10.8197 = 21.6395 * (10^-10) Joule

Carrier Power= (Eb/Tb)= Eb*Rb= 21.6395 * (10^-10) * (10^3) = 21.6395*(10^-7) Watt.

Q5a) Solution :

Suppose that a probabilistic experiment involves observation of the output emitted by a discrete source during every signaling interval.

The source output is modeled as a stochastic process, a sample of which is denoted by the discrete random variable S.

This random variable takes on symbols from the fixed finite alphabet,

$$\mathcal{S} = \{s_0, s_1, \ldots, s_{K-1}\} \tag{5.1}$$

with probabilities

$$\mathbb{P}(S=s_k) = p_k, \quad k = 0, 1, \ldots, K-1 \tag{5.2}$$

Of course, this set of probabilities must satisfy the normalization property

$$\sum_{k=0}^{K-1} p_k = 1, \quad p_k \geq 0 \tag{5.3}$$

We assume that the symbols emitted by the source during successive signaling intervals are statistically independent.

Given such a scenario, can we find a measure of how much information is produced by such a source?

To answer this question, we recognize that the idea of information is closely related to that of uncertainty or surprise, as described next.

Consider the event S = sk, describing the emission of symbol sk by the source with probability pk, as defined in (5.2).

Clearly, if the probability pk = 1 and pi = 0 for all , then there is no “surprise” and, therefore, no “information” when symbol sk is emitted, because we know what the message from the source must be.

If, on the other hand, the source symbols occur with different probabilities and the probability pk is low, then there is more surprise and, therefore, information when symbol sk is emitted by the source than when another symbol si, , with higher probability is emitted.

Thus, the words uncertainty, surprise, and information are all related.

Before the event S = sk occurs, there is an amount of uncertainty.

When the event S = sk occurs, there is an amount of surprise.

After the occurrence of the event S = sk, there is gain in the amount of information, the essence of which may be viewed as the resolution of uncertainty.

Most importantly, the amount of information is related to the inverse of the probability of occurrence of the event S = sk.

We define the amount of information gained after observing the event S = sk, which occurs with probability pk, as the logarithmic function

$$I(s_k) = \log\left(\frac{1}{p_k}\right) \tag{5.4}$$

which is often termed "self-information" of the event $S = s_k$. This definition exhibits the following important properties that are intuitively satisfying:

PROPERTY 1

$$I(s_k) = 0 \qquad \text{for } p_k = 1 \tag{5.5}$$

Obviously, if we are absolutely *certain* of the outcome of an event, even before it occurs, there is *no* information gained.

PROPERTY 2

$$I(s_k) \geq 0 \qquad \text{for } 0 \leq p_k \leq 1 \tag{5.6}$$

That is to say, the occurrence of an event $S = s_k$ either provides some or no information, but never brings about a *loss* of information.

PROPERTY 3

$$I(s_k) > I(s_i) \qquad \text{for } p_k < p_i \tag{5.7}$$

That is, the less probable an event is, the more information we gain when it occurs.

PROPERTY 4 $$I(s_k, s_l) = I(s_k) + I(s_l) \text{ if } s_k \text{ and } s_l \text{ are statistically independent}$$

This additive property follows from the logarithmic definition described in (5.4).

The base of the logarithm in (5.4) specifies the units of information measure. Nevertheless, it is standard practice in information theory to use a logarithm to base 2 with binary signaling in mind. The resulting unit of information is called the *bit*, which is a contraction of the words binary digit. We thus write

$$\begin{aligned} I(s_k) &= \log_2\left(\frac{1}{p_k}\right) \\ &= -\log_2 p_k \quad \text{for } k = 0, 1, \ldots, K-1 \end{aligned} \tag{5.8}$$

When $p_k = 1/2$, we have $I(s_k) = 1$ bit. We may, therefore, state:

One bit is the amount of information that we gain when one of two possible and equally likely (i.e., equiprobable) events occurs.

Note that the information I(sk) is positive, because the logarithm of a number less than one, such as a probability, is negative.

Note also that if pk is zero, then the self-information Isk assumes an unbounded value.

The amount of information I(sk) produced by the source during an arbitrary signaling interval depends on the symbol sk emitted by the source at the time.

The self-information I(sk) is a discrete random variable that takes on the values I(s0), I(s1), … , I(sK–1) with probabilities p0, p1, … , pK–1 respectively. The expectation of I(sk) over all the probable values taken by the random variable S is given by

$$H(S) = \mathbb{E}[I(s_k)]$$

$$= \sum_{k=0}^{K-1} p_k I(s_k) \qquad (5.9)$$

$$= \sum_{k=0}^{K-1} p_k \log_2\left(\frac{1}{p_k}\right)$$

The quantity H(S) is called the entropy, formally defined as follows: The entropy of a discrete random variable, representing the output of a source of information, is a measure of the average information content per source symbol.

Note that the entropy H(S) is independent of the alphabet $\mathcal{S}$; it depends only on the probabilities of the symbols in the alphabet $\mathcal{S}$ of the source.

