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Internal Assesment Test-I										
Sub:	Electromagnetic Theory							Code:	BEC401
Date:	26/03/2025	Duration:	90 mins	Max Marks:	50	Sem:	4th	Branch:	ECE(A,B,C,D)
Answer any **FIVE FULL** Questions									

		Marks	OBE CO	OBE RBT
1. a)	Transform the vector 10 $\mathbf{a}_x$ to spherical coordinate system at P(3,2,4)	[04]	CO1	L3
1. b)	Point charges of 50 nC each are located at A(1, 0, 0), B(−1, 0, 0), C(0, 1, 0), and D(0,−1, 0) in free space. Find the total force on the charge at A.	[06]	CO1	L3
2. a)	State and explain Coulomb's law in vector form.	[05]	CO1	L2
2. b)	Calculate **E** and **D** in rectangular coordinates at point P(2,−3, 6) produced by a point charge Q_A = 55 mC at A(−2, 3,−6).	[05]	CO1	L3
3.	Define line charge density. Obtain an expression for electric field intensity due to an infinitely long uniform line charge distribution.	[10]	CO1	L2
4.	Derive Maxwell's first equation of electrostatics. Also obtain the expression for Gauss's Divergence theorem.	[10]	CO2	L2
5. a)	Define surface charge density. Obtain an expression of electric field intensity due to an infinite sheet of charge from Gauss's law.	[06]	CO2	L3
5. b)	Define electric flux density. Derive the relation between electric flux density and electric field intensity.	[04]	CO2	L2
6.	Let $\mathbf{D} = 4xy\, \mathbf{a}_x + 2(x^2 + z^2)\, \mathbf{a}_y + 4yz\, \mathbf{a}_z$ C/m^2 and evaluate both sides of Divergence theorem to find the total charge enclosed in the rectangular parallelepiped $0 < x < 2, 0 < y < 3, 0 < z < 5$ m.	[10]	CO2	L3
7. a)	Derive an expression for the work done in moving a point charge Q in the presence of an electric field **E**.	[05]	CO3	L2
7. b)	Calculate the work done in moving a 2 μC charge from A (2,1,-1) to B (8,2,1) in electric field $\mathbf{E} = y\, \mathbf{a}_x + x\, \mathbf{a}_y$ along a straight line x = 6y- 4.	[05]	CO3	L3

CI CCI HoD

1. a) Transform the vector 10 $\mathbf{a_x}$ to spherical coordinate system at P(3,2,4) [04] CO1 L3

1) a)

$\vec{A} = 10\,\hat{a}_x$

P(3,2,4)

$$\boxed{\begin{aligned} A_x &= 10 \\ A_y &= 0 \\ A_z &= 0 \end{aligned}}$$

Convert to spherical co-ordinates

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$A_r = (\sin\theta\cos\phi)\,10 = 5.57$

$A_\theta = (\cos\theta\cos\phi)\,10 = 6.18$

$A_\phi = (-\sin\phi)\,10 = -5.546$

$$\boxed{\begin{aligned} \theta &= \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) = 42.03^\circ \\ \phi &= \tan^{-1}(y/x) = 33.69^\circ \end{aligned}}$$

$$\boxed{\vec{A} = 5.57\,\hat{a}_r + 6.18\,\hat{a}_\theta - 5.546\,\hat{A}_\phi}$$

1. b) Point charges of 50 nC each are located at A(1, 0, 0), B(−1, 0, 0), C(0, 1, 0), and D(0,−1, 0) in free space. Find the total force on the charge at A. [06] CO1 L3

Point charges of 50nC each are located at $A(1,0,0)$, $B(-1,0,0)$, $C(0,1,0)$, and $D(0,-1,0)$ in free space. Find the total force on the charge at A.

The force will be:

$$\mathbf{F} = \frac{(50\times10^{-9})^2}{4\pi\epsilon_0}\left[\frac{\mathbf{R}_{CA}}{|\mathbf{R}_{CA}|^3} + \frac{\mathbf{R}_{DA}}{|\mathbf{R}_{DA}|^3} + \frac{\mathbf{R}_{BA}}{|\mathbf{R}_{BA}|^3}\right]$$

where $\mathbf{R}_{CA} = \mathbf{a}_x - \mathbf{a}_y$, $\mathbf{R}_{DA} = \mathbf{a}_x + \mathbf{a}_y$, and $\mathbf{R}_{BA} = 2\mathbf{a}_x$. The magnitudes are $|\mathbf{R}_{CA}| = |\mathbf{R}_{DA}| = \sqrt{2}$, and $|\mathbf{R}_{BA}| = 2$. Substituting these leads to

$$\mathbf{F} = \frac{(50\times10^{-9})^2}{4\pi\epsilon_0}\left[\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{2}{8}\right]\mathbf{a}_x = \underline{21.5\mathbf{a}_x\ \mu\text{N}}$$

where distances are in meters.

