CBCS SCHEME

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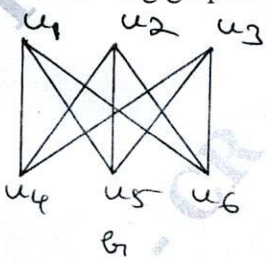
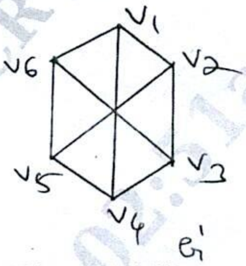
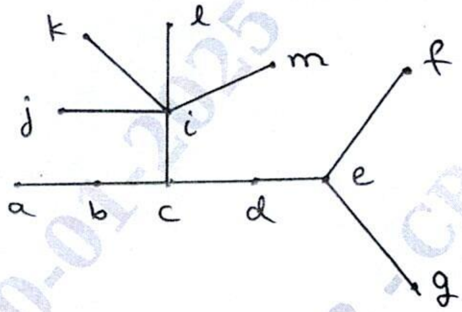
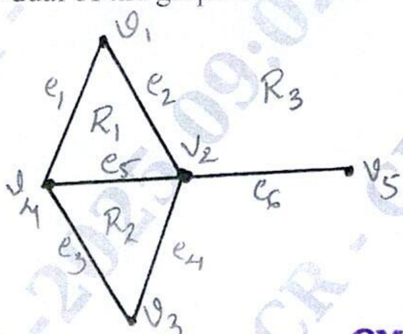
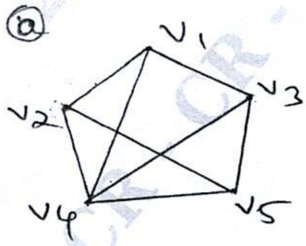
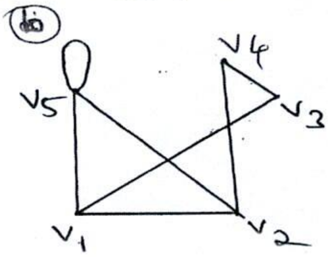
BCS405B

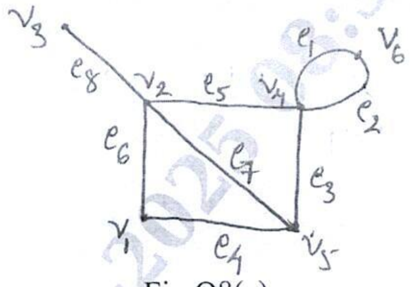
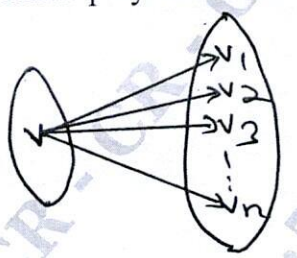
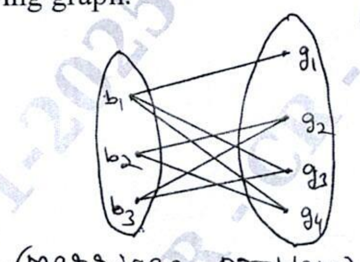
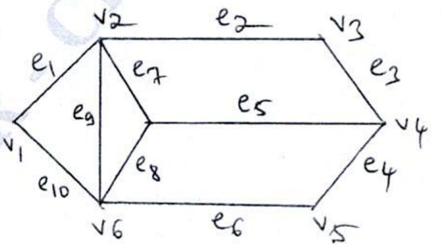
Fourth Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Graph Theory