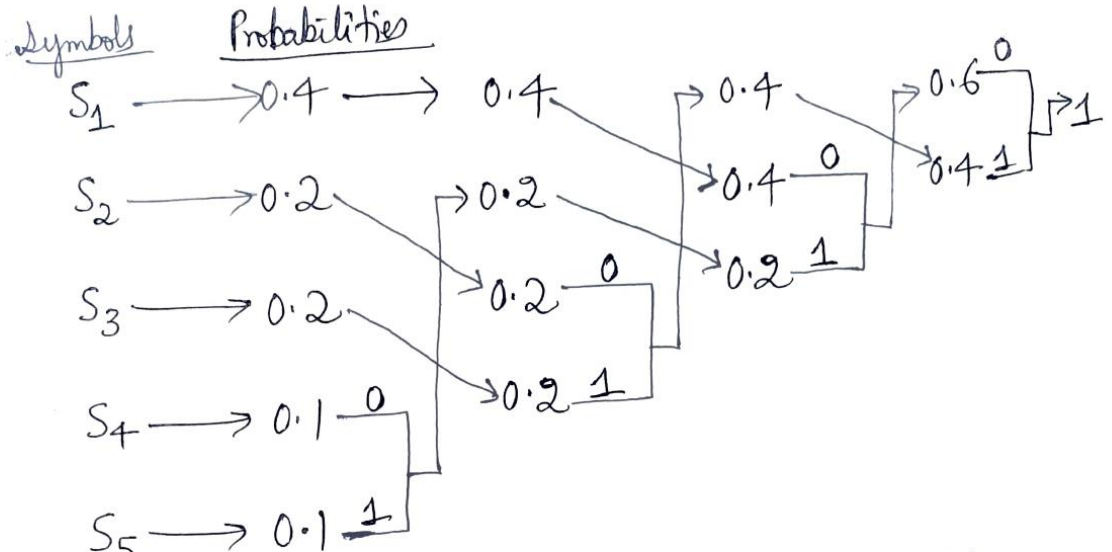
Properties of Entropy – Building on the definition of entropy given in (5.9), we find that entropy of the discrete random variable S is bounded as follows:

$$0 \leq H(S) \leq \log_2 K \qquad (5.10)$$

where K is the number of symbols in the alphabet $\mathcal{S}$.

Q5b) Solution :

$P(S_1) = 0.4,\ P(S_2) = 0.2,\ P(S_3) = 0.2,\ P(S_4) = 0.1,\ P(S_5) = 0.1$



The required codewords using Huffman Coding are

$S_1 \rightarrow 00;\ S_2 \rightarrow 10;\ S_3 \rightarrow 11;\ S_4 \rightarrow 010;\ S_5 \rightarrow 011$

Codeword length $(l_K) \Rightarrow S_1 = 2,\ S_2 = 2,\ S_3 = 2,\ S_4 = 3,\ S_5 = 3.$

Average length $(L) = \sum_{K=1}^{5} l_K p_K$

$= 2\times 0.4 + 2\times 0.2 + 2\times 0.2 + 3\times 0.1 + 3\times 0.1$

$= 0.8 + 0.4 + 0.4 + 0.3 + 0.3 = 2.2$ bits

Entropy $(H(S)) = \sum_{K=1}^{5} p_K \log_2\left(\frac{1}{p_K}\right) = \sum_{K=1}^{5} p_K \frac{\log_{10}\left(\frac{1}{p_K}\right)}{\log_{10}(2)}$

$$\Rightarrow H(S) = \frac{0.4\times\log_{10}\left(\frac{1}{0.4}\right) + 0.2\times 2\times\log_{10}\left(\frac{1}{0.2}\right) + 0.1\times 2\times\log_{10}\left(\frac{1}{0.1}\right)}{\log_{10}(2)}$$

$$= \frac{0.159 + 0.2795 + 0.2}{0.301} = 2.12126$$

Q5c) Solution :

An "instantaneous code" (also called a prefix code) is a type of code where each codeword can be uniquely decoded as soon as it is received, meaning you don't need to wait for the entire sequence to be received before identifying a codeword; the key property is that no codeword is a prefix of another codeword in the set,

Key properties of an instantaneous code:

Prefix-free:

The most important property is that no codeword can be the beginning (prefix) of another codeword.

Unique Decoding:

Due to the prefix-free nature, each codeword can be uniquely identified as soon as it is received, allowing for immediate decoding without ambiguity.

Easy Implementation:

Instantaneous codes are easy to decode using a simple "tree-like" structure where each branch represents a bit in the codeword.

Application in Variable-length Coding:

Instantaneous codes are particularly useful in situations where codewords can have different lengths, like in data compression algorithms like Huffman coding.

Example:

Consider the code: 0, 10, and 11.

This is an instantaneous code because no codeword is a prefix of another.

Why is it called "instantaneous"?

The term "instantaneous" reflects the fact that you can decode a codeword as soon as you receive it, without needing to wait for the entire sequence of bits to be received.

Question-6(a)

Derive expression for Mutual Information and summarize its properties.

MUTUAL INFORMATION

Given that we think of the channel output Y (selected from alphabet $\mathscr{Y}$) as a noisy version of the channel input X (selected from alphabet $\mathscr{X}$), and that the entropy $H(\mathscr{X})$ is a measure of the prior uncertainty about X, how can we measure the uncertainty about X after observing Y? To answer this question, we extend the ideas developed in Section 10.2 by defining the *conditional entropy* of X selected from alphabet $\mathscr{X}$, given that $Y = y_k$. Specifically, we write

$$H(\mathscr{X}|Y = y_k) = \sum_{j=0}^{J-1} p(x_j|y_k) \log_2\left[\frac{1}{p(x_j|y_k)}\right] \tag{10.40}$$

This quantity is itself a random variable that takes on the values $H(\mathscr{X}|Y = y_0), \ldots, H(\mathscr{X}|Y = y_{K-1})$ with probabilities $p(y_0), \ldots, p(y_{K-1})$, respectively. The mean of entropy $H(\mathscr{X}|Y = y_k)$ over the output alphabet $\mathscr{Y}$ is therefore given by

$$\begin{aligned} H(\mathscr{X}|\mathscr{Y}) &= \sum_{k=0}^{K-1} H(\mathscr{X}|Y = y_k)p(y_k) \\ &= \sum_{k=0}^{K-1}\sum_{j=0}^{J-1} p(x_j|y_k)p(y_k) \log_2\left[\frac{1}{p(x_j|y_k)}\right] \end{aligned} \tag{10.41}$$

$$= \sum_{k=0}^{K-1} \sum_{j=0}^{J-1} p(x_j, y_k) \log_2 \left[\frac{1}{p(x_j|y_k)} \right]$$

where, in the last line, we have made use of the relation

$$p(x_j, y_k) = p(x_j|y_k)\, p(y_k) \tag{10.42}$$

The quantity $H(\mathscr{X}|\mathscr{Y})$ is called a *conditional entropy*. It represents *the amount of uncertainty remaining about the channel input after the channel output has been observed.*

Since the entropy $H(\mathscr{X})$ represents our uncertainty about the channel input *before* observing the channel output, and the conditional entropy $H(\mathscr{X}|\mathscr{Y})$ represents our uncertainty about the channel input *after* observing the channel output, it follows that the difference $H(\mathscr{X}) - H(\mathscr{X}|\mathscr{Y})$ must represent our uncertainty about the channel input that is *resolved* by observing the channel output.