2. a) State and explain Coulomb's law in vector form. [05] CO1 L2

THE EXPERIMENTAL LAW OF COULOMB:

Coulomb's law states that the force between two very small objects separated in a vacuum or free space by a distance, which is large compared to their size, is proportional to the charge on each and inversely proportional to the square of the distance between them.

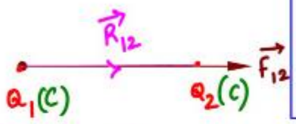
Force between Q1 and Q2: $\boxed{F = k\dfrac{Q_1 Q_2}{R^2}}$

Proportionality constant, $k = 9\times10^{9} = \dfrac{1}{4\pi\varepsilon_0}$

Permittivity of free space

$$\varepsilon_0 = \frac{10^{-9}}{36\pi} = 8.854\times10^{-12}\ (F/m)$$

The vector form of Coulomb's law is



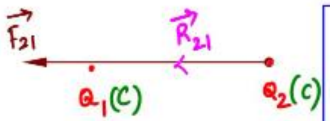
$\boxed{\vec{R}_{12} = -\vec{R}_{21}}$

Force on Q_2 by Q_1

$$\boxed{\vec{F}_{12} = k\frac{Q_1 Q_2}{|\vec{R}_{12}|^2}\hat{a}_{R_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\frac{\vec{R}_{12}}{|\vec{R}_{12}|^3}}\ N$$

where, $\hat{a}_{R_{12}}$ = unit vector in the direction of $\vec{R}_{12}$

$$\hat{a}_{R_{12}} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|}$$



Force on Q_1 by Q_2:

$$\boxed{\vec{F}_{21} = k\frac{Q_1 Q_2}{|\vec{R}_{21}|^2}\hat{a}_{R_{21}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\frac{\vec{R}_{21}}{|\vec{R}_{21}|^3}}\ N$$

$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$

where, $\hat{a}_{R_{21}}$ = unit vector in the direction of $\vec{R}_{21}$

$$\hat{a}_{R_{21}} = \frac{\vec{R}_{21}}{|\vec{R}_{21}|}$$

2. b) Calculate **E** and **D** in rectangular coordinates at point P(2,−3, 6) produced by a point charge Q_A = 55 mC at A(−2, 3,−6). [05] CO1 L3

$Q_A = 55mC$

$(-2, 3, -6)$

$\vec{R}_p = (4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z)$

P(2, −3, 6)

$R_p = \sqrt{4^2+6^2+12^2} = 14$

$$\vec{D} = \frac{Q}{4\pi}\frac{\vec{R}_p}{R_p^3}$$

$$\vec{D} = \frac{55\times10^{-3}}{4\pi}\times\frac{(4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z)}{(14)^3}$$

$$\boxed{\vec{D} = (6.38\hat{a}_x - 9.57\hat{a}_y + 19.14\hat{a}_z)\ \mu C/m^2}$$

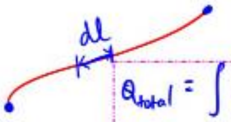
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{\vec{R}}{R^3}\right) = \frac{55\times10^{-3}\times9\times10^{9}(4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z)}{(14)^3}$$

$$\boxed{\vec{E} = 721.57\hat{a}_x - 1082.361\hat{a}_y + 2164.72\hat{a}_z\ V/m}$$

3. Define line charge density. Obtain an expression for electric field intensity due to an infinitely long uniform line charge distribution. [10] CO1 L2

Line charge density ρ_L C/m

It is defined as the Charge per unit length of the line charge distribution.



$$Q_{total} = \int_L \rho_L \cdot dl$$

$$\boxed{\rho_L = \lim_{\Delta L \to 0} \frac{Q}{\Delta L}} \quad C/m$$

Electric field Intensity due to a continuous line charge distribution with line charge density, ρ_L C/m:

(i) Finitely Long line charge

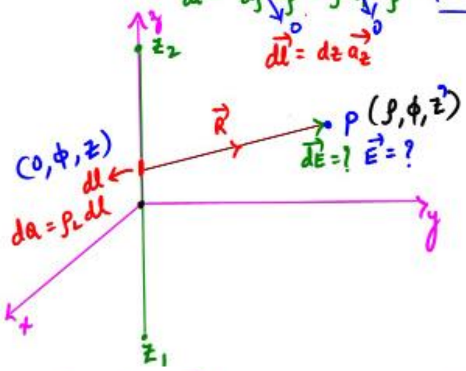
(ii) Infinitely long line charge

(i) Finitely Long line charge

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon |\vec{R}|^3} \cdot \vec{R}$$

$$\vec{dl} = d\rho\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + \boxed{dz\, \hat{a}_z}$$

$$\vec{dl} = dz\, \vec{a}_z$$



1) $dl = dz$

2) $dQ = \rho_L dl$

$dQ = \rho_L\, dz$

3) $\vec{R} = \rho\, \hat{a}_\rho + (z'-z)\hat{a}_z$

$|\vec{R}| = \left(\rho^2 + (z'-z)^2\right)^{1/2}$

$|\vec{R}|^3 = \left(\rho^2 + (z'-z)^2\right)^{3/2}$

4) $$d\vec{E} = \frac{dQ}{4\pi\varepsilon |\vec{R}|^3} \cdot \vec{R}$$

$dQ = \rho_L\, dz$

$\vec{R} = \rho\, \hat{a}_\rho + (z'-z)\hat{a}_z$

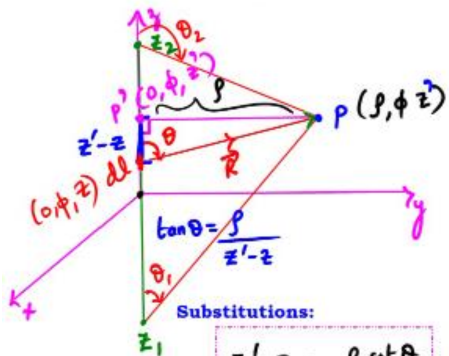
$|\vec{R}|^3 = \left(\rho^2 + (z'-z)^2\right)^{3/2}$

$$d\vec{E} = \frac{\rho_L\, dz\, \left[\rho\, \hat{a}_\rho + (z'-z)\, \hat{a}_z\right]}{4\pi\varepsilon \left[\rho^2 + (z'-z)^2\right]^{3/2}}$$

5) $$\vec{E} = \int_{z=z_1}^{z_2} d\vec{E}$$

$$\vec{E} = \int_{z=z_1}^{z_2} \frac{\rho_L\, dz\, \left[\rho\, \hat{a}_\rho + (z'-z)\, \hat{a}_z\right]}{4\pi\varepsilon \left[\rho^2 + (z'-z)^2\right]^{3/2}}$$

(i) Finitely Long line charge



Substitutions:

$$\boxed{z' - z = \rho \cot\theta}$$

Differentiating the above equation,

$$0 - dz = -\rho \operatorname{cosec}^2\theta \, d\theta$$

$$\boxed{dz = \rho \operatorname{cosec}^2\theta \, d\theta}$$

Change in limits

z:	z_1	z_2
θ:	θ_1	θ_2

$$\vec{E} = \int_{z=z_1}^{z_2} \frac{\rho_L \, dz \left[\rho \hat{a}_\rho + (z'-z)\hat{a}_z\right]}{4\pi\varepsilon \left[\rho^2 + (z'-z)^2\right]^{3/2}}$$

$$\boxed{z' - z = \rho \cot\theta}$$

$$\boxed{dz = \rho \operatorname{cosec}^2\theta \, d\theta}$$

After making all the subtitutions,

Change in limits

z:	z_1	z_2
θ:	θ_1	θ_2

$$\vec{E} = \int_{\theta=\theta_1}^{\theta_2} \frac{\rho_L \; \rho \operatorname{cosec}^2\theta \, d\theta \left[\rho \hat{a}_\rho + \rho \cot\theta \, \hat{a}_z\right]}{4\pi\varepsilon \left[\rho^2 + \rho^2 \cot^2\theta\right]^{3/2}}$$

