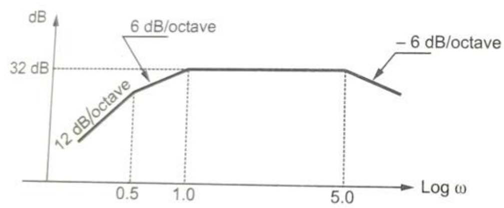
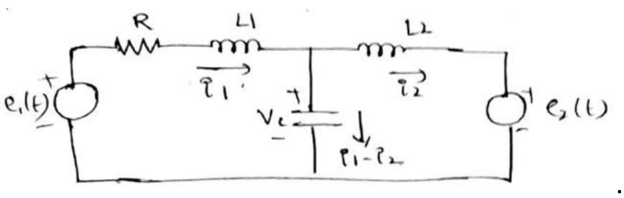
CMR
INSTITUTE OF
TECHNOLOGY

USN

Internal Assessment Test – II

Sub:	Control Systems							Code:	BEC403
Date:	24/ 05/ 2025	Duration:	90 mins	Max Marks:	50	Sem:	4th	Branch:	ECE
Q.1 is Compulsory. Answer **Any Four** from Remaining									

Q.1 is Compulsory		Marks	OBE	
			CO	RBT
1.	Sketch the bode plot for the following transfer function and determine phase margin and gain margin. G(s) = $\frac{50}{s(1+0.5s)(1+0.05s)}$	[10]	CO5	L3
2.	a. Make use of the response curve of second order underdamped system to define and derive the expression for (i) Peak time b. For a unity feedback system G(s) = $\frac{20(s+2)}{s(s+3)(s+4)}$, find the steady state error for r(t)=3u(t) + 5tu(t)	[06] [04]	CO3	L2
3.	Determine the ranges of k such that the characterstic equation is $s^3 + (2k+3)s^2 + (6k+7)s + (7k+8.5) = 0$ has roots more negative than s=-1.	[10]	CO3	L3
4.	The open loop transfer function of a unity feedback control system is given by $\frac{K}{(s+2)(s+4)(s^2+6s+25)}$. Determine the range of values of k for the system stability. What is the value of k which gives sustained oscillations and what is the oscillation frequency.	[10]	CO3	L3
5.	Sketch the root locus for a negative feedback control system with G(s)H(s) = $\frac{K}{S(s+4)(S^2+4S+20)}$	[10]	CO4	L3
6	a) Analyze the Bode plot shown in Fig below to estimate the transfer function of a control system: 	[10]	CO5	L3
7.	a. Develop a state model for the electrical network shown such that e1(t) and e2(t) are inputs and output is taken across the resistor R.  b. Find state transmission matrix for $\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$	[06] [04]	CO5	L3

CC CCI HOD

1.	Sketch the bode plot for the following transfer function and determine phase margin and gain margin. G(s) = $\frac{50}{s(1+0.5s)(1+0.05s)}$	[10]	CO5	L3

Solution

Putting $s = j\omega$, we get

$$GH(j\omega) = \frac{50}{j\omega(0.5j\omega + 1)(0.05j\omega + 1)}$$

Let
$$Y(j\omega) = \frac{50}{j\omega}$$

$$\Rightarrow \quad |Y(j\omega)| = \frac{50}{\omega}$$

$$\Rightarrow \quad 20\log|Y(j\omega)| = 20\log 50 - 20\log\omega \qquad (7.11)$$

Putting $\omega = 0.1$ (starting value of ω in the semilog sheet), we get

$$20\log|Y(j\omega)|_{\omega=0.1} = 54 \text{ dB}$$

Please note that the general from of $Y(j\omega)$ is $K(j\omega)^{\pm N}$, where N is any integer including 0.

The table shown below helps in the process of drawing the Bode magnitude plot. The various factors of $GH(j\omega)$ are entered into the table so that the corner frequencies of the individual factors are in the ascending order.

Factor	Corner frequency	Magnitude and slope characteristics of various asymptotes [2ex]
$\frac{50}{j\omega}$	–	This factor has a magnitude = 54 dB at $\omega = 0.1$ and slope = −20 dB/decade (cofficient of $\log\omega$ in equation (7.11)) upto next corner frequency, ω_1.
$\frac{1}{1+0.5j\omega}$	$\omega_1 = \frac{1}{0.5} = 2$	Net slope between ω_1 and ω_2 = slope contributed by $(1 + 0.5j\omega)^{-1}$ for $\omega > \omega_1$ + previous slope = −20 − 20 = −40 dB/decade.
$\frac{1}{1+0.05j\omega}$	$\omega_2 = \frac{1}{0.05} = 20$	Net slope ω_2 onwards = slope contributed by $(1 + 0.05j\omega)^{-1}$ for $\omega > \omega_2$ + previous slope = −20 − 40 = −60 dB/decade.