This important quantity is called the *mutual information* of the channel. Denoting the mutual information by $I(\mathscr{X};\mathscr{Y})$, we may thus write

$$I(\mathscr{X};\mathscr{Y}) = H(\mathscr{X}) - H(\mathscr{X}|\mathscr{Y}) \tag{10.43}$$

Similarly, we may write

$$I(\mathscr{Y};\mathscr{X}) = H(\mathscr{Y}) - H(\mathscr{Y}|\mathscr{X}) \tag{10.44}$$

where $H(\mathscr{Y})$ is the entropy of the channel output and $H(\mathscr{Y}|\mathscr{X})$ is the conditional entropy of the channel output given the channel input.

Properties of Mutual Information

The mutual information $I(\mathscr{X};\mathscr{Y})$ has the following important properties.

PROPERTY 1

The mutual information of a channel is symmetric; that is

$$I(\mathscr{X};\mathscr{Y}) = I(\mathscr{Y};\mathscr{X}) \tag{10.45}$$

where the mutual information $I(\mathscr{X};\mathscr{Y})$ is a measure of the uncertainty about the channel input that is resolved by *observing* the channel output, and the mutual information $I(\mathscr{Y};\mathscr{X})$ is a measure of the uncertainty about the channel output that is resolved by *sending* the channel input.

To prove this property, we first use the formula for entropy and then use Eqs. (10.36) and (10.38), in that order, to express $H(\mathscr{X})$ as

$$\begin{aligned}
H(\mathscr{X}) &= \sum_{j=0}^{J-1} p(x_j) \log_2\left[\frac{1}{p(x_j)}\right] \\
&= \sum_{j=0}^{J-1} p(x_j) \log_2\left[\frac{1}{p(x_j)}\right] \sum_{k=0}^{K-1} p(y_k|x_j) \\
&= \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(y_k|x_j)\, p(x_j) \log_2\left[\frac{1}{p(x_j)}\right] \\
&= \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(x_j, y_k) \log_2\left[\frac{1}{p(x_j)}\right]
\end{aligned} \tag{10.46}$$

Hence, substituting Eqs. (10.41) and (10.46) into Eq. (10.43) and then combining terms we get

$$I(\mathscr{X};\mathscr{Y}) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(x_j, y_k) \log_2\left[\frac{p(x_j|y_k)}{p(x_j)}\right] \tag{10.47}$$

From *Bayes' rule* for conditional probabilities, we have [see Eqs. (10.38) and (10.42)]

$$\frac{p(x_j|y_k)}{p(x_j)} = \frac{p(y_k|x_j)}{p(y_k)} \tag{10.48}$$

Hence, substituting Eq. (10.48) into Eq. (10.47), and interchanging the order of summation, we may write

$$\begin{aligned} I(\mathscr{X};\mathscr{Y}) &= \sum_{k=0}^{K-1} \sum_{j=0}^{J-1} p(x_j,y_k) \log_2\left[\frac{p(y_k|x_j)}{p(y_k)}\right] \\ &= I(\mathscr{Y};\mathscr{X}) \end{aligned} \tag{10.49}$$

which is the desired result.

PROPERTY 2

The mutual information is always nonnegative; that is

$$I(\mathscr{X};\mathscr{Y}) \geq 0 \tag{10.50}$$

To prove this property, we first note from Eq. (10.42) that

$$p(x_j|y_k) = \frac{p(x_j,y_k)}{p(y_k)} \tag{10.51}$$

Hence, a substituting Eq. (10.51) into Eq. (10.47), we may express the mutual information of the channel as

$$I(\mathscr{X};\mathscr{Y}) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(x_j,y_k) \log_2\left(\frac{p(x_j,y_k)}{p(x_j)p(y_k)}\right) \tag{10.52}$$

Next, a direct application of the fundamental inequality [defined by Eq. (10.12)] yields the desired result

$$I(\mathscr{X};\mathscr{Y}) \geq 0$$

with equality if, and only if,

$$p(x_j,y_k) = p(x_j)p(y_k) \quad \text{for all } j \text{ and } k \tag{10.53}$$

Property 2 states that *we cannot lose information, on the average, by observing the output of a channel.* Moreover, the average mutual information is zero if, and only if, the input and output symbols of the channel are statistically independent, as in Eq. (10.53).

PROPERTY 3

The mutual information of a channel is related to the joint entropy of the channel input and channel output by

$$I(\mathscr{X};\mathscr{Y}) = H(\mathscr{X}) + H(\mathscr{Y}) - H(\mathscr{X},\mathscr{Y}) \tag{10.54}$$

where the joint entropy $H(\mathscr{X},\mathscr{Y})$ is defined by

$$H(\mathscr{X};\mathscr{Y}) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(x_j,y_k) \log_2\left(\frac{1}{p(x_j,y_k)}\right) \tag{10.55}$$

To prove Eq. (10.54), we first rewrite the definition for the joint entropy $H(\mathcal{X},\mathcal{Y})$ as

$$H(\mathcal{X};\mathcal{Y}) = \sum_{j=0}^{J-1}\sum_{k=0}^{K-1} p(x_j,y_k)\log_2\left[\frac{p(x_j)p(y_k)}{p(x_j,y_k)}\right] + \sum_{j=0}^{J-1}\sum_{k=0}^{K-1} p(x_j,y_k)\log_2\left[\frac{1}{p(x_j)p(y_k)}\right] \tag{10.56}$$