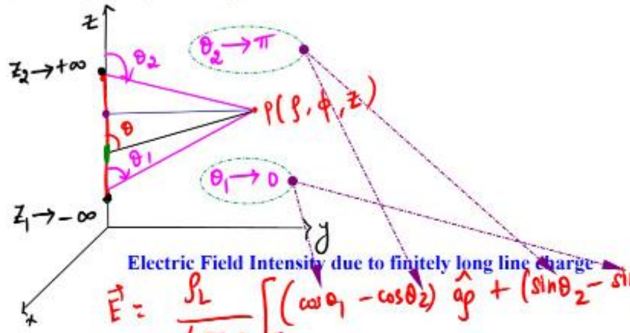
$$\vec{E} = \int_{\theta=\theta_1}^{\theta_2} \frac{\rho_L \; \rho^2 \operatorname{cosec}^2\theta \, d\theta \left[\hat{a}_\rho + \cot\theta \, \hat{a}_z\right]}{4\pi\varepsilon \left[\rho^2 + \rho^2 \cot^2\theta\right]^{3/2}}$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[\left[-\cos\theta\right]_{\theta_1}^{\theta_2} \hat{a}_\rho + \left[\sin\theta\right]_{\theta_1}^{\theta_2} \hat{a}_z \right]$$

$$\boxed{\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[(\cos\theta_1 - \cos\theta_2)\,\hat{a}_\rho + (\sin\theta_2 - \sin\theta_1)\,\hat{a}_z\right] \quad V/m}$$

Electric Field Intensity due to finitely long line charge

(ii) Infinitely Long line charge



Electric Field Intensity due to finitely long line charge

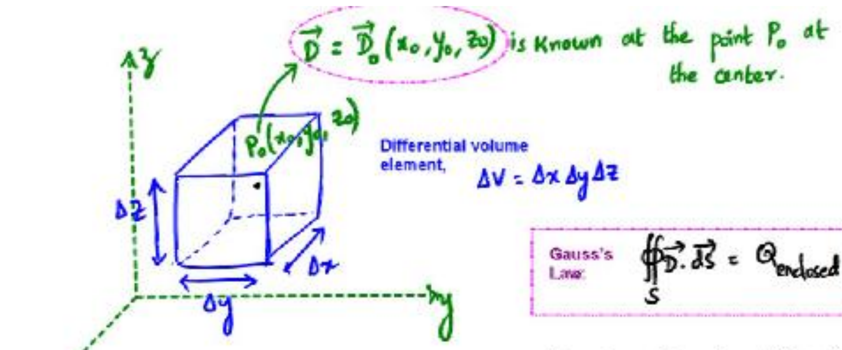
$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[(\cos\theta_1 - \cos\theta_2)\,\hat{a}_\rho + (\sin\theta_2 - \sin\theta_1)\,\hat{a}_z\right]$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[(1+1)\,\hat{a}_\rho + (0)\,\hat{a}_z\right]$$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon\rho}\,\hat{a}_\rho} \quad V/m$$

Electric Field Intensity due to infinitely long line charge

4. Derive Maxwell’s first equation of electrostatics. Also obtain the expression for Gauss’s Divergence theorem. [10] CO2 L2

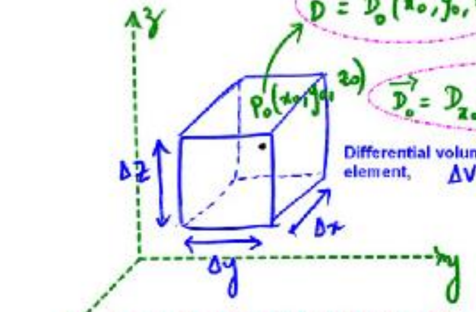


Gauss's Law: $\oint_S \vec{D}\cdot d\vec{s} = Q_{enclosed}$

$$\oint_S \vec{D}\cdot d\vec{s} = \iint_{\substack{\text{Front}\\\text{Surface}}} \vec{D}\cdot d\vec{s} + \iint_{\substack{\text{Back}\\\text{Surface}}} \vec{D}\cdot d\vec{s} + \iint_{\substack{\text{Left}\\\text{Surface}}} \vec{D}\cdot d\vec{s} + \iint_{\substack{\text{Right}\\\text{Surface}}} \vec{D}\cdot d\vec{s} + \iint_{\substack{\text{Top}\\\text{Surface}}} \vec{D}\cdot d\vec{s} + \iint_{\substack{\text{Bottom}\\\text{Surface}}} \vec{D}\cdot d\vec{s}$$

D is constant on each of the surfaces

$$\oint_S \vec{D}\cdot d\vec{s} = D_{front}\iint_{\substack{\text{Front}\\\text{Surface}}} d\vec{s} + D_{back}\iint_{\substack{\text{Back}\\\text{Surface}}} d\vec{s} + D_{left}\iint_{\substack{\text{Left}\\\text{Surface}}} d\vec{s} + D_{right}\iint_{\substack{\text{Right}\\\text{Surface}}} d\vec{s} + D_{top}\iint_{\substack{\text{Top}\\\text{Surface}}} d\vec{s} + D_{bottom}\iint_{\substack{\text{Bottom}\\\text{Surface}}} d\vec{s}$$



$\vec{D} = \vec{D}_0(x_0, y_0, z_0)$ is known at the point P_0 at the center.

