CBCS SCHEME

USN [ ] 18MAT21

Second Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, –2, –1) along $2i - j - 2k$. **(06 Marks)**

b. If $\vec{F} = \nabla(xy^3z^2)$ find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ at (1, –1, 1). **(07 Marks)**

c. Find the value of the constant 'a' such that the vector field, $\vec{F} = (axy - z^3)i + (a-2)x^2j + (1-a)xz^2k$ is irrotational and hence find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

2 a. If $\vec{F} = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int F \cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve given by $x = t$, $y = t^2$, $z = t^3$. **(06 Marks)**

b. Find the area between the parabola $y^2 = 4x$ and $x^2 = 4y$ using Green's theorem. **(07 Marks)**

c. Evaluate $\int_c xy\,dx + xy^2\,dy$ by using Stoke's theorem where c is the square in the xy-plane with vertices (1, 0), (–1, 0), (0, 1) and (0, –1). **(07 Marks)**

Module-2

3 a. Solve $(D^3 + 6D^2 + 11D + 6)y = 0$ **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} + y = \sec x \tan x$ by the method of variation of parameters. **(07 Marks)**

c. Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = (1+x)^2$ **(06 Marks)**

OR

4 a. Solve $(D^2 + 1)y = e^x + x^4 + \sin x$ **(06 Marks)**

b. Solve $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin 2\{\log(1+x)\}$ **(07 Marks)**

c. The current i and the charge q in a series containing an inductance L, capacitance C, emf $\in$, satisfy the differential equation $L\frac{d^2q}{dt^2} + \frac{q}{C} = \in$. Find q and i terms of 't' given that L, C, $\in$ are constants and the value of i and q are zero initially. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by elimination of arbitrary function from $\phi(x + y + z,\ xy + z^2) = 0$ **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$, when $x = 0$ and $z = 0$, when y is odd multiple of $\pi/2$. **(07 Marks)**

c. Derive one dimensional heat equation in the form $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ **(07 Marks)**

OR

6 a. Solve $\frac{\partial^2 z}{\partial y^2} = z$ given that $z = 0$ and $\frac{\partial z}{\partial y} = \sin x$ when $y = 0$. **(06 Marks)**

b. Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} + (mx - ly) = 0$ **(07 Marks)**

c. Find all possible solutions of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by variable separable method. **(07 Marks)**

Module-4

7 a. Test for convergence for series
$1 + \frac{2}{3}x + \frac{2.3}{3.5}x^2 + \frac{2.3.4}{3.5.7}x^3 + \ldots\ldots\ldots\ldots (x > 0)$ **(06 Marks)**

b. Solve Bessel's differential equation
$x^2 y'' + x y' + (x^2 - n^2)y = 0$ leading to $J_n(x)$. **(07 Marks)**

c. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Lengendre polynomials. **(07 Marks)**

OR

8 a. Discuss the nature of the series $\sum \frac{(n+1)^n x^n}{n^{n+1}}$ **(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$, prove that
$\int_0^1 x J_n(\alpha x) J_n(\beta x)\, dx = 0$, if $\alpha \neq \beta$. **(07 Marks)**

c. If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$, find the values of a, b and c. **(07 Marks)**

Module-5

9 a. Using Newton-Raphson method find the real root of the equation $3x = \cos x + 1$. Carry out three iterations. **(06 Marks)**

b. Using Newton's forward interpolation formula find f(3) given :

x	0	2	4	6	8	10
y = f(x)	0	4	56	204	496	980

(07 Marks)

c. Evaluate $\int_0^1 \frac{dx}{1+x}$ by applying Simpson's 3/8th rule considering 6 equal subintervals. Hence deduce the value of $\log_e 2$ (07 Marks)

OR

10 a. Using Newton's divided difference formula find $f(8)$ from the following data:

x :	4	5	7	10	11	13
y :	48	100	294	900	1210	2028

(06 Marks)

b. Using Regula-Falsi method find the real root of $x \log_{10} x - 1.2 = 0$.
Carry out three iterations. (07 Marks)

c. Evaluate $\int_4^{5.2} \log_e x \, dx$ taking 6 equal strips by applying Weddle's rule. (07 Marks)

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