The first double summation term on the right-hand side of Eq. (10.56) is recognized as the negative of the mutual information of the channel $I(\mathcal{X};\mathcal{Y})$, previously given in Eq. (10.52). As for the second summation term, we manipulate it as follows:

$$\begin{aligned}\sum_{j=0}^{J-1}\sum_{k=0}^{K-1} p(x_j,y_k)\log_2\left[\frac{1}{p(x_j)p(y_k)}\right] &= \sum_{j=0}^{J-1}\log_2\left[\frac{1}{p(x_j)}\right]\sum_{k=0}^{K-1}p(x_j,y_k) \\ &\quad + \sum_{k=0}^{K-1}\log_2\left[\frac{1}{p(y_k)}\right]\sum_{j=0}^{J-1}p(x_j,y_k) \\ &= \sum_{j=0}^{J-1}p(x_j)\log_2\left[\frac{1}{p(x_j)}\right] \\ &\quad + \sum_{k=0}^{K-1}p(y_k)\log_2\left[\frac{1}{p(y_k)}\right] \\ &= H(\mathcal{X}) + H(\mathcal{Y})\end{aligned} \tag{10.57}$$

Accordingly, using Eqs. (10.52) and (10.57) in Eq. (10.56), we get the result

$$H(\mathcal{X},\mathcal{Y}) = -I(\mathcal{X};\mathcal{Y}) + H(\mathcal{X}) + H(\mathcal{Y}) \tag{10.58}$$

Rearranging terms in this equation, we get the result given in Eq. (10.54), thereby confirming Property 3.

We conclude our discussion of the mutual information of a channel by providing a diagramatic interpretation of Eqs. (10.43), (10.44), and (10.54). The interpretation is given in Fig. 10.10. The entropy of channel input X is represented by the circle on the left. The entropy of channel output Y is represented by the circle on the right. The mutual information of the channel is represented by the overlap between these two circles.

Question-6(b)

Derive expression for channel capacity of binary symmetric channel.

To prove Eq. (10.54), we first rewrite the definition for the joint entropy $H(\mathscr{X},\mathscr{Y})$ as

$$H(\mathscr{X};\mathscr{Y}) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(x_j,y_k) \log_2\left[\frac{p(x_j)p(y_k)}{p(x_j,y_k)}\right] + \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(x_j,y_k) \log_2\left[\frac{1}{p(x_j)p(y_k)}\right] \tag{10.56}$$

The first double summation term on the right-hand side of Eq. (10.56) is recognized as the negative of the mutual information of the channel $I(\mathscr{X};\mathscr{Y})$, previously given in Eq. (10.52). As for the second summation term, we manipulate it as follows:

$$\begin{aligned}\sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(x_j,y_k) \log_2\left[\frac{1}{p(x_j)p(y_k)}\right] &= \sum_{j=0}^{J-1} \log_2\left[\frac{1}{p(x_j)}\right] \sum_{k=0}^{K-1} p(x_j,y_k) \\ &\quad + \sum_{k=0}^{K-1} \log_2\left[\frac{1}{p(y_k)}\right] \sum_{j=0}^{J-1} p(x_j,y_k) \\ &= \sum_{j=0}^{J-1} p(x_j) \log_2\left[\frac{1}{p(x_j)}\right] \\ &\quad + \sum_{k=0}^{K-1} p(y_k) \log_2\left[\frac{1}{p(y_k)}\right] \\ &= H(\mathscr{X}) + H(\mathscr{Y})\end{aligned} \tag{10.57}$$

Accordingly, using Eqs. (10.52) and (10.57) in Eq. (10.56), we get the result

$$H(\mathscr{X},\mathscr{Y}) = -I(\mathscr{X};\mathscr{Y}) + H(\mathscr{X}) + H(\mathscr{Y}) \tag{10.58}$$

Rearranging terms in this equation, we get the result given in Eq. (10.54), thereby confirming Property 3.

We conclude our discussion of the mutual information of a channel by providing a diagramatic interpretation of Eqs. (10.43), (10.44), and (10.54). The interpretation is given in Fig. 10.10. The entropy of channel input X is represented by the circle on the left. The entropy of channel output Y is represented by the circle on the right. The mutual information of the channel is represented by the overlap between these two circles.

Here we note that [see Eq. (10.38)]

$$p(x_j, y_k) = p(y_k|x_j)p(x_j)$$

Also, from Eq. (10.39), we have

$$p(y_k) = \sum_{j=0}^{J-1} p(y_k|x_j)p(x_j)$$

From these three equations we see that it is necessary for us to know the input probability distribution $\{p(x_j)|j = 0, 1, \ldots, J-1\}$ so that we may calculate the mutual information $I(\mathscr{X};\mathscr{Y})$. The mutual information of a channel therefore depends not only on the channel but also on the way in which the channel is used.

The input probability distribution $\{p(x_j)\}$ is obviously independent of the channel. We can then maximize the average mutual information $I(\mathscr{X};\mathscr{Y})$ of the channel with respect to $\{p(x_j)\}$. Hence, *we define the channel capacity of a discrete memoryless channel as the maximum average mutual information $I(\mathscr{X};\mathscr{Y})$ in any single use of the channel (i.e., signaling interval), where the maximization is over all possible input probability distributions $\{p(x_j)\}$ on $\mathscr{X}$.* The channel capacity is commonly denoted by C. We thus write

$$C = \max_{\{p(x_j)\}} I(\mathscr{X};\mathscr{Y}) \tag{10.59}$$

The channel capacity C is measured in *bits per channel use.*

Note that the channel capacity C is a function only of the transition probabilities $p(y_k|x_j)$, which define the channel. The calculation of C involves maximization of the average mutual information $I(\mathscr{X};\mathscr{Y})$ over J variables [i.e., the input probabilities $p(x_0), \ldots, p(x_{J-1})$] subject to two constraints:

$$p(x_j) \geq 0 \text{ for all } j$$

The channel capacity C varies with the probability of error (transition probability) p as shown in Fig. 10.11, which is symmetric about $p = 1/2$. Comparing the curve in this figure with that in Fig. 10.2, we may make the following observations:

1. When the channel is *noise free*, permitting us to set $p = 0$, the channel capacity C attains its maximum value of one bit per channel use, which is exactly the information in each channel input. At this value of p, the entropy function $H(p)$ attains its minimum value of zero.
2. When the conditional probability of error $p = 1/2$ due to noise, the channel capacity C attains its minimum value of zero, whereas the entropy function $H(p)$ attains its maximum value of unity; in such a case the channel is said to be *useless*.

