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	Internal Assessment Test I March 2025											

Sub:	**Discrete Mathematical Structures**							Code:	BCS405A
Date:	24/03/2025	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	**CSE/IS &AIML,CSDS**

Question 1 is compulsory and Answer any 6 from the remaining questions.		Marks	OBE CO	OBE RBT
1	a) $1.3+2.4+3.5+\ldots\ldots\ldots\ldots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$ b) Let $a_0=1$, $a_1=2$, $a_2=3$ and $a_n= a_{n-1}+a_{n-3}+a_{n-3}$ for $n\geq 3$ prove that $a_n\leq 3^n \; \forall \, n \in Z^+$	[8]	CO1	L1,L3
2	Define tautology. Determine whether the following compound statement is a tautology or not. $[(p\vee q) \rightarrow r] \leftrightarrow [(p\rightarrow r)\wedge (q\rightarrow r)]$	[7]	CO1	L3
3	Define Compound Proposition and by constructing truth table show that $[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$	[7]	CO1	L3

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4	a) Define direct and indirect proof and prove that for all integers k and l if k and l are both odd them k+l and kl is odd by direct proof. b) Prove that" If n is odd integer, then n+9 is even integer" by indirect proof.	[7]	CO2	L3
5	Check whether the following statement is a valid argument by Rules of inference. If I study, then I will not fail in the examination If I don't watch TV in the evening, I will study I failed in the examination .... $\therefore$ *I must watched TV in the evening*	[7]		
6	For $n \geq 0$ let Fn denote the nth Fibonacci number Prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$	[7]	CO3	L1,L3
7	a) Find the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$ b) Find the coefficient of x^{12} in the expansion for $x^3(1-2x)^{10}$	[7]	CO2	L3
8	Obtain the recursive definition of the sequence $\{a_n\}$ in each of the fallowing case a) $a_n=6^n$ b) $a_n= 2-(-1)^n$ b) Find the explicit form of the recursive expression $a_1=7$ and $a_n=2a_{n-1}+1$	[3+4]	CO2	L1,L3

(1) (a) $1\cdot3 + 2\cdot4 + 3\cdot5 + \dots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$

Let $S(n) = 1\cdot3 + 2\cdot4 + 3\cdot5 + \dots + n(n+2) = \frac{n(n+1)(2n+7)}{6}.$

Basic Step: $S(n)$ is true when $n = 1$.

$S(1) = 1\cdot3 + 2\cdot4 + 3\cdot5 + \dots + 1(1+2) = \frac{1(1+1)(2+7)}{6}.$

$3 = \frac{2(9)^{3}}{\cancel{6}\,3}$

$1 = 1.$

$\therefore$ LHS = RHS.

It can be proved that $S(n)$ is true for $n = 1$.

Induction Step: Let us assume $S(n)$ is true for $n = k$, where k is an integer greater than or equal to one $(k \geq 1)$

$S(k)$: $1\cdot3 + 2\cdot4 + 3\cdot5 + \dots + k(k+2) = \frac{k(k+1)(2k+7)}{6}.$

Now, we need to prove that $S(n)$ is true for $n = (k+1)(k+2)$ as well. Adding $(k+1)(k+2)$ on both sides.

$1\cdot3 + 2\cdot4 + 3\cdot5 + \dots + k(k+2) + (k+1)(k+2) = \frac{k(k+1)(2k+7)}{6} + (k+1)($

$= \frac{k(k+1)(2k+7) + 6(k+1)(k+2)}{6}$

$$= \frac{k(k+1)(2k+7) + 6(k+1)(k+2)}{6}$$

$$= \frac{(k+1)(k(2k+7) + 6(k+2))}{6}$$

$$= \frac{(k+1)(2k^2+7k+6k+18)}{6}$$

$$= \frac{(k+1)(2k^2+13k+18)}{6}$$

$$= \frac{(k+1)(2k^2+4k+9k+18)}{6}$$

$$= \frac{(k+1)(2k(k+2)+9(k+2))}{6}$$

$$= \frac{(k+1)(2k+9)(k+2)}{6}$$

Hence, we can prove that $S(n)$ is true using mathematical induction.

(b) Let $a_0 = 1$, $a_1 = 2$, $a_2 = 3$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$ prove that $a_n \leq 3^n$ $\forall$ $n \in Z^+$.

