CBCS SCHEME

USN

Second Semester B.E. Degree Examination, Dec.2024/Jan.2025
Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x} - \cos^2 x$ **(06 Marks)**

b. Solve by inverse differential operator method $(D^4 + 4D^3 - 5D^2 - 36D - 36)\, y = 0$. **(05 Marks)**

c. Solve by the method of undetermined coefficients $y'' + 4y = e^{-x} + x^2$. **(05 Marks)**

OR

2 a. Solve $y'' - 4y = \sin h^2 x$ by inverse differential operator method. **(06 Marks)**

b. Solve $(D^2 + 3)y = x^2 e^{3x} + \cos 3x$ by inverse differential operator method. **(05 Marks)**

c. Solve by the method of variation of parameters $y'' + y = \operatorname{cosec} x$ **(05 Marks)**

Module-2

3 a. Solve $x^2y'' - xy' + 2y = x \sin(\log x)$. **(06 Marks)**

b. Solve $p^2 + p(x + y) + xy = 0$ **(05 Marks)**

c. Find general and singular solution of $(a^2 - x^2)\, p^2 + 2xyp + b^2 - y^2 = 0$. **(05 Marks)**

OR

4 a. Solve $(x + a)\frac{d^2y}{dx^2} - 4(x + a)\frac{dy}{dx} + 6y = x$ **(06 Marks)**

b. Solve $y = 2px + \tan^{-1}(xp^2)$ **(05 Marks)**

c. Find the general and singular solution of $(px - y)(py + x) = a^2p$ by using the substitution $u = x^2$, $v = y^2$ **(05 Marks)**

Module-3

5 a. Obtain the partial differential equation by eliminating the arbitrary function from $xyz = f(x^2 + y^2 + z^2)$ **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\pi/2$. **(05 Marks)**

c. Derive one dimensional heat equation $\frac{\partial u}{\partial f} = c^2 \frac{\partial^2 y}{\partial x^2}$ **(05 Marks)**

OR

6 a. Find the partial differential equation by eliminating constants a and b from $(x - a)^2 + (y - b)^2 + z^2 = 16$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2} - 5\frac{\partial z}{\partial y} + 6z = 0$ given that $z = x$ and $\frac{\partial z}{\partial y} = 0$ when $y = 0$. **(05 Marks)**

c. Find the various possible solutions of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. **(05 Marks)**

Module-4

7 a. Evaluate $\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2 + y^2)\, dy\, dx$ by changing the order of integration. **(06 Marks)**

b. Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **(05 Marks)**

c. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m + n)}$ **(05 Marks)**

OR

8 a. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$ by changing to polar coordinates. **(06 Marks)**

b. Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2yz\, dx\, dy\, dz$ **(05 Marks)**

c. Prove that $\int_0^a \sqrt{x}\, e^{-x^2} dx \times \int_0^\infty \frac{e^{-x^2}}{\sqrt{x}} dx = \frac{\pi}{2\sqrt{2}}$ **(05 Marks)**

Module-5

9 a. Find : i) $L[te^{-t} \sin 3t]$ ii) $L\left[\frac{\sin 3t \cos 2t}{t}\right]$ **(06 Marks)**

b. Show that $L[f(t)] = \frac{1}{s}\tanh\left(\frac{as}{2}\right)$ if $f(t) = \begin{cases} 1 & 0 < t < a \\ -1 & a \le t \le 2a \end{cases}$ and $f(t + 2a) = f(t)$ **(05 Marks)**

c. Using Laplace transform solve $y'' - 2y' + y = e^{2t}$ with $y(0) = 0$, $y'(0) = 1$ **(05 Marks)**

OR

10 a. Express $f(t) = \begin{cases} t & 0 < t < 2 \\ 2 & t \ge 2 \end{cases}$ in terms of unit step function and hence find Laplace transform. **(05 Marks)**

b. Find : i) $L^{-1}\left[\frac{s+1}{(s-2)^2}\right]$ ii) $L^{-1}\left[\log\left(1 - \frac{a}{s}\right)\right]$ **(06 Marks)**

c. Find $L^{-1}\left[\frac{1}{s(s^2 + a^2)}\right]$ by using convolution theorem. **(05 Marks)**

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