Procedure and tips for drawing the magnitude plot:

1. It is not possible to plot the magnitude plot down to zero frequency becacuse of the logarithmic frequency values ($\log 0 = -\infty$), the minimum value of ω on the log scale is normally taken as 0.1 (one may start from 0.01 as well depending on the requirement).
2. The construction lines of slope = −20, −40 and −60 dB/decade are drawn as shown in the semilog sheet.
3. Draw a straight line having an intercept of 54 dB at $\omega = 0.1$ and it must be parallel to the construction line of slope = −20 dB/decade. This line should terminate at $\omega_1 = 2$ rad/sec.
4. Draw a line parallel to the construction line of slope = −40 dB/decade between the corner frequencies ω_1 and ω_2.
5. Finally draw a line parallel to the construction line of slope = −60 dB/decade

Phase plot:

$$\phi(\omega) = -90^\circ - \tan^{-1} 0.5\omega - \tan^{-1} 0.05\omega$$

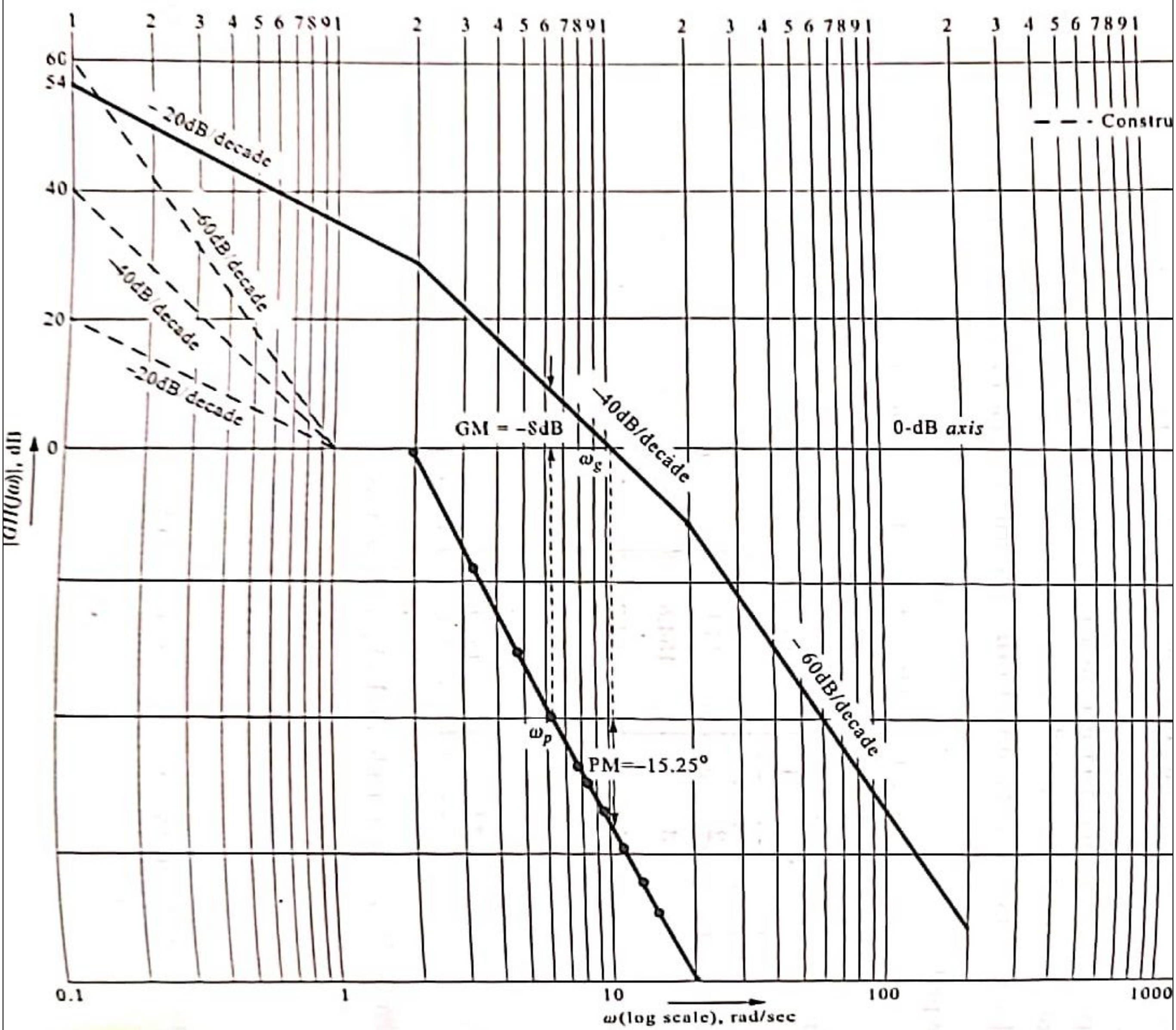
ω (rad/sec)	$\phi(\omega)$(deg)	ω (rad/sec)	$\phi(\omega)$(deg)
2	−140	7	−183.3
3	−154.8	8	−187.8
4	−164.7	9	−191.7
5	−172.2	10	−195.25
6	−178.2	20	−219.3
6.32	−180.0		