$\vec{D}_0 = D_{x_0}\,\hat{a}_x + D_{y_0}\,\hat{a}_y + D_{z_0}\,\hat{a}_z$

Differential volume element, $\Delta V = \Delta x\,\Delta y\,\Delta z$

The front face is at a distance of $\Delta x/2$ from P0

$\therefore\ (x - x_0) = \frac{\Delta x}{2}$

Because the surface element is very small, D is essentially constant (over this portion of the entire closed surface) and we have only to approximate the value of Dx at this front face

$$D_{front} = D_{x_0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x} + \text{higher order terms}$$

negligible

Taylor Series (Three Dimensional):

$$f(x,y,z) = f(x_0,y_0,z_0) + \frac{(x-x_0)}{1!}\frac{\partial f}{\partial x} + \frac{(y-y_0)}{1!}\frac{\partial f}{\partial y} + \frac{(z-z_0)}{1!}\frac{\partial f}{\partial z} + \text{higher order terms}$$

$$\boxed{D_{back} = D_{x_0} - \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}}$$

If we combine these two integrals, we have

$$D_{front}\iint_{\substack{\text{Front}\\\text{Surface}}} d\vec{S} + D_{back}\iint_{\substack{\text{Back}\\\text{Surface}}} d\vec{S} = \left[D_{x_0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}\right](\Delta y\,\Delta z) + \left[D_{x_0} - \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}\right](-\Delta y\,\Delta z)$$

$$\boxed{D_{front}\iint_{\substack{\text{Front}\\\text{Surface}}} d\vec{S} + D_{back}\iint_{\substack{\text{Back}\\\text{Surface}}} d\vec{S} = \frac{\partial D_x}{\partial x}\,\Delta x\,\Delta y\,\Delta z}$$

By exactly the same process we find that,

$$D_{left}\iint_{\substack{\text{Left}\\\text{Surface}}} d\vec{S} + D_{right}\iint_{\substack{\text{Right}\\\text{Surface}}} d\vec{S} = \left[D_{y_0} + \frac{\Delta y}{2}\frac{\partial D_y}{\partial y}\right](\Delta x\,\Delta z) + \left[D_{y_0} - \frac{\Delta y}{2}\frac{\partial D_y}{\partial y}\right](-\Delta x\,\Delta z)$$

$$\boxed{D_{left}\iint_{\substack{\text{Left}\\\text{Surface}}} d\vec{S} + D_{right}\iint_{\substack{\text{Right}\\\text{Surface}}} d\vec{S} = \frac{\partial D_y}{\partial y}\,\Delta x\,\Delta y\,\Delta z}$$

By exactly the same process we find that,

$$D_{top}\iint_{\substack{\text{Top}\\\text{Surface}}} d\vec{S} + D_{bottom}\iint_{\substack{\text{Bottom}\\\text{Surface}}} d\vec{S} = \left[D_{z_0} + \frac{\Delta z}{2}\frac{\partial D_z}{\partial z}\right](\Delta x\,\Delta y) + \left[D_{z_0} - \frac{\Delta z}{2}\frac{\partial D_z}{\partial z}\right](-\Delta x\,\Delta y)$$

$$\boxed{D_{top}\iint_{\substack{\text{Top}\\\text{Surface}}} d\vec{S} + D_{bottom}\iint_{\substack{\text{Bottom}\\\text{Surface}}} d\vec{S} = \frac{\partial D_z}{\partial z}\,\Delta x\,\Delta y\,\Delta z}$$

$$\oint_S \vec{D}\cdot\vec{ds} = \iint_{\text{Front Surface}} \vec{D}\cdot\vec{ds} + \iint_{\text{Back Surface}} \vec{D}\cdot\vec{ds} + \iint_{\text{Left Surface}} \vec{D}\cdot\vec{ds} + \iint_{\text{Right Surface}} \vec{D}\cdot\vec{ds} + \iint_{\text{Top Surface}} \vec{D}\cdot\vec{ds} + \iint_{\text{Bottom Surface}} \vec{D}\cdot\vec{ds}$$

$$\boxed{\oint_S \vec{D}\cdot\vec{ds} = \left[\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right] \Delta x\, \Delta y\, \Delta z}$$

