CBCS SCHEME

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21MAT21

Second Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Advanced Calculus & Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Evaluate $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)dydx$. **(06 Marks)**

b. Change the order of integration and evaluate $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dydx$. **(07 Marks)**

c. Derive $\beta(m,n)=\frac{\Gamma m\Gamma n}{\Gamma(m+n)}$. **(07 Marks)**

OR

2 a. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)dzdydx$. **(06 Marks)**

b. Find the area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by double integration. **(07 Marks)**

c. Evaluate $\int_0^{\frac{\pi}{2}}\sqrt{\cot\theta}\,d\theta$ by expressing in terms of gamma functions. **(07 Marks)**

Module-2

3 a. Find the angle between the normals to the surface $xy=z^2$ at the points (4, 1, 2) and (3, 3, -3) **(06 Marks)**

b. If $\vec{F}=\nabla(xy^3z^2)$ find div $\vec{F}$ and curl $\vec{F}$ at the point (1, -1, 1). **(07 Marks)**

c. Show that $\vec{F}=(2xy^2+yz)i+(2x^2y+xz+2yz^2)j+(2y^2z+xy)k$ is a conservative force field. **(07 Marks)**

OR

4 a. If $\vec{F}=xyi+yzj+zxk$, evaluate $\int_C \vec{F}dr$ where C is the curve represented by x = t, y = t^2, $z=t^3$, $-1\le t\le 1$. **(06 Marks)**

b. Using Green's theorem, evaluate $\int_C(xy-x^2)dx+x^2ydy$ where C is the closed curve formed by y = 0, x = 1 and y = x. **(07 Marks)**

c. Using stoke's theorem, evaluate $\int_C xydx+xy^2dx$ where C is the square in the x-y plane with vertices (1, 0), (-1, 0), (0, 1), (0, -1). **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-3

5 a. For the function $f(xy+z^2, x+y+z)=0$ form the partial differential equation. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2}+z=0$ given that when x = 0, $z=e^y$ and $\frac{\partial z}{\partial x}=1$. **(07 Marks)**

c. Derive one dimensional Heat equation. **(07 Marks)**

OR

6 a. Find the PDE by eliminating arbitrary function $z=f(x+at)+g(x-at)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2}=xy$ subject to the condition that $\frac{\partial z}{\partial x}=\log(1+y)$ when x = 1 and z = 0 when x = 0. **(07 Marks)**

c. Derive one-dimensional wave equation. **(07 Marks)**

Module-4

7 a. Show that a root of the equation $x^3+5x-11=0$ lies between 1 and 2. Find the root by Newton's Raphson method (carryout 3 iterations). **(06 Marks)**

b. Given f(40) = 184, f(50) = 204, f(60) = 226, f(70) = 250, f(80) = 276, f(90) = 304, find f(38) using suitable interpolation formula. **(07 Marks)**

c. Evaluate $\int_0^1\frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}^{rd}$ rule taking four equal parts and hence deduce an approximate value of π. **(07 Marks)**

OR

8 a. Compute the real root of $x\log_{10}x-1.2=0$ by the method of false position. Carryout 3 iterations. **(06 Marks)**

b. Use Lagrange's interpolation formula to find y at x = 10 given,

x	5	6	9	11
y	12	13	14	16

(07 Marks)

c. Evaluate $\int_0^1\frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}^{th}$ rule. Hence deduce the value of $\log_e 2$. **(07 Marks)**

Module-5

9 a. Find y at x = 1.02 correct to four decimal places given $dy=(xy-1)dx$ and y = 2 at x = 1 apply Taylor's series method. **(06 Marks)**

b. Using Runge-Kutta method of 4^{th} order, find y(0.2) for the equation, $\frac{dy}{dx}=\frac{y-x}{y+x}$, y(0) = 1, taking h = 0.2. **(07 Marks)**

c. Apply Milne's method to compute y(1.4) correct to 4 decimal places. Given $\frac{dy}{dx}=x^2+\frac{y}{2}$ and the following data : y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514. Use corrector formula twice. **(07 Marks)**

OR

10 a. Use Taylor's series method to find y at x = 0.1, considering terms upto the third degree given that $\frac{dy}{dx} = x^2 + y^2$ and y(0) = 1. **(06 Marks)**

b. Using modified Euler's method find y(0.1). Correct to 4 decimal places solving the equation, $\frac{dy}{dx} = x - y^2$, y(0) = 1, h = 0.1 **(07 Marks)**

c. Use Fourth order Runge-Kutta method to solve $(x + y)\frac{dy}{dx} = 1$, y(0.4) = 1 at x = 0.5. Correct to 4 decimal places. **(07 Marks)**

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