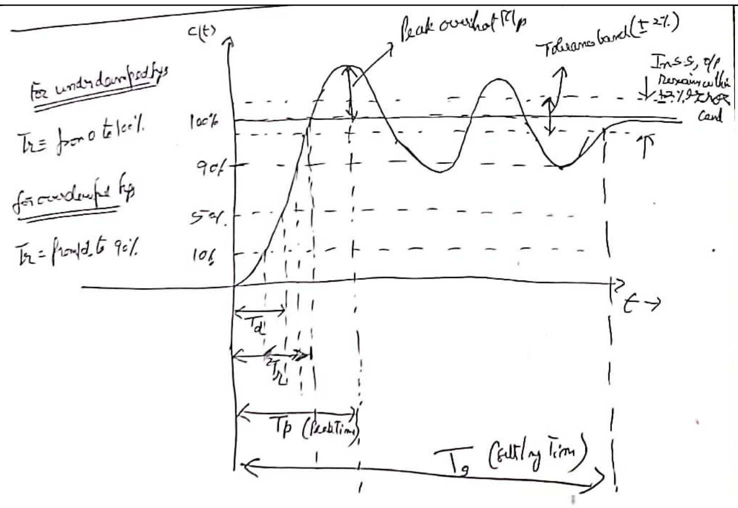
From the Bode magnitude and phase diagrams:

$$\text{GM} = -8 \text{ dB}$$

$$\text{PM} = \phi_M = -15.25^o$$

Comment on stability: Since GM in dB and PM in degree are negative, the closed-loop system is unstable.

	 Fig. EX. 7.1 Bode magnitude and phase diagrams for $GH(s) = \frac{50}{s(0.1s+1)(0.05s+1)}$			
2.	a. Make use of the response curve of second order underdamped system to define and derive the expression for (i) Peak time b. For a unity feedback system $G(s) = \frac{20(s+2)}{s(s+3)(s+4)}$, find the steady state error for $r(t)=3u(t) + 5tu(t)$	[06] [04]	CO3	L2
	a.			



3] **Peak time (T_p)** :→ It is the time required for the response to reach its peak value. It is also defined as the time at which response undergoes the first overshoot which is always peak overshoot.

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \text{ sec.}$$

Derivation of Peak Time (T_p) :→

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \text{ where}$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}.$$

As at $t = T_p$, $c(t)$ will achieve its maxima.

According to Maxima Theorem,

$$\left.\frac{d}{dt}c(t)\right|_{t=T_p} = 0. \longrightarrow (I)$$

So differentiating $c(t)$ w.r.t. time, we get:

$$-\frac{e^{-\xi\omega_n t_p}(-\xi\omega_n)\sin(\omega_d t_p+\theta)}{\sqrt{1-\xi^2}} - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\omega_d\cos(\omega_d t_p+\theta)=0$$

Substituting $\omega_d = \omega_n\sqrt{1-\xi^2}$.

$$e^{-\xi\omega_n t_p}\frac{\xi\omega_n}{\sqrt{1-\xi^2}}\sin(\omega_d t_p+\theta) - \frac{\omega_n\sqrt{1-\xi^2}}{\sqrt{1-\xi^2}}\cdot e^{-\xi\omega_n t_p}\cdot\cos(\omega_d t_p+\theta)=0$$

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\times\omega_n\left[\xi\sin(\omega_d t_p+\theta) - \sqrt{1-\xi^2}\cos(\omega_d t_p+\theta)\right]=0$$

Now

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\times\omega_n \neq 0.$$

$$\therefore \left[\xi\sin(\omega_d t_p+\theta) - \sqrt{1-\xi^2}\cos(\omega_d t_p+\theta)\right] = 0 \longrightarrow (1)$$

But from our earlier analysis we know:

$$\cos\theta = \xi \quad \& \quad \sin\theta = \sqrt{1-\xi^2} \longrightarrow (2)$$