Gauss's Law: $\oint_S \vec{D}\cdot\vec{ds} = Q_{enclosed}$

$$\oint_S \vec{D}\cdot\vec{ds} = \left[\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right] \Delta x\, \Delta y\, \Delta z \qquad Q_{enc} = \rho_v\, \Delta V = \rho_v\, \Delta x\, \Delta y\, \Delta z$$

Equating the two sides of Gauss's Law: $\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_v$

$$\text{div}\, \vec{D} = \vec{\nabla}\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_v$$

$\boxed{\text{div}\, \vec{D} = \vec{\nabla}\cdot\vec{D} = \rho_v}$ Maxwell's First Equation of Electrostatics

This equation is also called Point (Differential) form of Gauss's law

Divergence Theorem:

This theorem applies to any vector field for which the appropriate partial derivatives exist.

This theorem can be derived from Gauss's law.

Gauss's Law: $\oint_S \vec{D}\cdot\vec{ds} = Q_{enclosed}$

$$\oint_S \vec{D}\cdot\vec{ds} = Q_{enclosed}$$

Maxwell's 1st Equation of electrostatics: $\rho_v = \vec{\nabla}\cdot\vec{D}$

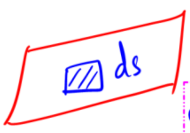
$$\oint_S \vec{D}\cdot\vec{ds} = Q_{enclosed} = \iiint_V \rho_v\, dv = \iiint_V \vec{\nabla}\cdot\vec{D}\, dv$$

$$\boxed{\oint_S \vec{D}\cdot\vec{ds} = \iiint_V \vec{\nabla}\cdot\vec{D}\, dv}$$ **Gauss's Divergence Theorem**

5. a) Define surface charge density. Obtain an expression of electric field intensity due to an infinite sheet of charge from Gauss's law. [06] CO2 L3

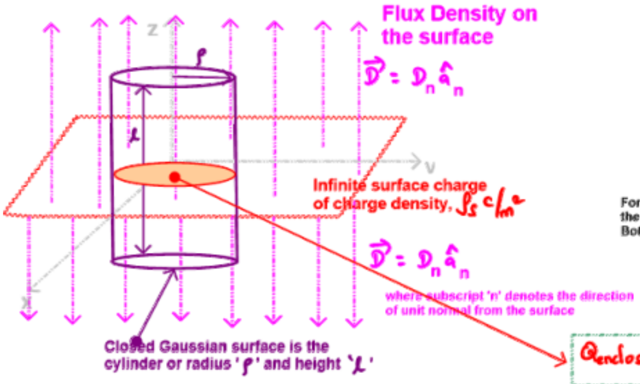
Surface charge density ρ_s C/m^2 It is defined as the Charge per unit surface area of the surface charge distribution.

$$\boxed{\rho_s = \lim_{\Delta S \to 0} \frac{Q}{\Delta S}} \; C/m^2$$



$$q_{total} = \iint_S \rho_s \cdot ds$$

iii) Application of Gauss's Law to an infinite sheet/surface charge:



Gauss's law: $\oint\!\!\oint_S \vec{D}\cdot\vec{ds} = Q_{enclosed}$

For Cylinder,
the number of surface integrals = 3 (Top Surface, Bottom Surface, Lateral Curved SUrface)

$Q_{enclosed} = \rho_s \times \text{Area} = \rho_s \times \pi\rho^2$

$$\oint\!\!\oint_S \vec{D}\cdot\vec{ds} = \underbrace{\iint_{\text{Top Surface}} \vec{D}\cdot\vec{ds} + \iint_{\text{Bottom Surface}} \vec{D}\cdot\vec{ds}}_{\vec{D}\text{ is normal to the surface}} + \underbrace{\iint_{\text{Lateral Curved Surface}} \vec{D}\cdot\vec{ds}}_{\vec{D}\text{ is tangential to the surface}}$$