<u>Question-7(a)</u>

Advantages and disadvantages of error control control

Error control coding (ECC) is a technique used in digital communication and data storage to detect and correct errors. It improves reliability but comes with trade-offs. Here are its advantages and disadvantages:

Advantages:

1. **Improved Data Integrity** – ECC helps detect and correct errors, ensuring accurate data transmission and storage.
2. **Reliable Communication** – It enhances communication over noisy channels (e.g., wireless networks, deep space communication).
3. **Efficient Storage Systems** – Used in RAM, SSDs, and other storage devices to protect against data corruption.
4. **Extended Transmission Distance** – Enables data transmission over long distances without significant loss (e.g., satellite and fiber-optic communication).
5. **Reduces Retransmissions** – Error correction reduces the need for retransmission, improving system efficiency.

Disadvantages:

1. **Increased Redundancy** – ECC requires extra bits for error detection and correction, increasing data size.
2. **Higher Processing Overhead** – Encoding and decoding require additional computation, slowing down processing speed.
3. **More Complex Hardware** – Implementing ECC requires sophisticated circuits, increasing system cost and design complexity.
4. **Limited Error Correction Capability** – Some codes can only correct a limited number of errors, making them ineffective against severe noise.
5. **Energy Consumption** – Extra processing and memory usage increase power consumption, which is critical in battery-powered devices.

Question-7(b)

Given:

- C is a valid codeword of a linear block code.
- H is the **parity-check matrix** of the code.

Parity-Check Matrix Property:
The parity-check matrix HHH of a code is an (n−k)×n(n-k) \times n(n−k)×n matrix that defines the set of valid codewords. A codeword CCC belongs to the code if and only if it satisfies the fundamental equation:

C H^T = 0

where:

- C is a **row vector** of length n
- H is an **(n−k)×n(n-k) \times n(n−k)×n** matrix,
- H^T is the transpose of H, making it an n×(n−k)n \times (n-k)n×(n−k) matrix,
- The product C H^T results in a zero vector of length (n−k)(n-k)(n−k).

2. **Explanation of C H^T = 0:**
   - The rows of H define a set of linear constraints that all valid codewords must satisfy.
   - Since the code is linear, any valid codeword is formed as a linear combination of the generator matrix rows.

- By construction, all codewords are orthogonal to the rows of H, meaning their dot product results in zero.

3. **Conclusion:**
   Since C satisfies the parity-check equation, the syndrome SSS computed as:

   S = C H^T

results in a zero vector. This confirms that C is a valid codeword.

Thus, we have proved that for any valid code vector C, the equation C H^T =0 holds.

(80)

<u>To prove that $CH^T = 0$:</u>

We know that, $CH^T = DGH^T$

Now, $G = [I_k \mid P]_{k \times n}$

& $H = [P^T \mid I_{n-k}]_{(n-k)\times n}$

$\therefore\ H^T = \begin{bmatrix} P \\ \hline I_{n-k} \end{bmatrix}_{n \times (n-k)}$

$\therefore\ GH^T = [I_k \mid P] \begin{bmatrix} P \\ \hline I_{n-k} \end{bmatrix}_{k \times (n-k)}$ ——— (1)

Consider an element P_{ij} in submatrix P in the above equation. When the particular row in G is multiplied with the particular column in H^T, due to the identity matrices in both of them,

$$(1 \times P_{ij}) \oplus (P_{ij} \times 1) = 0.$$

Hence, the result is a null matrix.

i.e, $GH^T = [O]_{k \times (n-k)}$

Multiplying both sides by D of size $(1 \times k)$,

$DGH^T = 0$

i.e, $\boxed{CH^T = 0}$ of size $1 \times (n-k)$.

Question-7(c) Design an encoder for the (7, 4) binary cyclic code generated by : $g(x)= 1+x^2 + x^3$ for the message vector 1001.

Understand the (7,4) Cyclic Code

(7,4) code means 4-bit message vectors are encoded into 7-bit codewords. The generator polynomial g divides the message polynomial multiplied by x^r, where $r=7-4=3$

Now let us Represent the Message as a Polynomial

The given message vector is **1001**, which we interpret as a polynomial:

$m(x) = x^3 + 1$

Multiply by x^r

Shift the message polynomial left by $r=3$:

$x^3 m(x) = x^6 + x^3$

Perform Modulo Division by $g(x)$

Let us find the remainder when $x^6 + x^3$ is divided by $g(x)$:

$(x^6 + x^3) \div (1 + x^2 + x^3)$

Using polynomial long division:

1. Divide x^6 by x^3, quotient is x^3.
2. Multiply: $x^3(1 + x^2 + x^3) = x^3 + x^5 + x^6$.
3. Subtract: $(x^6 + x^3) - (x^6 + x^5 + x^3) = x^5$.
4. Divide x^5 by x^3, quotient is x^2.
5. Multiply: $x^2(1 + x^2 + x^3) = x^2 + x^4 + x^5$.
6. Subtract: $x^5 - (x^5 + x^4 + x^2) = x^4 + x^2$.
7. Divide x^4 by x^3, quotient is x.
8. Multiply: $x(1 + x^2 + x^3) = x + x^3 + x^4$.
9. Subtract: $(x^4 + x^2) - (x^4 + x^3 + x) = x^3 + x^2 + x$.