Substituting (2) in (1) we get

$$\cos\theta \cdot \sin(\omega_d t_p + \theta) - \sin(\theta) \cdot \cos(\omega_d t_p + \theta) = 0 \longrightarrow (3)$$

$$\left.\begin{aligned} \sin(A+B) &= \sin A \cdot \cos B + \cos A \cdot \sin B \\ \sin(A-B) &= \sin A \cdot \cos B - \sin B \cdot \cos A \end{aligned}\right]$$

$\Longrightarrow$ Here $A \equiv (\omega_d t_p + \theta)$ & $B = \theta$

$\Longrightarrow$ Hence eq. (3) can be written as:

$$\sin\left[(\omega_d t_p + \theta) - \theta\right] = 0$$

$$\Longrightarrow \sin(\omega_d t_p) = 0$$

$$\Longrightarrow \omega_d t_p = n\pi \quad ; \quad n = 1, 2, 3, \ldots$$

Hence for the 1st Peak overshoot, $n = 1$.

$$\Longrightarrow \omega_d t_p = \pi$$

$$\Longrightarrow \boxed{t_p = \frac{\pi}{\omega_d}}$$

$$\Longrightarrow \boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

//

b.

	$r(t) = 3u(t) + 5t\,u(t)$ Sol: $R(s) = \left[\frac{3}{s} + \frac{5}{s^2}\right]$ $e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1+G(s)H(s)} = \lim_{s \to 0} \frac{s \times \frac{1}{s}\left[3 + \frac{5}{s}\right]}{1 + \frac{20(s+2)}{s(s+3)(s+4)}}$ $= \lim_{s \to 0} \frac{\frac{3s+5}{s}}{\frac{s(s+3)(s+4)+20(s+2)}{s(s+3)(s+4)}} = \lim_{s \to 0} \frac{(3s+5)(s+3)(s+4)}{s(s+3)(s+4)+20(s+2)}$ $= \frac{5 \times 3 \times 4}{20 \times 2} = \frac{3}{2} = 1.5$ $\Rightarrow \boxed{e_{ss} = 1.5}$			
3.	Determine the ranges of k such that the characterstic equation is $s^3 + (2k+3)s^2 + (6k+7)s + (7k+8.5) = 0$ has roots more negative than s=-1.	[10]	CO3	L3
4.	The open loop transfer function of a unity feedback control system is given by $\frac{K}{(s+2)(s+4)(s^2+6s+25)}$. Determine the range of values of k for the system stability. What is the value of k which gives sustained oscillations and what is the oscillation frequency.	[10]	CO3	L3

To have relative stability at s = -1, put s = (x - 1) in char. eq.

$\Longrightarrow$

$$(x-1)^3 + (2k+3)(x-1)^2 + (6k+7)(x-1) + 7k + 8.5 = 0$$

~~$x^2 - 2x + 1 + (2k+3)$~~

$$(a-b)^3 = [a^3 - 3a^2b + 3ab^2 - b^3]$$

$\Longrightarrow$ $x^3 - 3x^2 + 3x - 1 + (2k+3)(x^2 - 2x + 1) + 6k(x) + 7x$

$$-6k - 7 + 7k + 8.5 = 0$$

$$x^3 - 3x^2 + 3x - 1 + 2k(x^2) - 4kx + 2k + 3x^2 - 6x + 3 + 6k(x)$$

$$+ 7x - 6k - 7 + 7k + 8.5 = 0$$

$$x^3 + x^2(-3 + 2k + 3) + x(3 - 4k - 6 + 6k + 7)$$

$$+ (-1 + 2k + 3 - 6k - 7 + 7k + 8.5) = 0$$

$$x^3 + 2k(x^2) + (4 + 2k)x + (3.5 + 3k) = 0$$

	x^3	1	$4+2K$	0
	x^2	$2K$	$(3.5+3K)$	0
	x^1	$\frac{4K^2+5K-3.5}{2K}$	0	
	x^0	$(3.5+3K)$		

From Routh-Hurwitz stability requirements, we have

$3.5+3K \geq 0$

$\Longrightarrow K \geq -1.16 \longrightarrow (1)$

Illy from the conditions of marginal stability, we have:

$\frac{4K^2+5K-3.5}{2K} = 0 \Longrightarrow 4K^2+5K-3.5 = 0$

Solving the above equation we get the roots for 'K' as

$K = -1.75$ or $K = 0.5$ $\} \longrightarrow (2)$

Hence checking the less conditions expressed in (1) & (2) we get that

$K \geq 0.5$ for stability of the system

Hence the range of K should be

$0.5 \leq K \leq \infty$

5.	Sketch the root locus for a negative feedback control system with G(s)H(s) = $\frac{K}{S(s+4)(S^2+4S+20)}$	[10]	CO4	L3

