CBCS SCHEME

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17MATDIP41

Fourth Semester B.E. Degree Examination, Dec.2024/Jan.2025
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of the matrix, $\begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$ by applying elementary row transformations. **(06 Marks)**

b. Solve the following system of linear equations by Gauss Elimination method.
$2x + y + 4z = 12$
$4x + 11y - z = 33$
$8x - 3y + 2z = 20$ **(07 Marks)**

c. Find the Inverse of the matrix, $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ by Cayley Hamilton theorem. **(07 Marks)**

OR

2 a. Reduce the matrix $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$ into its echelon form and hence find its rank. **(06 Marks)**

b. Find all the Eigen values and Eigen vectors corresponding to smallest eigen values of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **(07 Marks)**

c. Solve the system of linear equations $x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$ by applying Gauss Elimination method. **(07 Marks)**

Module-2

3 a. Solve : $(D^2 + D + 1)y = (1 - e^x)^2$ where $D = \frac{d}{dx}$. **(06 Marks)**

b. Solve : $y'' + 9y = \cos 2x . \cos x$ **(07 Marks)**

c. Solve : $(D^2 + 1)y = \tan x$ by the method of variation of parameters. **(07 Marks)**

OR

4 a. Solve : $y'' + 2y' + y = 2x + x^2$. **(06 Marks)**

b. Solve : $y'' - 4y' = \cosh(2x - 1) + 3^x$. **(07 Marks)**

c. Solve : $y'' - 2y' - 3y = e^{3x} + e^{2x}$ by the method of undetermined co-efficient. **(07 Marks)**

Module-3

5 a. Find the Laplace transform of,
(i) $e^{-2t} \sinh 4t$
(ii) $\sin 5t . \cos 2t$ **(06 Marks)**

b. Find (i) $L(e^{-t} \cos^2 3t)$
(ii) $L(t^2 \sin at)$ **(07 Marks)**

c. Find the Laplace transform of,
$f(t) = \begin{cases} E, & 0 \le t \le \frac{a}{2} \\ -E, & \frac{a}{2} \le t \le a \end{cases}$ where $f(t + a) = f(t)$. **(07 Marks)**

OR

6 a. Find the Laplace transform of,
(i) $(3t + 4)^3 + 5^t$
(ii) $\left(3\sqrt{t} + \frac{4}{\sqrt{t}}\right)$ **(06 Marks)**

b. Find (i) $L(t \cos at)$ (ii) $L(e^t \sin^2 t)$ **(07 Marks)**

c. Express $f(t) = \begin{cases} \sin 2t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$ interms of Heaviside unit step function and hence find their Laplace transform. **(07 Marks)**

Module-4

7 a. Find the Inverse Laplace transform of,
(i) $L^{-1}\left[\frac{(s+2)^3}{s^6}\right]$.
(ii) $L^{-1}\left[\frac{s+1}{s^2 + 6s + 9}\right]$ **(06 Marks)**

b. Find (i) $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$ (ii) $L^{-1}\left[\frac{s+2}{s^2(s+3)}\right]$ **(07 Marks)**

c. Using Laplace transforms, solve $y'' + 4y' + 3y = e^{-t}$ subject to the initial conditions, $y(0) = 1 = y'(0)$. **(07 Marks)**

OR

8 a. Find the Inverse Laplace transform of,
(i) $\left[\frac{1}{s\sqrt{s}} + \frac{3}{s^2\sqrt{s}} - \frac{8}{\sqrt{s}}\right]$
(ii) $\left(\frac{2s+1}{s^2 + 3s + 1}\right)$ **(06 Marks)**

b. Find (i) $L^{-1}\left[\frac{2s^2 + 5s - 4}{s^3 + s^2 - 2s}\right]$ (ii) $L^{-1}\left[\log\left(1 - \frac{a^2}{s^2}\right)\right]$. **(07 Marks)**

c. By applying Laplace transform, solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 10 \sin t$, subject to the initial conditions $y(0) = 0 = y'(0)$. **(07 Marks)**

Module-5

9 a. State and prove Baye's theorem. **(06 Marks)**

b. Three machines A, B, C produces 50%, 30% and 20% of the items in a factory. The percentage of defective outputs are 3, 4, 5. If an item is selected at random, what is the probability that it is defective? What is the probability that it is from A. **(07 Marks)**

c. A shooter can hit a target in 3 out of 4 shots and another shooter can hit the target in 2 out of 3 shots. Find the probability that the target is being hit.
   (i) When both of them try
   (ii) By only one shooter. **(07 Marks)**

OR

10 a. Prove that,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(A \cap B \cap C) - P(A \cap B) - P(B \cap C) - P(C \cap A)$$

(06 Marks)

b. A bag contains three coins, one of which is two headed and the other two are normal and fair. A coin is chosen at random from the bag and tossed four times in succession, if head turns up each time, what is the probability that this is the two headed coin. **(07 Marks)**

c. The probability that 3 students A, B, C, solve a problem are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. If the problem is simultaneously assigned to all of them what is the probability that the problem is solved. **(07 Marks)**

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