The remainder is:

$x^3 + x^2 + x$

Construct the Codeword

The codeword is formed by appending the remainder to $x^3 m(x)$:

c(x) = x^6 + x^3 + x^3 + x^2 + x

Simplifying:

c(x)=x6+x2+xc(x) = x^6 + x^2 + xc(x)=x6+x2+x

let's Convert to Codeword Vector

The coefficient representation in 7-bit binary format is:

0100011

Final Answer:

The encoded **(7,4) cyclic codeword** for message **1001** is **0100011**.

Question-8(b)

A **(7,4) linear block code** means:

- **4-bit message vectors** are encoded into **7-bit codewords.**
- The **parity-check matrix** P is given as:

$$P = \begin{bmatrix} 110 \\ 011 \\ 111 \\ 101 \end{bmatrix}$$

The **generator matrix** G in standard form is:

$$G = [I_4 | P^T] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

This generator matrix encodes any **4-bit message** into a **7-bit codeword.**

Each message vector m (4-bit) is encoded as:

$$c = mG$$

The 16 possible message vectors and their corresponding codewords are:

Message m	Codeword c	Hamming Weight
0000	0000000	0
0001	0001011	3
0010	0010111	4
0011	0011100	3
0100	0100110	3
0101	0101101	4
0110	0110001	3
0111	0111010	4
1000	1001100	3
1001	1000111	4
1010	1011011	5
1011	1010000	2
1100	1101010	4
1101	1100001	3
1110	1111101	5
1111	1110110	4

Thus, the Hamming weights of the codewords range from **0 to 5**.

The **parity-check matrix** H is derived from P:

$$H = [P^T | I_3] = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The received vector is:

$$r = [1100010]$$

To find the syndrome s:

$$s = Hr^T$$

Performing the matrix multiplication modulo 2:

$$s = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

S=110. So we will see H^T and try to match this value in the r given.

So as per us it's first position . so we will flip the 1st bit. So correct r is 0100010.

Question-8(a)

Describe the block diagram of generator and parity check matrix with equation. Also write syndrome equation and list its properties.

Let us represent the msg. block as a row-vector of data-bits or k-tuple as,

$$D = (d_1\ d_2 \dots d_k)$$

Let the code-word be represented as,

$$C = (c_1\ c_2 \dots c_k\ c_{k+1} \dots c_n)$$

This code-vector contains $(n-k)$ check bits at the end, & hence the "rate efficiency" of this (n,k) block code is k/n. These $(n-k)$ check bits are generated according to a predetermined rule, as–

$$c_{k+1} = p_{11}d_1 \oplus p_{21}d_2 \oplus \dots \oplus p_{k1}d_k$$

$$c_{k+2} = p_{12}d_1 \oplus p_{22}d_2 \oplus \dots \oplus p_{k2}d_k$$

$$\vdots$$

$$c_n = p_{1,n-k}d_1 \oplus p_{2,n-k}d_2 \oplus \dots \oplus p_{k,n-k}d_k$$

The coefficients p_{ij} are 0's and 1's, which are predetermined, & the addition operation is performed using modulo-2 arithmetic.

Therefore, the matrix form for the code-vectors is written as,

$$[c_1\, c_2 \dots c_n] = [d_1\, d_2 \dots d_k] \begin{bmatrix} 1\,0\,0 \dots 0\,0 & P_{11}\, P_{12} \dots P_{1,n-k} \\ 0\,1\,0 \dots 0\,0 & P_{21}\, P_{22} \dots P_{2,n-k} \\ \vdots & \\ 0\,0\,0 \dots 0\,1 & P_{k1}\, P_{k2} \dots P_{k,n-k} \end{bmatrix}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad \underbrace{\qquad\qquad}_{k} \quad \underbrace{\qquad\qquad}_{n-k}$$

Or, $[C] = [D][G]$

where $[G] = [I_k \mid P]_{k \times n}$

Here, G = Generator matrix of order $(k \times n)$

I_k = Identity matrix of order $(k \times k)$

P = Parity matrix of order $(k \times n-k)$

When the parity matrix is specified, the (n,k) block code is fully defined.

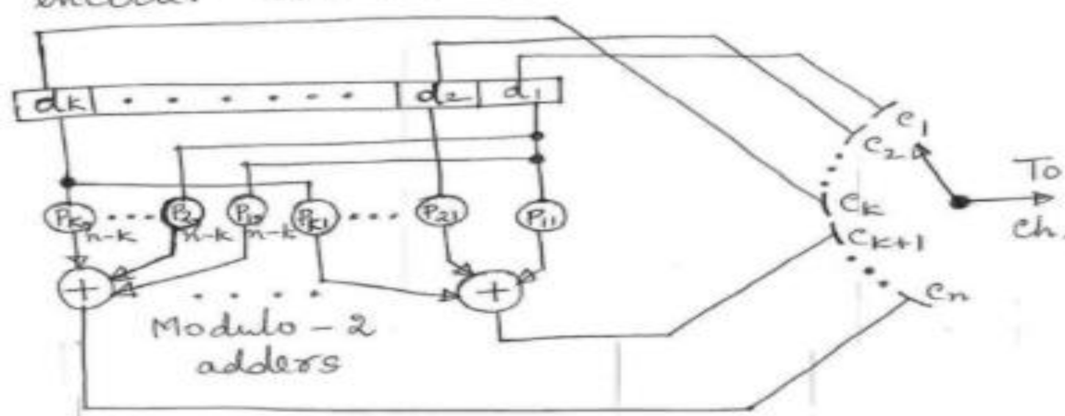
Parity check matrix (H): The generator matrix is utilized for the encoding operation, by using the stored submatrix, P. In a similar fashion, the receiver requires a matrix for decoding, which is called as parity check matrix, H. This matrix is defined as —

$$H = \begin{bmatrix} P_{11} & P_{21} & \dots & P_{k1} & 1\,0\,0 \dots 0\,0 \\ P_{12} & P_{22} & \dots & P_{k2} & 0\,1\,0 \dots 0\,0 \\ \vdots & & & & \\ P_{1,n-k} & P_{2,n-k} & \dots & P_{k,n-k} & 0\,0\,0 \dots 0\,1 \end{bmatrix}$$

or, $H = [P^T \mid I_{n-k}]_{(n-k) \times n}$

This matrix is also called as "Hamming"

The encoder ckt can be realized as-



matrix, and this is used for error detection & correction, at the receiver.

