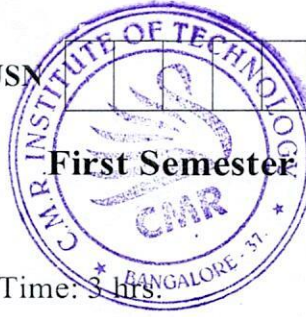


CBCS SCHEME

USN

18MAT11



First Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025

Calculus and Linear Algebra

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. With usual notation, prove that $\tan \phi = r \frac{d\theta}{dr}$. (06 Marks)
- b. Find the radius of curvature for the Folium of De-Cartes $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on it. (06 Marks)
- c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. (08 Marks)

OR

- 2 a. Show that the pair of curves $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$ intersect each other orthogonally. (06 Marks)
- b. Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$. (06 Marks)
- c. Show that for the curve $r = a(1 + \cos \theta)$, $\frac{\rho^2}{r}$ is a constant. (08 Marks)

Module-2

- 3 a. Using Maclaurin's series prove that $\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} \dots$. (06 Marks)
- b. Evaluate (i) $\lim_{x \rightarrow 1} x^{1/1-x}$ (ii) $\lim_{x \rightarrow \pi/2} (\cos x)^{\pi/2-x}$. (07 Marks)
- c. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. (07 Marks)

OR

- 4 a. If $u = f(x - y, y - z, z - x)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (06 Marks)
- b. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$. (07 Marks)
- c. Find the maximum and minimum distance of the point $(1, 2, 3)$ from the sphere $x^2 + y^2 + z^2 = 56$. (07 Marks)

Module-3

- 5 a. Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$ (06 Marks)
- b. Find by double integration the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (07 Marks)
- c. Show that $\beta(m, n) = \frac{\sqrt{m} \sqrt{n}}{(m+n)}$ (07 Marks)

OR

- 6 a. Evaluate $\int_{x=0}^{\infty} \int_{y=x}^{\infty} \frac{e^{-y}}{y} dy dx$ by changing the order of integration. (06 Marks)
- b. Find the volume generated by the revolution of the cardioid $r = a(1 + \cos \theta)$ about the initial line. (07 Marks)
- c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \int_0^{\pi/2} \sqrt{\sin \theta} \cdot d\theta = \pi$ (07 Marks)

Module-4

- 7 a. Solve $(x^2 + y^2 + x) dx + xy dy = 0$ (06 Marks)
- b. Find the orthogonal trajectories of the family of curves $r^n = a^n \cos n\theta$. (07 Marks)
- c. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C . (07 Marks)

OR

- 8 a. Solve $y(2xy + e^x) dx - e^x dy = 0$ (06 Marks)
- b. Solve the equation $y^2(y - xp) = x^4 p^2$ by reducing into Clairaut's form, taking the substitution $X = \frac{1}{x}$ and $Y = \frac{1}{y}$. (07 Marks)
- c. A series circuit with resistance R , inductance L and electromotive force E is governed by the differential equation $L \frac{di}{dt} + Ri = E$, where L and R are constant and initially the current i is zero. Find the current at any time t . (07 Marks)

Module-5

- 9 a. Find the rank of the matrix $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by applying elementary row operations. (06 Marks)

- b. Apply Gauss-Jordan method to solve the following system of equations:

$$2x + y + 3z = 1$$

$$4x + 4y + 7z = 1$$

$$2x + 5y + 9z = 3$$

(07 Marks)

- c. Find the dominant eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

by powers method taking the initial eigen vector as $[1, 1, 1]^T$. Carry out 5 iterations.

(07 Marks)

OR

- 10 a. Investigate the values of λ and μ such that the system of equations

$$x + y + z = 6, \quad x + 2y + 3z = 10, \quad x + 2y + \lambda z = \mu$$

may have (i) unique solution (ii) infinite solution (iii) No solution.

(06 Marks)

- b. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form.

(07 Marks)

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- c. Solve the following system of equations by Gauss Seidel method. Carry out three iterations.

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

(07 Marks)
