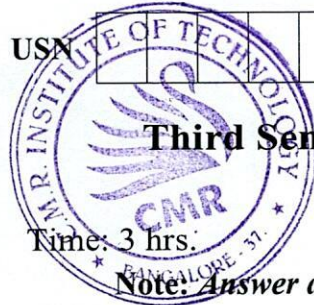


USN

15MATDIP31



Third Semester B.E. Degree Examination, Dec.2024/Jan.2025

Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the modulus and amplitude of $1 + \cos \theta + i \sin \theta$. (05 Marks)
- b. If n is positive integer, prove that $(\sqrt{3} + i)^n + (\sqrt{3} - i)^n = 2^{n+1} \cos\left(\frac{n\pi}{6}\right)$. (06 Marks)
- c. Find a unit vector perpendicular to both the vectors $\vec{A} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\vec{B} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$. (05 Marks)

OR

- 2 a. Find the cube root of $1 + i$. (05 Marks)
- b. If $\vec{a}, \vec{b}, \vec{c}$ are any three vectors, prove that :
- i) $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$
- ii) $[\vec{b} \times \vec{c}, \vec{c} \times \vec{a}, \vec{a} \times \vec{b}] = [\vec{a}, \vec{b}, \vec{c}]^2$. (06 Marks)
- c. Find the value of λ so that the vectors $\vec{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\vec{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\vec{c} = \mathbf{j} + \lambda\mathbf{k}$ are coplanar. (05 Marks)

Module-2

- 3 a. Find the n^{th} derivative of $e^{ax} \cos(bx + c)$. (05 Marks)
- b. If $y = e^{m \cos^{-1} x}$, prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + m^2)y_n = 0$. (06 Marks)
- c. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$. (05 Marks)

OR

- 4 a. Find the Pedal equation for the curve $r = a(1 + \cos \theta)$. (05 Marks)
- b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$. (06 Marks)
- c. If $x = r \cos \theta$, $y = r \sin \theta$, find the value $\frac{\partial(x, y)}{\partial(r, \theta)}$. (05 Marks)

Module-3

- 5 a. Evaluate: $\int_0^{\pi/2} \cos^n x dx$. (05 Marks)
- b. Evaluate: $\int_0^1 x^6 \sqrt{1-x^2} dx$. (05 Marks)
- c. Evaluate: $\int_1^2 \int_3^4 (xy + e^y) dy dx$. (06 Marks)

OR

- 6 a. Evaluate: $\int_0^{\infty} \frac{dx}{(1+x^2)^{3/2}}$. (05 Marks)
- b. Evaluate: $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$. (06 Marks)
- c. Evaluate: $\int_0^1 \int_0^2 \int_0^2 x^2 yz dx dy dz$. (05 Marks)

Module-4

- 7 a. A particle moves along the curve $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$, where t is the time variable. Determine the velocity and acceleration and also magnitudes of velocity and acceleration at $t = 2$. (06 Marks)
- b. Find the unit vector normal to the surface $x^2 + y^2 + z^2 = 9$ at the point $(2, -1, 2)$. (05 Marks)
- c. Find the directional derivative of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of vector $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. (05 Marks)

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OR

- 8 a. Find the curl of the vector $\vec{F} = (3x^2y - z)\mathbf{i} + (xz^3 + y^4)\mathbf{j} - (2x^3z^2)\mathbf{k}$. (05 Marks)
- b. If $\vec{V} = 2xy\mathbf{i} + 3x^2y\mathbf{j} - 3ayz\mathbf{k}$ is solenoidal at $(1, 1, 1)$. Find a . (06 Marks)
- c. Show that $\vec{F} = \frac{xi + yj}{x^2 + y^2}$ is irrotational. (05 Marks)

Module-5

- 9 a. Solve: $x \frac{dy}{dx} = y - x \cos^2\left(\frac{y}{x}\right)$. (06 Marks)
- b. Solve $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$. (05 Marks)
- c. Solve $\frac{dy}{dx} = \frac{x + y - 1}{x + y + 1}$. (05 Marks)
- OR
- 10 a. Solve $(x^2 - ay)dx + (y^2 - ax)dy = 0$. (05 Marks)
- b. Solve $[x \tan\left(\frac{y}{x}\right) - y \sec^2\left(\frac{y}{x}\right)]dx + x \sec^2\left(\frac{y}{x}\right)dy = 0$. (06 Marks)
- c. Solve $x \frac{dy}{dx} + y = x^3 y^6$. (05 Marks)

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