Syndrome & Error correction: Let C be the code-vector transmitted & let R be the code-vector received. Due to noise in the ch., the vector R may be different from the valid vector C. Hence, the error vector is, $E = R \oplus C$. The error vector can be represented as –

$$E = (e_1\ e_2 \ldots e_n)$$

The error-vector is a n-tuple where $e_i = 1$ if $r_i \neq c_i$ and $e_i = 0$ if $r_i = c_i$. Hence, the 1's present in E represent the error caused by noise in the ch. In order to find E, the receiver utilizes an (n-k) vector S defined as,

$$S = R.H^T = (s_1\ s_2 \ldots s_{n-k})$$

As R is $1 \times n$ and H^T is $n \times (n-k)$, the resultant vector, S is $1 \times (n-k)$. The vector "S" is called as "Error Syndrome" of R, which is used to obtain E.

$$\therefore S = (C \oplus E) H^T$$
$$= CH^T \oplus EH^T$$
$$= 0 \oplus EH^T \quad (\because CH^T = 0)$$
$$\therefore \boxed{S = EH^T}$$

As both S & H^T are known, the receiver can compute E, using this equation. When E is obtained, C can be easily obtained as, $\boxed{C = R \oplus E}$

Note: If $R = C$, then $S = 0$.

Syndrome calculation ckt:

Let $R = (r_1\ r_2 \ldots r_n)$

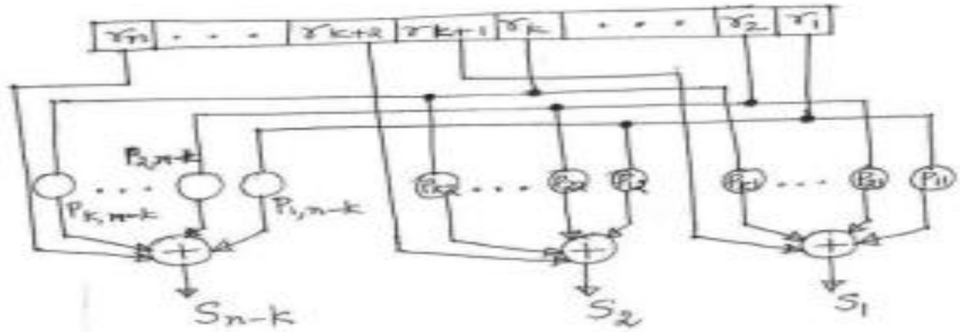
& $S = (s_1\ s_2 \ldots s_{n-k})$

We know that, $S = RH^T$

$$\therefore [s_1\ s_2 \ldots s_{n-k}] = [r_1\ r_2 \ldots r_n] \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1,n-k} \\ P_{21} & P_{22} & \cdots & P_{2,n-k} \\ \vdots & & & \vdots \\ P_{k1} & P_{k2} & \cdots & P_{k,n-k} \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\therefore s_1 = r_1 P_{11} \oplus r_2 P_{21} \oplus \ldots \oplus r_k P_{k1} \oplus r_{k+1}$$
$$s_2 = r_1 P_{12} \oplus r_2 P_{22} \oplus \ldots \oplus r_k P_{k2} \oplus r_{k+2}$$
$$\vdots$$
$$s_{n-k} = r_1 P_{1,n-k} \oplus r_2 P_{2,n-k} \oplus \ldots \oplus r_k P_{k,n-k} \oplus r_n$$

$\therefore$ The ckt. can be written as—



PROPERTIES OF SYNDROME:

1. For all the single-error patterns, the syndromes generated are unique.
2. The syndromes generated for double-error patterns are different from those of single-error patterns.

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3. Depending upon the length of the code, the syndromes of double-error patterns may not be unique, which means to say that, the same syndrome can be generated for two different code-vectors received.
4. If the syndromes are not unique, then the error is detected, but it cannot be corrected.

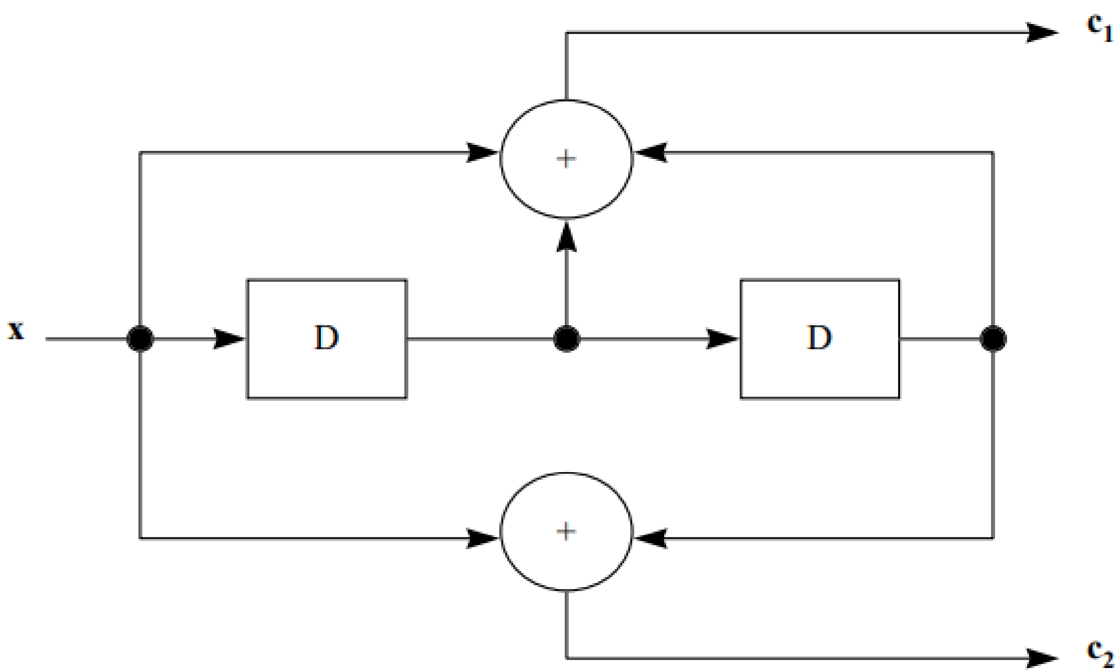
<u>Question-9(b)</u>

Describe the recursive convolutional code encoder with an example.