Solution : The open loop poles are located at

$$s = 0, -4 \text{ and } s = \frac{-4 \pm \sqrt{16-80}}{2} = -2 \pm j\,4$$

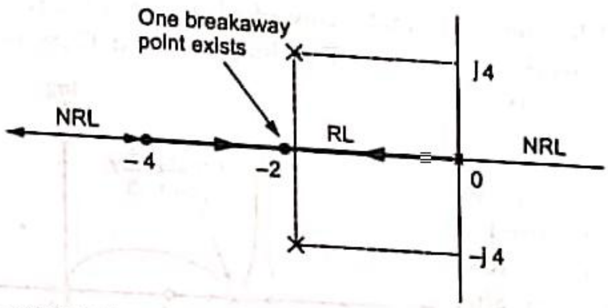
Step 1 : Initial data : P = 4, Z = 0, N = P = 4 branches,

P − Z = 4 approaching to ∞.

Starting points = 0, − 4, −2 + j 4, − 2 − j 4

Terminating points = ∞, ∞, ∞, ∞.

Step 2 : Section of real axis



Step 3 : Angles of asymptotes

$$\theta = \frac{(2q+1)180°}{P-Z}, \; q = 0, 1, 2, 3$$

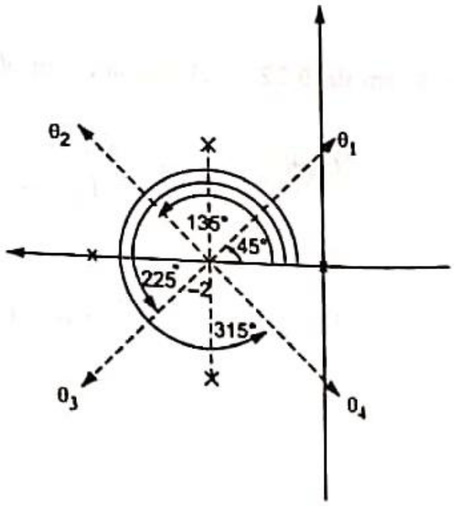
$$\theta_1 = \frac{180°}{4} = 45° \qquad \theta_2 = \frac{3\times180°}{4} = 135°$$

$$\theta_3 = \frac{5\times180°}{4} = 225° \quad \theta_4 = \frac{7\times180°}{4} = 315°$$

Step 4 : Centroid

$$\sigma = \frac{\sum \text{R.P. of poles} - \sum \text{R.P. of zeros}}{P-Z}$$

$$= \frac{(0-4-2-2)-(0)}{4} = -2$$



Step 5 : Breakaway points

$$1 + G(s)H(s) = 0 \text{ i.e. } 1 + \frac{K}{s(s+4)\left(s^2+4s+20\right)} = 0$$

$\therefore\ s^4 + 8s^3 + 36s^2 + 80s + K = 0$ i.e. $K = -s^4 - 8s^3 - 36s^2 - 80s$ ... (1)

$\frac{dK}{ds} = -4s^3 - 24s^2 - 72s - 80$ i.e. $s^3 + 6s^2 + 18s + 20 = 0$

Solving, $s = -2$ and $-2 \pm j2.45$

All are valid breakaway points. The validity of $-2 \pm j\,2.45$ as a breakaway point can be confirmed by using angle condition. (Refer example 9.17)

At $s = -2$, $K = 64$, from equation (1).

Step 6 : Intersection with imaginary axis

The characteristic equation is already obtained as,

$$s^4 + 8s^3 + 36\,s^2 + 80\,s + K = 0$$

The Routh's array is,

s^4	1	36	K
s^3	8	80	0
s^2	26	K	
s^1	$\frac{2080 - 8K}{26}$	0	
s^0	K		

$\therefore\ 2080 - 8\,K = 0$

$\therefore\ K_{mar} = \frac{2080}{8} = 260$

$A(s) = 26\,s^2 + K = 0$

$\therefore\ 26\,s^2 + 260 = 0$

$\therefore\ s^2 = -10$

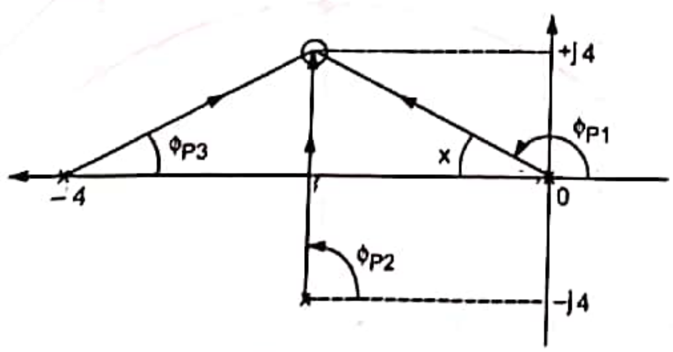
$\therefore\ s = \pm j\sqrt{10} = \pm j\,3.162$

These are the intersection points with imaginary axis.