$\vec{D}\cdot\vec{ds} = 0$

$$\oint\!\!\oint_S \vec{D}\cdot\vec{ds} = \iint_{\text{Top Surface}} \vec{D}\cdot\vec{ds} + \iint_{\text{Bottom Surface}} \vec{D}\cdot\vec{ds} = D_s\left[\iint_{S,\ \text{Top Surface}} ds\right] + D_s\left[\iint_{S,\ \text{Bottom Surface}} ds\right] = D_s \times \pi\rho^2 + D_s \times \pi\rho^2$$

$$\oint\!\!\oint_S \vec{D}\cdot\vec{ds} = 2\left(D_s \times \pi\rho^2\right)$$

Equating the two sides of Gauss's Law:

$$2D_s\,\cancel{\pi\rho^2} = \rho_s\,\cancel{\pi\rho^2}$$

$$\oint\!\!\oint_S \vec{D}\cdot\vec{ds} = 2\left(D_s \times \pi\rho^2\right)$$

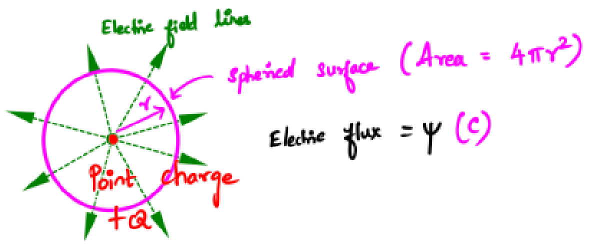
$$D_s = \frac{\rho_s}{2}$$

Electric Flux Density due to an infinite surface charge, $\vec{D} = \frac{\rho_s}{2}\hat{a}_n$ C/m^2 where subscript 'n' denotes the direction of unit normal from the surface

Electric Field Intensity due to an infinite line charge, $\vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{\rho_s}{2\varepsilon}\hat{a}_n$ V/m

5. b) Define electric flux density. Derive the relation between electric flux density and electric field intensity. [04] CO2 L2

Electric Flux is defined as the number of electric field lines crossing a given area.
The electric flux is denoted by `ψ´ (psi). Its unit is C.

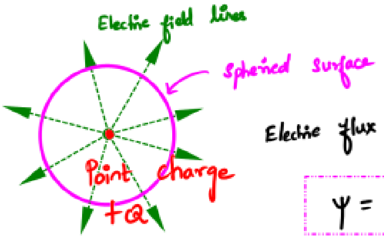


Electric Flux density is defined as the number of electric field lines crossing per unit area.
Electric Flux Density is also called as Displacement Flux Density. It is denoted by $\vec{D}$. Its unit is C/m^2.

$$\vec{D} = \frac{\Psi}{\text{Area}} \quad C/m^2$$

Electric Flux Density, $\vec{D} = \frac{\Psi}{4\pi r^2}\hat{a}_r$ C/m^2

Gauss's Law:
The electric flux passing through any closed surface is equal to the total charge enclosed by that surface.



Gauss's law

$\Psi = Q_{enclosed}$

Electric flux $= \Psi$ (C)

$\Psi = Q$

Electric Flux Density, $\vec{D} = \dfrac{\Psi}{4\pi r^2}\,\hat{a}_r$ C/m^2

$\vec{D} = \dfrac{Q}{4\pi r^2}\,\hat{a}_r$ C/m^2

Electric Field Intensity due to point charge, $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\,\hat{a}_r$ V/m

Comparing the above two expressions, we can get the relation between electric field intensity and electric flux density.

$\vec{D} = \varepsilon_0\vec{E}$ (for Free Space)

6. Let $\mathbf{D} = 4xy\,\mathbf{a_x} + 2(x^2 + z^2)\,\mathbf{a_y} + 4yz\,\mathbf{a_z}$ C/m^2 and evaluate both sides of Divergence theorem to find the total charge enclosed in the rectangular parallelepiped $0 < x < 2$, $0 < y < 3$, $0 < z < 5$ m. [10] CO2 L3

b) $\text{RHS} = \iiint_V \vec{\nabla}\cdot\vec{D}\, dV$ $\qquad \vec{D} = 4xy\,\hat{a}_x + 2(x^2+z^2)\,\hat{a}_y + 4yz\,\hat{a}_z$

$$\vec{\nabla}\cdot\vec{D} = \frac{\partial}{\partial x}(4xy) + \frac{\partial}{\partial y}(2x^2+2z^2) + \frac{\partial}{\partial z}(4yz)$$