Recursive Convolutional Code Encoder

A **Recursive Convolutional Code (RCC) Encoder** is a type of convolutional encoder that includes feedback in its structure. Unlike a non-recursive convolutional encoder, where the input directly affects the output, in an RCC, the output depends not only on the current input but also on past outputs due to the feedback loop.

The recursive systematic convolutional (RSC) encoder is obtained from the nonrecursive nonsystematic (conventional) convolutional encoder by feeding back one of its encoded outputs to its input. Below Figure shows a conventional convolutional encoder.

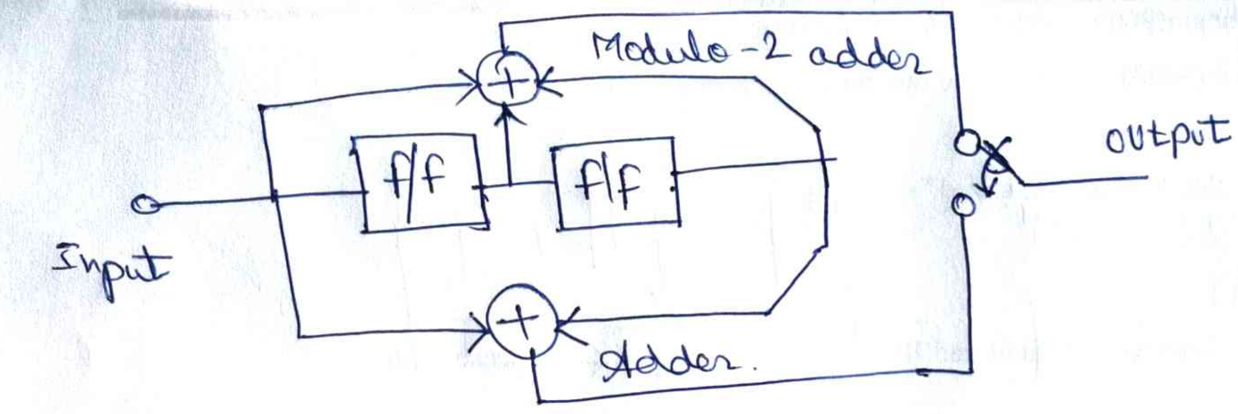


The conventional convolutional encoder is represented by the generator sequences g1 =[111] and g2 =[101] and can be equivalently represented in a more compact form as G=[g1, g2]. The RSC encoder of this conventional convolutional encoder is represented as G=[1, g2 / g1] where the first output (represented by g1) is fed back to the input. In the above representation, 1 denotes the systematic output, g2 denotes the feedforward output, and g1 is the feedback to the input of the RSC encoder.

<u>Question-9(a)</u>

To compute the output sequence of a convolutional encoder with parameters (2,1,3)(2,1,3)(2,1,3) using the **transform domain approach**, follow these steps:

Q-9 a) $D = 10011$



a) Output Sequence.

$$K=1,\ n=2,\ r=\frac{1}{2},\ m=2\ \&\ K=3$$

$$(2,1,3)$$

Generator Polynomials :→

$$G_1(D) = g_1^0 + g_1^1 D + g_1^2 D^2$$

$$G_2(D) = g_2^0 + g_2^1 D + g_2^2 D^2$$

∴ let's assume the generator polynomials for this convolutional encoder are :-

$$G_1(D) = 1 + D + D^2 \quad —(1)$$

$$G_2(D) = 1 + D^2 \quad —(2)$$

Transform Domain approach computes the output sequence as :-

$$X_i(D) = D(D) \cdot G_i(D) \quad —(3)$$

where $D(D)$ is the data sequence treated as a polynomial.

$$\therefore \quad D = 10011$$

$$D(D) = 1 + 0 \cdot D^1 + 0 \cdot D^2 + 1 \cdot D^3 + 1 \cdot D^4$$

$$= 1 + D^3 + D^4 \quad - (4)$$

from (3) & (4),

$\therefore$ $x_1(D)$

$$x_1(D) = D(D) \cdot G_1(D)$$

$$= (1 + D^3 + D^4) \oplus (1 + D + D^2)$$

$$= 1 + D + D^2 + D^3 + D^4 + D^6 + D^4 + D^5 + D^6$$

$$= 1 \oplus D + D^2 + D^3 + 2D^4 + 2D^5 + D^6$$

Since binary arithmetic follows modulo-2 Addition, we get :-

$$x_1(D) = 1 + D + D^2 + D^3 + D^6 \quad - (5)$$

Again, from (4) & (2)

$$X_2(D) = D(D) \cdot G_2(D)$$

$$= (1+D^3+D^4)(1+D^2)$$

$$= 1+D^2+D^3+D^5+D^4+D^6$$

$$= 1+D^2+D^3+D^4+D^5+D^6 \quad \text{— (6)}$$

Output Sequences from (5) & (6),

$X_1(D) = 1,1,1,1,0,0,1$

$X_2(D) = 1,0,1,1,1,1,1$

So, the final encoded bit sequences (interleaved) is :-

(11, 10, 11, 11, 01, 01, 11)

or, in a ~~sequence~~ Sequence,

11101111010111