Step 7 : Angle of departure

$$\phi_{P1} = 180° - x = 180° - \tan^{-1}\left(\frac{4}{2}\right) = 180° - 63.43° = +\ 116.56°$$

$$\phi_{P2} = +\ 90° \quad \text{and} \quad \phi_{P3} = \tan^{-1}\left(\frac{4}{2}\right) = +63.43°$$



$$\therefore \quad \sum \phi_P = 116.56° + 90° + 63.43° = 270°, \quad \sum \phi_Z = 0°$$

$$\therefore \quad \phi = \sum \phi_P - \sum \phi_Z = 270°$$

$$\therefore \quad \phi_d = 180° - \phi = 180° - 270° = -90° \quad \text{at } -2 + j4$$

While ϕ_d at $-2 - j4$ is, $\phi_d = +90°$

The entire root locus as is shown in Fig. 9.30.

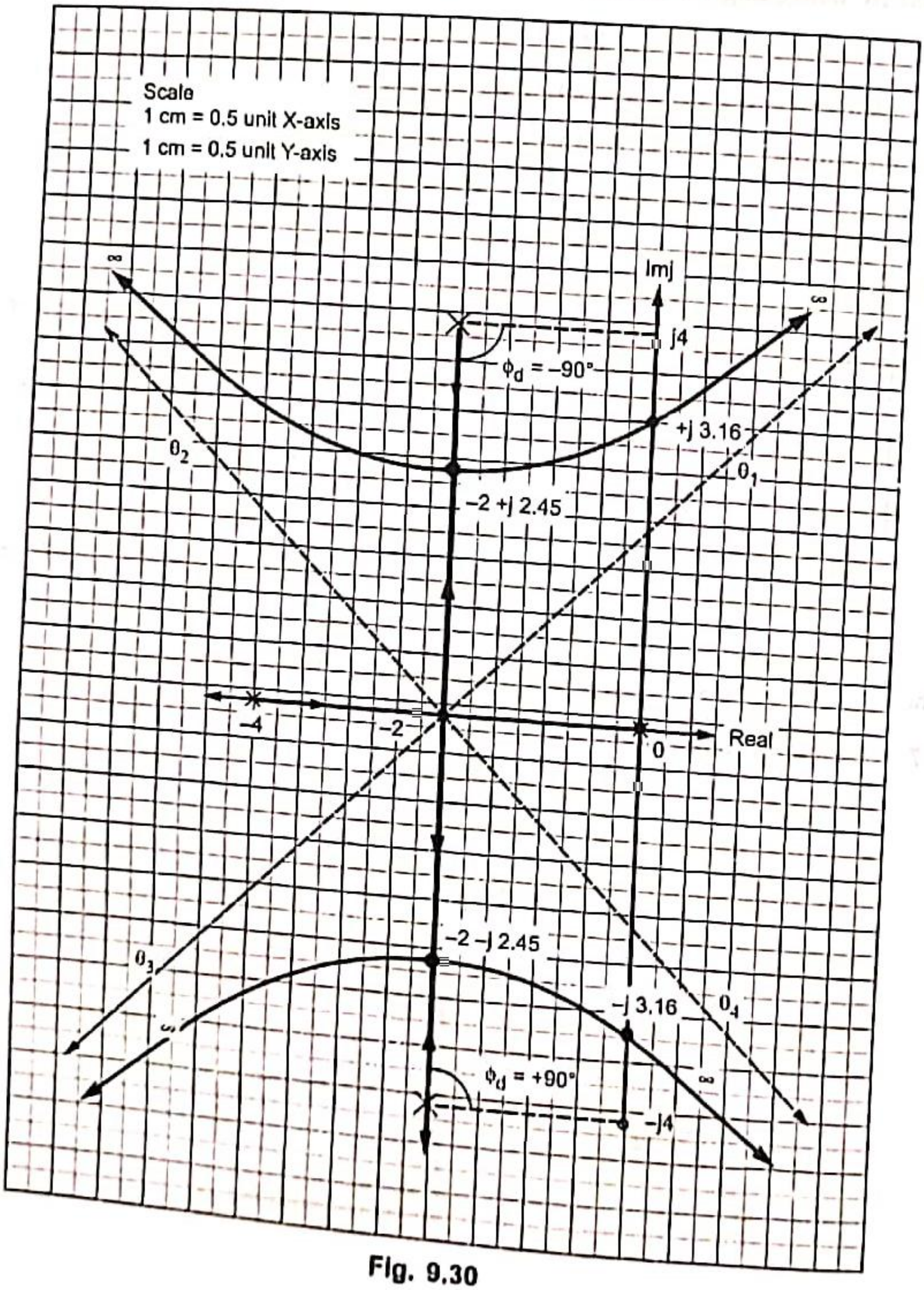
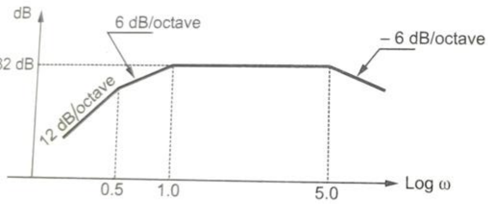