Let $S(n) = a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$.

~~Basic step: S(k) is true for n=1.~~

~~$S(1) = a_1 = a_0 + a$~~

Let $n = 3$,

$a_3 = a_{3-1} + a_{3-2} + a_{3-3}$

$= a_2 + a_1 + a_0$.

$= 3 + 2 + 1$

$= 6$

$\therefore$ $a_n \leq 3^n$ is true for values other than $0, 1, 2, 3, 4, 5$.

(2) TAUTOLOGY.

In a given compound proposition, if the truth value is true for all the possible truth values, it is known as a tautology.

p	q	r	$p \vee q$	$(p \vee q) \to r$
0	0	0	0	1
0	0	1	0	1
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

p	q	r	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
0	0	0	1	~~0~~1	1
0	0	1	1	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	1	1	1

Since, $[(p \vee q) \to r] \leftrightarrow [(p \to r) \wedge (q \to r)]$ is hence proved true, the given compound statement is a tautology.

(3)

p	q	r	$p \vee q$	$(p \vee q) \to r$	$(p \to r)$	$(q \to r)$	$(p \to r) \wedge (q \to r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1.

Hence proved,

Compound Preposition is a preposition where two propositions are connected using logical connectives.

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4 (a) DIRECT PROOF

In this proof, a statement/declaration is given with two propositions. We need to prove the conditional connector. Let us take the two prepositions as p and q. We need to prove that if p is true then q is true. $(p \rightarrow q)$.

INDIRECT PROOF

In this case, a statement/declaration is given with two propositions. We shall assume them as p and ~~p~~ q. We must negate the q proposition and prove if not q then p. $(\neg q \rightarrow p)$

p : k and l are odd.

q : k+l is even and kl is odd.

Let us assume,

$k = 2m+1$

$l = 2n+1$

as k and l are given as odd terms.

Now, let us substitute k and l values in the sum to prove that it is even.

$= k+l$

$= (2m+1)+(2n+1)$

$= 2m+2n+2$

$= 2(m+n+1)$. [Taking $(m+n+1)$ as a].

$= \underline{2a}$

Since 2a is divisible by 2, it can be proved that k+l is even.

Now, substitute k, l values in kl to prove odd.

$= kl$

$= (2m+1)(2n+1)$

$= (4mn + 2m + 2n + 1)$

$= 2(2mn + m + n) + 1$ [Taking $(2mn+m+n)$ as b]

$= \underline{2b+1}$.

Since 2b+1 will now be divisibly by 2, it can be proved that kl is odd.

$\Rightarrow$ $\boxed{\text{Refer Last page}}$ (b).

(6) For $n \geq 0$. let Fn denote the nth Fibonnaci number. prove that.

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

Basic step: Let us assume $n=0$ & $n=1$.

$$F_0 = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^0 - \left(\frac{1-\sqrt{5}}{2}\right)^0\right]$$

$$= \frac{1}{\sqrt{5}}\left[0-0\right] = 0.$$

$$F_1 = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^1 - \left(\frac{1-\sqrt{5}}{2}\right)^1\right] = \frac{1}{\sqrt{5}}\left[\frac{\cancel{1}+\sqrt{5}-\cancel{1}+\sqrt{5}}{2}\right]$$

$$= \frac{1}{\cancel{\sqrt{5}}}\left[\frac{\cancel{2}\cancel{\sqrt{5}}}{\cancel{2}}\right] = 1.$$

Hence we know that F_0 and F_1 are true for values $n=0$ and $n=1$.

<u>Induction</u> <u>Step</u>:

we know that in fibonnaci sequence,

$$F_n = F_{n-1} + F_{n-2}$$

Let us assume $n=k$ where k is an integer greater than or equal to 1.

