



First Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025

Calculus and Differential Equations

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. With usual notations prove that

$$\tan \phi = r \frac{d\theta}{dr}$$

(06 Marks)

- b. Show that curves $\frac{2a}{r} = 1 + \cos \theta$ and $\frac{2a}{r} = 1 - \cos \theta$ cuts orthogonally.

(07 Marks)

- c. Prove that for the Cardioids $r = a(1 + \cos \theta)$, $\frac{\rho^2}{r}$ is constant.

(07 Marks)

OR

- 2 a. If p be the perpendicular from the pole on the tangent, then show that

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta} \right)^2$$

(06 Marks)

- b. Find the pedal equation of the curve $r^n = a^n \cos n\theta$

(07 Marks)

- c. Find the radius of curvature of the curve $x^3 + y^3 = 3xy$ at $(3/2, 3/2)$

(07 Marks)

Module-2

- 3 a. Expand $\log(1 + \sin x)$ by Maclaurin's series upto the term containing x^4 .

(06 Marks)

- b. If $u = \tan^{-1} \left(\frac{x}{y} \right)$, where $x = e^t - e^{-t}$ and $y = e^t + e^{-t}$, find the total derivative $\frac{du}{dt}$.

(07 Marks)

- c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, evaluate $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$

(07 Marks)

OR

- 4 a. Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x + d^x}{4} \right)^{1/x}$

(06 Marks)

- b. If $u = f(y - z, z - x, x - y)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

(07 Marks)

- c. Examine the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ for extreme values.

(07 Marks)

Module-3

- 5 a. Solve $[1 + e^{x/y}]dx + e^{x/y} \left[1 - \frac{x}{y} \right] dy = 0$

(06 Marks)

- b. Show that the family of curves $y^2 = 4a(x + a)$ is self orthogonal.

(07 Marks)

- c. Solve $x^2 \left(\frac{dy}{dx} \right)^2 + xy \frac{dy}{dx} - 6y^2 = 0$

(07 Marks)

OR

- 6 a. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$.

(06 Marks)

- b. A body originally at 80°C cools down to 60°C in 20 minutes in the surroundings of temperature 40°C . Find the temperature of the body after 40 minutes from the original instant.

(07 Marks)

CMRIT LIBRARY
BANGALORE - 560 037

- c. Find the general and singular solution of $y = xp + \sqrt{4 + p^2}$

(07 Marks)

Module-4

- 7 a. Solve $(D^4 + 4D^3 - 5D^2 - 36D - 36)y = 0$.

(06 Marks)

- b. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = e^{2x} + \sin x$

(07 Marks)

- c. Solve $\frac{d^2y}{dx^2} + y = \sec x$ by using method of variation of parameters.

(07 Marks)

OR

- 8 a. Solve $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = e^{2x} + e^x$

(06 Marks)

- b. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x^2 + 3x + 1$

(07 Marks)

- c. Solve $x^2y'' - 4xy' - 6y = \cos(2 \log x)$

(07 Marks)

Module-5

- 9 a. Find the rank of the matrix

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

by using elementary row transformation.

(06 Marks)

- b. Solve the system of equations by using Gauss - Jordan method.
 $x + 2y + z = 8$, $2x + 3y + 4z = 20$, $4x + 3y + 2z = 16$

(07 Marks)

- c. Find the largest eigen values and the corresponding eigen vector of

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

with the initial approximation eigen vector $[1 \ 1 \ 1]^T$. Perform five iterations. (07 Marks)

OR

- 10 a. Find the rank of the matrix

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

by using elementary row transformation.

(06 Marks)

- b. Test the consistency and solve

$$x + y + z = -3, \quad 3x + y - 2z = -2, \quad 2x + 4y + 2z = 7$$

(07 Marks)

- c. Solve the system of equations $2x - 3y + 20z = 25$, $20x + y - 2z = 17$, $3x + 20y - z = -18$ using Gauss - Seidel method taking $(0, 0, 0)$ as an initial approximate. Carry out 4 iterations. (07 Marks)
