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21MAT11

First Semester B.E. Degree Examination, June/July 2024

Calculus and Differential Equations

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the angle of intersection between the curves, $r = a\theta$ and $r = \dfrac{a}{\theta}$. (06 Marks)

b. With usual notations, prove the following :

(i) $p = r\sin\phi$ (ii) $\dfrac{1}{p^2} = \dfrac{1}{r^2} + \dfrac{1}{r^4}\left(\dfrac{dr}{d\theta}\right)^2$. (07 Marks)

c. Show that the radius of curvature for the curve $r^2 \sec 2\theta = a^2$ is $\dfrac{a^2}{3r}$. (07 Marks)

OR

2 a. Find the angle between the radius vector and the tangent for the curve $r = ae^{\theta\cot\alpha}$. (06 Marks)

b. For the curve $r^n = a^n \sin n\theta + b^n \cos n\theta$, show that the pedal equation is $p^2\left(a^{2n} + b^{2n}\right) = r^{2n+2}$ (07 Marks)

c. Find the radius of curvature of the curve $x^2y = a\left(x^2 + y^2\right)$ at the point (-2a, 2a). (07 Marks)

Module-2

3 a. Obtain Maclaurin's series expansion of $\log(1+\sin x)$ upto the term containing x^4. (06 Marks)

b. If $z = e^{ax+by} f(ax - by)$, prove that $b\dfrac{\partial z}{\partial x} + a\dfrac{\partial z}{\partial y} = 2abz$. (07 Marks)

c. Find the extreme values of the function, $f(x, y) = x^3 + y^3 - 63x - 63y + 12xy$. (07 Marks)

OR

4 a. Evaluate the following : $\underset{x\to 0}{\text{Lt}}\left(\dfrac{a^x + b^x + c^x}{3}\right)^{\frac{1}{x}}$. (06 Marks)

b. If $z = e^{ax-by}\sin(ax + by)$ then prove that $b\dfrac{\partial z}{\partial x} - a\dfrac{\partial z}{\partial y} = 2abz$. (07 Marks)

c. If $u = x^2 - 2y^2$, $v = 2x^2 - y^2$, find $\dfrac{\partial(u, v)}{\partial(x, y)}$. (07 Marks)

Module-3

5 a. Solve $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$. (06 Marks)

b. If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes, find the temperature of the body after 24 minutes. (07 Marks)

c. Solve $(px - y)(py + x) = a^2p$ by using the substitution $X = x^2$ and $Y = y^2$. (07 Marks)

OR

6 a. Solve $x^3\frac{dy}{dx} - x^2y = -y^4\cos x$. **(06 Marks)**

b. Find the orthogonal trajectories of the family of curves $r = 4a(\sec\theta + \tan\theta)$, where a is the parameter. **(07 Marks)**

c. Solve $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$. **(07 Marks)**

Module-4

7 a. Solve $\frac{d^2y}{dx^2} - 4y = e^{3x}$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = x^2 + 7x + 9$. **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \tan x$ by the method of variation of parameters. **(07 Marks)**

OR

8 a. Solve $(4D^4 - 4D^3 - 23D^2 + 12D + 36)y = 0$. **(06 Marks)**

b. Solve $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x}$. **(07 Marks)**

c. Solve $(2x-1)^2\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} - 2y = 8x^2 - 2x + 3$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ by reducing it to the echelon form. **(06 Marks)**

b. Test for consistency and solve the following system of equations,
$x + 3y - 2z = 0$, $2x - y + 4z = 0$, $x - 11y + 14z = 0$ **(07 Marks)**

c. Use the Gauss-Seidel iterative method to solve the system of equations,
$x + 4y + 2z = 15$, $x + 2y + 5z = 20$, $5x + 2y + z = 12$
Carryout four iterations, taking the initial approximation to the solution as (1, 0, 3). **(07 Marks)**

OR

10 a. Apply Gauss elimination method to solve the system of equations,
$2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$ **(06 Marks)**

b. Investigate the values λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$, have (i) a unique solution, (ii) infinitely many solutions (iii) no solution. **(07 Marks)**

c. Find the largest Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking $[1 \quad 1 \quad 1]^T$ as initial Eigen vector by Rayleigh's power method. **(07 Marks)**

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