CBCS SCHEME

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21MATCS41

Fourth Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Mathematical Foundations for Computing, Probability and Statistics

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define tautology. Determine whether the following compound statement is a tautology or not.
$[(p \vee q) \to r] \leftrightarrow [\sim r \to \sim (p \vee q)]$ **(06 Marks)**

b. Prove the following using the laws of logic
$[\sim p \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$ **(07 Marks)**

c. Give direct proof and proof by contradiction for the statement "If n is an odd integer then n + q is an even integer". **(07 Marks)**

OR

2 a. Define : i) Open Statement ii) Quantifiers. **(06 Marks)**

b. Test the validity of the arguments using rules of inference
$p \to (q \to r)$
$p \vee \sim s$
q
$\therefore s \to r$ **(07 Marks)**

c. If $p(x) : x \geq 0$, $q(x) : x^2 \geq 0$, $r(x) : x^2 - 3x - 4 = 0$, $s(x) : x^2 - 3 > 0$. Determine the truth or falsity of the following statement:
i) $\exists x\, [p(x) \wedge q(x)]$
ii) $\forall x\, [p(x) \to q(x)]$
iii) $\forall x\, [q(x) \to s(x)]$
iv) $\forall x\, [r(x) \wedge s(x)]$
v) $\exists x\, [p(x) \wedge r(x)]$
vi) $\forall x\, [r(x) \to p(x)]$
vii) $\exists x[r(x) \to \sim p(x)]$ **(07 Marks)**

Module-2

3 a. Let f and g be functions from R to R defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$ if $(gof)(x) = 9x^2 - 9x + 3$. Determine a and b. **(06 Marks)**

b. Let A = {1, 2, 3, 4} and let R be the relation on A defined by xRy if and only if "x divides y" write down the relation R, relation matrix M_R and draw its diagraph. **(07 Marks)**

c. Prove that in every graph the number of vertices of odd degree is even. **(07 Marks)**

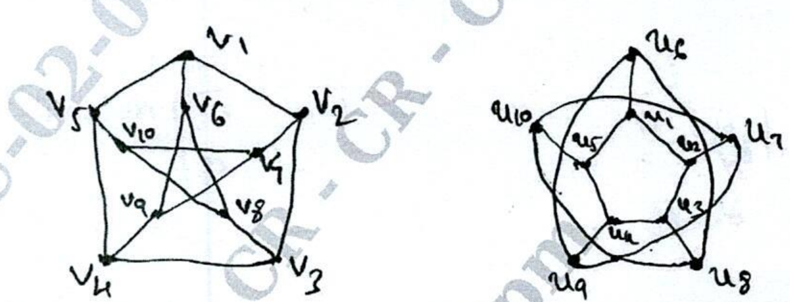
OR

4 a. Draw the Hasse diagram of the relation R on A = {1, 2, 3, 4, 5} whose matrix is as given below

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$ **(06 Marks)**

b. Consider the function $f : R \to R$ defined by $f(x) = 2x + 5$. Let a function $g : R \to R$ be defined by $g(x) = \frac{1}{2}(x - 5)$. Prove that g is an inverse of f. **(07 Marks)**

c. Define graph isomorphism. Determine whether the following graphs are isomorphic or not. **(07 Marks)**



Module-3

5 a. Calculate the coefficient of correlation and obtain the lines of regression for the following data:

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(06 Marks)

b. Fit a straight line in the least square sense for the following data:

x	5	10	15	20	25
y	16	19	23	26	30

(07 Marks)

c. Fit a curve $y = ax^b$ for the following data:

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(07 Marks)

OR

6 a. The following are the percentage of marks in mathematics (x) and statistics (y) of nine students. Calculate the rank correlation coefficient.

x	38	50	42	61	43	55	67	46	72
y	41	64	70	75	44	55	62	56	60

(06 Marks)

b. Fit a parabola $y = ax^2 + bx + c$ for the data

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

(07 Marks)

c. With usual notation, compute means $\bar{x}, \bar{y}$ and correlation coefficient r from the following lines of regression, $2x + 3y + 1 = 0$, $x + 6y - 4 = 0$. **(07 Marks)**

Module-4

7 a. A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find k, and evaluate $p(x < 6)$, $p(x \geq 6)$, $p(0 < x < 5)$ **(06 Marks)**

b. Find the mean and S.D of binomial distribution. **(07 Marks)**

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and S.D. 5. Find the number of students whose marks will be i) less than 65 ii) more than 75 iii) between 65 and 75 ($\phi(1) = 0.3413$). **(07 Marks)**

OR

8 a. Find the constant for such that $f(x) = \begin{cases} kx^2, & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$ is a p.d.f. Also compute $p(1 < x < 2)$, $p(x \leq 1)$, $p(x > 1)$. **(06 Marks)**

b. A shop has 4 diesel generator sets which it hires every day. The demand for a genset on an average is a poisson variate with value 5/2. Obtain the probability that on a particular day i) there was no demand ii) A demand had to be refused. **(07 Marks)**

c. In a normal distribution 31% of the iterms are under 45 and 8% of the items are over 64. Find the mean and S.D of the distribution: $(p(0.5) = 0.19, p(1.4) = 0.42)$ **(07 Marks)**

Module-5

9 a. The joint distribution of two random variables x and y is as follows:

x/y	3	4	5
2	1/6	1/6	1/6
5	1/12	1/12	1/12
7	1/12	1/12	1/12

Compute i) E(x) and E(y) ii) E(xy) iii) Cov (x, y). Are they independent random variables? **(06 Marks)**

b. A coin was tossed 400 times and head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance. **(07 Marks)**

c. In experiments on pea breeding, the following frequencies of seeds were obtained:

Round and Yellow	Wrinkled and Yellow	Round and Green	Wrinkled and Green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9:3:3:1. Examine the correspondence between theory and experiment $\left(\psi^2_{0.05} = 7.815 \text{ for } 3\text{df}\right)$ **(07 Marks)**

OR

10 a. Explain the terms:
i) Null hypothesis
ii) Significance level
iii) Type I and Type II errors **(06 Marks)**

b. The mean life of 100 fluorescent tube lights manufactured by a company is found to be 1570 hrs with a S.D of 120 hrs. Test the hypothesis that the mean life time of the lights produced by the company is 1600 hrs at 0.01 level of significance. **(07 Marks)**

c. The nine items of a sample have the following values:
45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5? ($t_{0.05} = 2.31$ for 8 d.f). **(07 Marks)**

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