$$= 4y + 4y$$

$$\vec{\nabla}\cdot\vec{D} = 8y \quad C/m^3 \qquad 0<x<2,\; 0<y<3,\; 0<z<5$$

$$Q_{enc} = \iiint_V 8y\, dx\, dy\, dz$$

$$= [x]_0^2 \left[\frac{8y^2}{2}\right]_0^3 [z]_0^5$$

$$= 2\times 4(9)\times 5$$

$$\boxed{Q_{enc} = 360\ C}$$

$$\text{LHS} = \iint \vec{D}\cdot d\vec{s}\Big|_{x=0} + \iint \vec{D}\cdot d\vec{s}\Big|_{x=2} + \iint \vec{D}\cdot d\vec{s}\Big|_{y=0} + \iint \vec{D}\cdot d\vec{s}\Big|_{y=3} + \iint \vec{D}\cdot d\vec{s}\Big|_{z=0} + \iint \vec{D}\cdot d\vec{s}\Big|_{z=5}$$

$$= -\iint \cancel{4xy}\, dy\, dz\Big|_{x=0} + \iint 4xy\, dy\, dz\Big|_{x=2} + \iint \cancel{2(x^2+z^2)}\, dx\, dz - \iint \cancel{2(x^2+z^2)}\, dx\, dz - \iint \cancel{4yz}\, dx\, dy\Big|_{z=0} + \iint 4yz\, dx\, dy\Big|_{z=5}$$

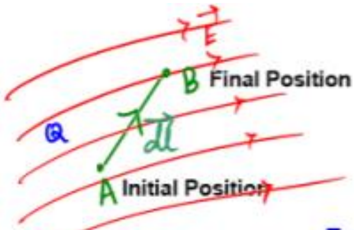
$$= \int_0^5\int_0^3 4\times 2\times y\, dy\, dz + 4\times 5\int_0^3\int_0^2 y\, dx\, dy$$

$$\text{LHS} = 8\left[\frac{y^2}{2}\right]_0^3 [z]_0^5 + 4\times 5\left[\frac{y^2}{2}\right]_0^3 [x]_0^2$$

$$= 4\times 9\times 5 + 10\times 9\times 2$$

$$\text{LHS} = 360\ C$$

7\. a) Derive an expression for the work done in moving a point charge Q in the presence of an electric field **E**. [05] CO3 L2



If we attempt to move the test charge against the electric field, we have to exert a force equal and opposite to that exerted by the field, and this requires us to expend energy or do work.

Work Done, $dW = \vec{F}\cdot\vec{dl}$ J

$$W = \int_{Initial}^{Final} \vec{F}\cdot\vec{dl}$$

Force on the point charge in the electric field,

$$\vec{F} = Q\vec{E}$$

Force applied to move the charge against electric field,

$$\vec{F}_{applied} = -Q\vec{E}$$

Work Done, $dW = -Q\vec{E}\cdot\vec{dl}$

$$\boxed{W = -\int_{Initial}^{Final} Q\vec{E}\cdot\vec{dl} = -Q\int_{A}^{B} \vec{E}\cdot\vec{dl} \text{ J}}$$

$$dW = -Q\vec{E}\cdot\vec{dl}$$

$$dW = -Q\vec{E}\cdot\vec{dl_1} - Q\vec{E}\cdot\vec{dl_2} - Q\vec{E}\cdot\vec{dl_3} - Q\vec{E}\cdot\vec{dl_4} - Q\vec{E}\cdot\vec{dl_5}$$

$$dW = -Q\vec{E}\cdot(\underbrace{\vec{dl_1} + \vec{dl_2} + \vec{dl_3} + \vec{dl_4} + \vec{dl_5}}_{\vec{dl}})$$

$$dW = -Q\vec{E}\cdot\vec{dl}$$

$$\boxed{W = -Q\int_{A}^{B} \vec{E}\cdot\vec{dl}}$$

*) Work done remains same irrespective of the path chosen in moving the charge from A to B

*) Work done around a closed path is zero

$$\boxed{W = -Q\oint_{L} \vec{E}\cdot\vec{dl} = 0}$$

*) In general, vectors whose line integral does not depend on the path of integration are called conservative. Thus, E is conservative field.

7. b) Calculate the work done in moving a 2 μC charge from A (2,1,-1) to B (8,2,1) in electric field $\mathbf{E} = y\,\mathbf{a_x} + x\,\mathbf{a_y}$ along a straight line x = 6y- 4. [05] CO3 L3

$$W = -Q \int_A^B \vec{E} \cdot \vec{dl} \Rightarrow W = -2\mu \left\{ \int_1^2 (6y-4)\,dy + \int_2^8 \left(\frac{x+4}{6}\right) dx \right\}$$

$x = 6y - 4$

$y = \frac{x+4}{6}$

$$\boxed{W = -28\,\mu J}$$