Fig. 9.30

6	a) Analyze the Bode plot shown in Fig below to estimate the transfer function of a  control system:	[10]	CO5	L3
	Sol: **Solution :** The slope dB/octave is to be converted to dB/decade. Initial slope is +12 dB/octave, hence there are two zeros at the origin. This is because **6 dB/octave = 20 dB/decade** **12 dB/octave = 40 dB/decade**			

7.

Now the equation of line after $\omega = 0.5$ is,

$$M = 20 \log \omega + C$$

at $\omega = 1, M = +32$ dB shown

$\therefore$ $32 = 20 \text{ Log } 1 + C$

$\therefore$ $C = 32$

At $\omega = 0.5, M = 20 \text{ Log } 0.5 + 32 = +26$ dB

Now $\omega = 0.5$, M = 26 dB is also on the initial line whose equation is

$$M = +40 \log \omega + C_1$$

$$\text{At } \omega = 0.5, +26 = 40 \log 0.5 + C_1$$

$\therefore$ $C_1 = 38.0412$ dB

Now this line must have M = 0 dB at $\omega = 1$ for K =1.

But at ω =1, M = 40 log 1+38.0412

i.e. $M = 38.0412$ dB

This is due to contribution of system gain constant K.

$\therefore$ $20 \text{ Log } K = 38.0412$

$\therefore$ $K = 79.8$

At $\omega_C = 0.5$, slope changed by –20, there is simple pole.

$$\text{Factor} = \frac{1}{(1+T_1 s)}, \text{ where } T_1 = \frac{1}{\omega_C} = \frac{1}{0.5} = 2 = \frac{1}{(1+2s)}$$

At $\omega_C = 1$, slope changed by –20, there is simple pole.

$$\text{Factor} = \frac{1}{1+T_2 s}, \text{ where } T_2 = \frac{1}{\omega_C} = \frac{1}{1} = 1 = \frac{1}{1+s}$$

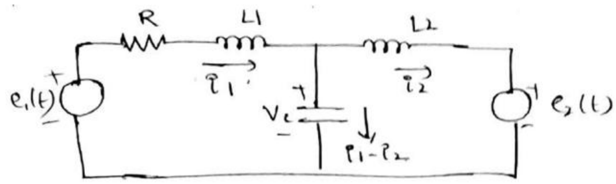
At $\omega_C = 5$, slope further changed by –20, there is simple pole.

$$\text{Factor} = \frac{1}{1+T_3 s}, \text{ where } T_3 = \frac{1}{\omega_C} = \frac{1}{5} = 0.2 = \frac{1}{(1+0.2s)}$$

Hence the transfer function is

$$G(s)H(s) = \frac{79.8 s^2}{(1+2s)(1+s)(1+0.2s)}$$

7. a. Develop a state model for the electrical network shown such that e1(t) and e2(t) are inputs and output is taken across the resistor R. [06] CO5 L3



b. Find state transmission matrix for $\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$ [04]

Let:

- $i_1(t)$: current through L_1
- $i_2(t)$: current through L_2
- $v_C(t)$: voltage across capacitor C

We'll choose the **state variables:**

$$x_1 = i_1(t), \quad x_2 = i_2(t), \quad x_3 = v_C(t)$$

Inputs:

$$u_1 = e_1(t), \quad u_2 = e_2(t)$$

Inputs:

$$u_1 = e_1(t), \quad u_2 = e_2(t)$$

Output:

$$y = \text{voltage across } R = R \cdot i_1 = R \cdot x_1$$

- **Left loop (KVL):**

$$e_1(t) = Ri_1 + L_1 \frac{di_1}{dt} + v_C \Rightarrow u_1 = Rx_1 + L_1\dot{x}_1 + x_3 \Rightarrow \dot{x}_1 = \frac{1}{L_1}(u_1 - Rx_1 - x_3)$$