$$F_k = F_{k-1} + F_{k-2}.$$

$$F_{k-1} = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1} - \left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\right]$$

$$F_{k-2} = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-2} - \left(\frac{1+\sqrt{5}}{2}\right)^{k-2}\right]$$

$$F_k = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{k-1}\right] + \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-2} - \left(\frac{1-\sqrt{5}}{2}\right)^{k-2}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{k-1} + \left(\frac{1+\sqrt{5}}{2}\right)^{k-2} - \left(\frac{1-\sqrt{5}}{2}\right)^{k-2}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1} + \left(\frac{1+\sqrt{5}}{2}\right)^{k-2} - \left(\left(\frac{1-\sqrt{5}}{2}\right)^{k-1} + \left(\frac{1-\sqrt{5}}{2}\right)^{k-2}\right)\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-2}\left(\frac{1+\sqrt{5}}{2}+1\right) - \left(\frac{1-\sqrt{5}}{2}\right)^{k-2}\left(\frac{1-\sqrt{5}}{2}+1\right)\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-2}\left(\frac{3+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^{k-2}\left(\frac{3-\sqrt{5}}{2}\right)\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-2}\left(\frac{3+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^{k-2}\left(\frac{3-\sqrt{5}}{2}\right)\right]$$

multiplying and dividing by 2 on both terms to rationalise

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-2}\left(\frac{6+2\sqrt{5}}{4}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^{k-2}\left(\frac{6-2\sqrt{5}}{4}\right)\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-2}\left(\frac{6+2\sqrt{5}}{2^2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^{k-2}\left(\frac{6-2\sqrt{5}}{2^2}\right)\right]$$

$$6+2\sqrt{5} = (1+\sqrt{5})^2$$

$$6-2\sqrt{5} = (1-\sqrt{5})^2$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-2}\left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right)^{k-2}\left(\frac{1-\sqrt{5}}{2}\right)^2\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-\cancel{2}+\cancel{2}} - \left(\frac{1-\sqrt{5}}{2}\right)^{k-\cancel{2}+\cancel{2}}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k} - \left(\frac{1-\sqrt{5}}{2}\right)^{k}\right]$$

Hence, proved.

$$\boxed{F_k = F_{k-1} + F_{k-2}}.$$

8 (a) $a_n = 6^n$.

$a_0 = 6^0 = 1$

$a_1 = 6^1 = 6$

$a_2 = 6^2 = 36$

$a_3 = 6^3 = 216$.

$a_4 = 6^4 = 1296$.

Hence, the recursive definition will be 6^n.

(b) $a_n = 2 - (-1)^n$.

$(-1)^n \to \begin{cases} -1 & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even.} \end{cases}$

$a_0 = 2 - (-1)^0 = 1$

$a_1 = 2 - (-1)^1 = 3$

$a_2 = 2 - (-1)^2 = 1$

$a_3 = 2 - (-1)^3 = 3 \ldots\ldots$

$a_n = a_0, a_1, a_2, a_3 \ldots\ldots$

$= a_0, 2 + a_0, a_1 - 2, 2 - a_2, \ldots\ldots$

$\therefore$ The recursive definition is $a_{n-1} - 2 + 2 - a_n$.

(b) . $a_1 = 7$ and $a_n = 2a_{n-1} + 1$.

$a_1 = 7$

$a_2 = 2a_{2-1} + 1 = 2a_1 + 1 = 2(7) + 1 = 15$

$a_3 = 2a_{3-1} + 1 = 2a_2 + 1 = 2(15) + 1 = 3\cancel{0}$.

$a_4 = 2a_{4-1} + 1 = 2a_3 + 1 = 2(30) + 1 = 61$.

$7, 15, 31, 61 \ldots\ldots$

$$a_n = 2a_{n-1}+1$$
$$= 2[2a_{n-2}+1]+1$$
$$= 2^2 a_{n-2} + 2+1$$
$$= 2^2[2a_{n-3}+1]+2+1$$
$$= 2^3 a_{n-3} + 2^2+2+1$$
$$= 2^3[2a_{n-3}+1]+2^2+2+1$$
$$= 2^4 a_{n-3} + 2^3+2^2+2+1$$
$$\vdots$$
$$\Rightarrow a_n = 2^{n-1} a_{n-(n-1)} + \underbrace{2^{n-2}+2^{n-1}+\cdots+2^3+2^2+2^1}+1$$
$$= 2^{n-1} a_1 + \left[\frac{2(2^{n-2}-1)}{2-1}\right]+1$$

Because $2^1+2^2+2^3+\cdots+2^{n-1}+2^{n-2}$ is a G.P. with first term, $a=2$ and common ratio, $r=2$.

$$\therefore S_{n-2} = \frac{a(r^{n-2}-1)}{r-1}$$

$$\Rightarrow a_n = 2^{n-1}(7)+2^{n-1}-2+1 \qquad [\because a_1=7]$$
$$= 8.2^{n-1}-1$$
$$= 2^3.2^{n-1}-1$$
$$\boxed{a_n = 2^{n+2}-1}$$

which is the required explicit formula.

