

**Third Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025**  
**Transform Calculus, Fourier Series and Numerical Techniques**

Time: 3 hrs.

Max. Marks: 100

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

**Module-1**

- 1 a. Find the Laplace transform of  $te^{-2t} \sinh 3t$  (06 Marks)
- b. Find the Laplace transform of the triangular wave of period '2a' given by  

$$f(t) = \begin{cases} t & \text{in } 0 < t < a \\ 2a - t & \text{in } a < t < 2a \end{cases}$$
 (07 Marks)
- c. Using convolution theorem, find the inverse Laplace transform of  $\frac{1}{s(s^2 + a^2)}$  (07 Marks)

**OR**

- 2 a. Using unit step function, find the Laplace transform of  

$$f(t) = \begin{cases} 1 & \text{in } 0 < t < 1 \\ t & \text{in } 1 < t \leq 2 \\ t^2 & \text{in } t > 2 \end{cases}$$
 (06 Marks)
- b. Find the inverse Laplace transform of  
 (i)  $\frac{4s-18}{9-s^2}$  (ii)  $\frac{s-2}{s^2+7s+12}$  (07 Marks)
- c. Solve by using Laplace transform techniques,  
 $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 5e^{2x}$ , given  $y(0) = 2$ ,  $y'(0) = 1$ . (07 Marks)

**Module-2**

- 3 a. Obtain Fourier series expansion of  $f(x) = x(2\pi - x)$  in  $0 \leq x \leq 2\pi$  (06 Marks)
- b. Obtain the half range cosine series for the function  $f(x) = x^2$ , over the interval in  $(0, \pi)$ . (07 Marks)
- c. Obtain the constant term and coefficient of first cosine and sine terms in the Fourier series expansion of  $y$  from the table.

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(07 Marks)

**OR**

- 4 a. Find the Fourier series expansion of  $f(x) = 2x - x^2$  in  $(0, 2)$  (06 Marks)
- b. Find sine half range series of  $f(x)$  in  $(0, 1)$  where  $f(x) = \begin{cases} \frac{1}{4} - x & \text{in } (0, 1/2) \\ x - \frac{3}{4} & \text{in } (1/2, 1) \end{cases}$  (07 Marks)
- c. Obtain Fourier series expansion upto first harmonics of  $y = f(x)$ . Also find their amplitudes for the following data :

x	0	60	120	180	240	300	360
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(07 Marks)

**Module-3**

- 5 a. Find the Fourier transform of  

$$f(x) = \begin{cases} 1 & \text{in } |x| \leq a \\ 0 & \text{in } |x| > a \end{cases}$$
 (06 Marks)
- b. Find the Fourier cosine and sine transform of  $f(x) = 2e^{-3x}$ . (07 Marks)
- c. Find the Z-transform of  $\cos n\theta$  and  $\sin n\theta$ . (07 Marks)

**OR**

- 6 a. Find the complex fourier transform of the function  

$$f(x) = \begin{cases} 1 & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$$
  
 Hence evaluate  $\int_0^\infty \frac{\sin x}{x} dx$  (06 Marks)
- b. Find the inverse Z - transform of  $\frac{3z^2 - 8z}{(z-2)(z-3)}$  (07 Marks)
- c. Using Z - transform, solve the difference equation  $y_{n+2} - 5y_{n+1} + 6y_n = u_n$  when  $y_0 = 0$ ,  $y_1 = 1$  and  $u_n = 1$ . (07 Marks)

**Module-4**

- 7 a. Classify the following partial differential equation  
 (i)  $u_{xx} + u_{xy} + (x^2 + 4y^2)u_{yy} = 2 \sin xy$   
 (ii)  $x^2u_{yy} + 2xy u_{xy} + y^2 u_{xx} = 0$   
 (iii)  $(1 + x^2)u_{xx} + (5 + 2x^2)u_{xt} + (4 + x^2)u_{tt} = 0$   
 (iv)  $x^2u_{xx} + (1 - y^2)u_{yy} = 0$   $-\infty < x < \infty$ ;  $-1 < y < 1$  (10 Marks)
- b. Solve the wave equation  $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$  subject to the condition  $u(0, t) = 0$ ,  $u(4, t) = 0$ ,  $u_t(x, 0) = 0$  and  $u(x, 0) = x(4 - x)$  by taking  $h = 1$ ,  $k = 0.5$  upto four steps. (10 Marks)



OR

- 8 a. Evaluate the pivotal values of the equation  $u_{tt} = 16u_{xx}$ , taking  $h = 1$  upto  $t = 1.25$ . The boundary conditions are  $u(0, t) = u(5, t) = 0$ ,  $u_t(x, 0) = 0$  and  $u(x, 0) = x^2(5 - x)$ . (10 Marks)
- b. Given the values of  $u(x, y)$  on the boundary of the square in the figure. Evaluate the function  $u(x, y)$  satisfying the Laplace equation  $u_{xx} + u_{yy} = 0$  at the pivotal points of the Fig.Q8(b).

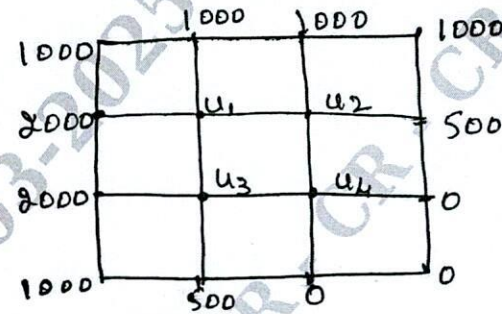


Fig.Q8(b)

(10 Marks)

**Module-5**

- 9 a. Using Runge-Kutta method of order four, solve

$$\frac{d^2y}{dx^2} = x \left( \frac{dy}{dx} \right)^2 - y^2 \text{ for } x = 0.2. \text{ Given that } y(0) = 1, y'(0) = 0$$

(06 Marks)

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- b. Derive Euler's equation.

(07 Marks)

- c. Prove that the geodesics on a plane are straight line.

(07 Marks)

OR

- 10 a. Apply Milne's method to compute  $y(0.8)$  given that

$$\frac{d^2y}{dx^2} = 1 - 2y \frac{dy}{dx} \text{ and the initial values are}$$

$$y(0) = 0, \quad y(0.2) = 0.02, \quad y(0.4) = 0.0795, \quad y(0.6) = 0.1762$$

$$y'(0) = 0, \quad y'(0.2) = 0.1996, \quad y'(0.4) = 0.3937, \quad y'(0.6) = 0.5689.$$

(06 Marks)

- b. Find the extremal of the functional  $\int_{x_1}^{x_2} (y' + x^2 y')^2 dx$

(07 Marks)

- c. Solve the variational problem.

$$\delta \int_0^1 (x + y + y'^2) dx = 0 \text{ under the conditions } y(0) = 1; y(1) = 2.$$

(07 Marks)

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