- **Right loop (KVL):**

$$e_2(t) = L_2 \frac{di_2}{dt} + v_C \Rightarrow u_2 = L_2\dot{x}_2 + x_3 \Rightarrow \dot{x}_2 = \frac{1}{L_2}(u_2 - x_3)$$

Capacitor Current (KCL at center node):

Current into capacitor:

$$i_C = i_1 - i_2 = C\frac{dv_C}{dt} \Rightarrow \dot{x}_3 = \frac{1}{C}(x_1 - x_2)$$

State-Space Equations

$$\dot{x}_1 = \frac{1}{L_1}(u_1 - Rx_1 - x_3)$$

$$\dot{x}_2 = \frac{1}{L_2}(u_2 - x_3)$$

$$\dot{x}_3 = \frac{1}{C}(x_1 - x_2)$$

$$y = Rx_1$$

☑ Matrix Form (State-Space Model)

Let:

- $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$
- $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$
- $y = Rx_1$

State Equation:

$$\dot{\mathbf{x}} = \begin{bmatrix} -\frac{R}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & 0 & -\frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \end{bmatrix} \mathbf{u}$$

Output Equation:

$$y = \begin{bmatrix} R & 0 & 0 \end{bmatrix} \mathbf{x}$$

b.

Q: State Transition matrix for $\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$

we have $A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$

$$\Rightarrow [sI - A] = \begin{bmatrix} s & -1 \\ 3 & s+4 \end{bmatrix}$$

$$\Rightarrow [sI - A]^{-1} = \frac{Adj\,[sI - A]}{det\,(sI - A)} = \frac{\begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix}}{(s+1)\,(s+3)}$$

$$= \begin{bmatrix} \frac{(s+4)}{(s+1)\,(s+3)} & \frac{1}{(s+1)\,(s+3)} \\ \frac{-3}{(s+1)\,(s+3)} & \frac{s}{(s+1)\,(s+3)} \end{bmatrix}$$

Here we have

$$\frac{(s+4)}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$$ using partial fraction we get $A = \frac{3}{2}$, $B = \frac{-1}{2}$

$$\Rightarrow \frac{(s+4)}{(s+1)(s+3)} = \frac{3/2}{(s+1)} + \frac{(-1/2)}{(s+3)} = \frac{3}{2(s+1)} - \frac{1}{2(s+3)} \longrightarrow (1)$$

$$\text{Ily.} \frac{1}{(s+1)(s+3)} = \frac{1}{2(s+1)} - \frac{1}{2(s+3)} \longrightarrow (2)$$

$$\text{Ily.} \frac{-3}{(s+1)(s+3)} = \frac{-3}{2(s+1)} + \frac{3}{2(s+3)} \longrightarrow (3)$$

and,

$$\frac{s}{(s+1)(s+3)} = \frac{-1}{2(s+1)} + \frac{3}{2(s+3)} \longrightarrow (4)$$

$\Longrightarrow$ Taking Laplace inverse of (1), (2), (3) & (4) we get

$$L^{-1}\left[\frac{3}{2(s+1)} - \frac{1}{2(s+3)}\right] = \left[\frac{3}{2}e^{-t} - \frac{1}{2}e^{-3t}\right]$$

$$\text{Ily } L^{-1}\left[\frac{1}{2(s+1)} - \frac{1}{2(s+3)}\right] = \left[\frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t}\right]$$

$$\text{Ily } L^{-1}\left[\frac{-3}{2(s+1)} + \frac{3}{2(s+3)}\right] = \left[\frac{-3}{2}e^{-t} + \frac{3}{2}e^{-3t}\right]$$

$$\text{Ily } L^{-1}\left[\frac{-1}{2(s+1)} + \frac{3}{2(s+3)}\right] = \left[\frac{-1}{2}e^{-t} + \frac{3}{2}e^{-3t}\right]$$

Also we know that

$$\therefore \phi(t) = e^{At} = \begin{bmatrix} \frac{3}{2}e^{-t} - \frac{1}{2}e^{-3t} & \frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t} \\ \frac{-3}{2}e^{-t} + \frac{3}{2}e^{-3t} & \frac{-1}{2}e^{-t} + \frac{3}{2}e^{-3t} \end{bmatrix}$$

is the state Transition matrix.