7b. p: n is odd integer

q: n + 9 is even integer

Hypothesis : Assume $\sim q$ is true i.e n+9 is an odd integer

Analysis : We know that n+9 is an odd integer

$$n + 9 = 2x + 1 \quad \text{where } x \in Z.$$

$$n = 2x + 1 - 9$$

$$n = 2x - 8$$

$$n = 2(x - 4)$$

∴ n is an even integer i.e $\sim p$ is true.

5. p : I study

q : I will not fail in the examination

r : I watch TV in the evening

Tabular form: $p \to q$

$\neg r \to p$

$\neg q$

$\therefore r$

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$\equiv (p \to q) \wedge (\neg r \to p) \wedge \neg q$

$\equiv (\neg r \to p) \wedge (p \to q) \wedge \neg q$ (commutative law)

$\equiv (\neg r \to q) \wedge \neg q$ (Rule of Syllogism)

$\equiv (\neg(\neg r) \vee q) \wedge \neg q$ (Law of Conditional)

$\equiv (r \vee q) \wedge \neg q$ (~~Law~~ Double Negation)

$\equiv \neg q \wedge (r \vee q)$ (commutative)

$\equiv (\neg q \wedge r) \vee (\neg q \wedge q)$ (Distributive Law)

$\equiv (\neg q \wedge r) \vee (F_0)$ (Inverse Law)

$\equiv (\neg q \wedge r)$ (Identity Law)

$\equiv r$ (Rule of Conjuctive Simplification)

Thus the given statement is a valid argument.

7. a) $a^2 b^3 c^2 d^5$ in $(a+2b-3c+2d+5)^{16}$

we have from multinomial theorem,

$$(x_1+x_2+x_3+\dots x_n)^n = \sum_{r=r}^{n} \binom{n}{n_1, n_2, n_3 \dots n_r} x_1^{n_1} . x_2^{n_2} \dots x_r^{n_r}$$

for above question we have

$$(a+2b-3c+2d+5)^{16} = \sum_{r=r}^{16} \binom{16}{n_1 n_2 n_3 n_4 n_5} x_1^{n_1} x_2^{n_2} x_3^{n_3} x_4^{n_4} x_5^{n_5}$$

$$a^{n_1} . (2b)^{n_2} (-3c)^{n_3} . (2d)^{n_4} (5)^{n_5}$$

For $r = 4$

$n_1 = 2,\ n_2 = 3,\ n_3 = 2,\ n_4 = 5,\ n_5 = 4$

$$(a+2b-3c+2d+5)^{16} = \sum_{i=4}^{16} \binom{16}{2\times 3\times 2\cdot 5\cdot 4} a^2 \cdot (2b)^3, (-3c)^2 (2d)^5\, 5^4$$

$$= a^2 \cdot b^3 \cdot c^2 \cdot d^5 \sum_{i=4}^{16} \frac{16!}{2!3!2!5!4!} \times 2^3 \times (-3)^2 \times (2)^5 \times 5^4$$

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$$= a^2 b^3 c^2 d^5 \times 4.35891456 \times 10^{14}$$

$\therefore$ The cofficient of $a^2 b^3 c^2 d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$ is $4.35891456 \times 10^{14}$

b) x^{12} in $x^3 (1-2x)^{10}$

we have for binomial theorem

$$(x+y)^n = \sum_{i=r} {}^{n}C_r \cdot x^{n-r} y^r$$

for above question we have

$$x^3(1-2x)^{10} = x^3 \sum_{i=r}^{10} {}^{10}C_r\, x^{10-r}\, y^r$$

$$\sum {}^{10}C_r\,(1)^{10-r}\cdot(-2x)^r$$

$\therefore r=9$

$$= x^3 \sum_{i=9}^{10} {}^{10}C_9\,(1)^1\cdot(-2x)^9$$

$$= x^3 x^9 \sum_{i=9} {}^{10}C_9 \times 1 \times (-2)^9$$

$$= x^3 x^9\cdot(-5120)$$

$\therefore$ The coefficient of x^{12} in the expansion of $x^3(1-2x)^{10}$ is -5120