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define the following with an example: i) Regular graph ii) Complete graph iii) Complete Bipartite graph	06	L1	CO1
	b.	Show that the number of vertices of odd degree is always even.	07	L3	CO1
	c.	Find the number of vertices for the graph G = (V, E) in the following cases: i) G has 9 edges and all the vertices of degree 3 ii) G is a cubic graph with 9 edges. iii) G has 10 edges with 2 vertices of degree 4 and others of degree 3.	07	L2	CO1
		OR			
Q.2	a.	Define the following : i) walk ii) open walk iii) path iv) circuit v) cycle vi) Trail	06	L1	CO1
	b.	Explain Konigsberg Bridge problem of graph theory.	07	L2	CO1
	c.	Show that the following graphs are isomorphic:  Fig.Q2(c)(i)  Fig.Q2(c)(ii)	07	L2	CO1
		Module – 2			
Q.3	a.	Define the following with an example: i) Euler circuit ii) Euler trail iii) Euler graph	06	L1	CO2
	b.	If all the vertices in a connected graph G are of even degree then show that G is Eulerian graph.	07	L3	CO2
	c.	Define Hamilton cycle and Hamilton path. In a complete graph with n vertices, where n is odd and ≥ 3, show that $\left(\frac{n-1}{2}\right)$ edge-disjoint Hamilton cycles exist.	07	L3	CO2
		OR			
Q.4	a.	Define Hamilton graph. By specifying the walk draw a graph which contains the following : i) Both Euler circuit and Hamilton cycle. ii) Euler circuit but no Hamilton cycle. iii) Hamilton cycle but no Euler circuit. iv) Neither a Hamilton cycle nor an Euler circuit.	06	L2	CO2
	b.	Explain Travelling – Salesman problem of graph theory.	07	L3	CO2
	c.	i) Define directed graph and draw a diagraph with 5 vertices and 10 edges. ii) Prove that in any diagraph the sum of the outdegrees of all the vertices is equal to sum of their indegrees and this sum is equal to the number of edges in the diagraph.	07	L3	CO2

		Module – 3			
Q.5	**a.**	Define tree and show that a tree with n vertices has n – 1 edges.	**06**	**L3**	**CO3**
	b.	Define rooted tree and binary tree. Draw all rooted trees with four vertices.	**07**	**L2**	**CO3**
	c.	Show that for any graph G, the vertex connectivity cannot exceed the edge connectivity and the edge connectivity cannot exceed the degree of the vertex with the smallest degree in G.	**07**	**L3**	**CO3**
		OR			
Q.6	**a.**	For the following graph shown in Fig.Q6(a), find the eccentricities of any three vertices. Also find its centre radius and diameter.  Fig.Q6(a)	**06**	**L3**	**CO3**
	b.	Define spanning tree. Show that every connected graph has atleast one spanning tree.	**07**	**L2**	**CO3**
	c.	Explain the problem of counting structural isomers by using counting trees.	**07**	**L3**	**CO3**
		Module – 4			
Q.7	**a.**	i) Define planar and nonplanar graphs. ii) Show that the complete graph k_5 is nonplanar	**06**	**L1**	**CO4**
	b.	i) Define geometric dual of a graph G. ii) Draw the geometric dual of the graph G.  Fig.Q7(b)	**07**	**L2**	**CO4**
	c.	i) Define adjacency matrix. ii) Find the adjacency matrices for the following graphs:   Fig.Q7(c)	**07**	**L2**	**CO4**

		OR			
Q.8	**a.**	Show that Kuratowski's second graph is a nonplanar graph.	**06**	**L3**	**CO4**
	b.	Show that a connected planar graph G with n vertices and m edges has $m-n+2$ regions.	**07**	**L3**	**CO4**
	c.	i) Define path matrix and circuit matrix of a graph. ii) Find the path matrix from V_3 to V_5 for the following graph.  Fig.Q8(c)	**07**	**L2**	**CO4**
		Module – 5			
Q.9	**a.**	Prove that a graph of order ($n \geq 2$) consisting of a single circuit is 2 chromatic if n is even and 3 chromatic if n is odd.	**06**	**L2**	**CO5**
	b.	Define chromatic number and chromatic polynomial of a graph. Find the chromatic number and chromatic polynomial for following graph.  Fig.Q9(b)	**07**	**L1**	**CO5**
	c.	State and prove Five color theorem.	**07**	**L2**	**CO5**
		OR			
Q.10	**a.**	Prove that every tree with two or more vertices is 2 chromatic.	**06**	**L2**	**CO5**
	b.	Define matching and complete matching. Find all the possible sets of matching for the following graph.  Fig.Q10(b)	**07**	**L1**	**CO5**
	c.	Define covering and minimal covering of a graph. Find the minimal vertex covering and minimal edge covering of the following graph.  Fig.Q10(c)	**07**	**L1**	**CO5**

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