Fourth Semester B.E/B.Tech. Degree Examination, June/July 2025

Electromagnetics Theory

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, choosing ONE full question from each module.*
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
1	a.	Derive an expression for electric field intensity due to infinite the charge.	8	L2	CO1
	b.	Define Coulomb's law in the vector from and explain.	5	L1	CO1
	c.	Transform the vector field $\bar{W} = 10\bar{a}_x - 8\bar{a}_y + 6\bar{a}_z$ to cylindrical co-ordinate system at point P(10, –8, 6).	7	L3	CO1
		OR			
2	a.	Define position vector and distance vector with an illustration in Cartesian system.	5	L1	CO1
	b.	A change of $1\mu c$ is at A(2, 0, 0), what charge must be placed at print B(–2, 0, 0), which will make 'y' component of total force per unit charge is zero at point C(1, 2, 2). Assume that the media is free space.	7	L3	CO1
	c.	Electric charge lies in the plane at $z = -2$m in the form of a square sheet described by $-2 \le x \le +2$m and $-2 \le y \le +2$m with charge density P_s of $2(x^2 + y^2 + 4)^{3/2}\eta$ c/m². Determine electric field intensity $\bar{E}$ at the origin.	8	L3	CO1
		Module – 2			
3	a.	If $\bar{E} = -8xy\bar{a}_x - 4x^2\bar{a}_y + \bar{a}_z$ V/m, the charge of 6C is to be moved from B(1, 8, 5) to A(2, 18, 6). Find the work done. Selected path is $y = 3x^2 + z$ and $Z = x + 4$.	9	L3	CO2
	b.	State and prove Gauss law.	5	L2	CO2
	c.	Derive the expression for current continuity equation.	6	L2	CO2
		OR			
4	a.	Obtain $\bar{E}$ and $\bar{D}$ for infinite sheet of charge using Gauss law.	8	L2	CO2
	b.	Let $\bar{D} = 5r^2\bar{a}_r$ m C/m² for $r < 0.08$m and $\bar{D} = 0.1/r^2\bar{a}_r$ m C/m² for $r > 0.1$m, find : i) Volume charge density for r = 0.06m, ii) Volume charge density for r = 0.1m. Assume that $\bar{D}$ is in spherical system.	6	L3	CO2
	c.	The current density vector is given by $\bar{J} = \frac{2}{r}\cos\theta\,\bar{a}_r + 20e^{-2r}\sin\theta\,\bar{a}_\theta$, find i) $\bar{J}$ at $(r = 3\text{m}, \theta = 0^\circ, \phi = \pi)$ ii) Total current passing through the sphere with r = 3m, $0 \le \theta \le 20^\circ$ and $0 \le \phi \le 2\pi$ in $\bar{a}_r$ direction.	6	L3	CO2
		Module – 3			
5	a.	Find $\bar{E}$ at P(3, 1, 2) for the field of two co-axial conducting cylinders with v = 50V at r = 2m and v = 20V at r = 3 m using Laplace's equation.	9	L3	CO3
	b.	Calculate the value of $\bar{J}$ if $\bar{H} = \frac{1}{\sin}\bar{a}_\theta$ at P(2, 30°, 20°).	5	L3	CO3
	c.	Deduced Poisson's and Laplace's equation using Gauss law in point form. Write Laplacian operation on 'V' for different co-ordinate system.	6	L2	CO3

		OR			
6	a.	Derive the expression for magnetic field $\overline{H}$ due to infinite long straight line using Biot – Savart law.	10	L2	CO3
	b.	A Co-axial cable with radius of inner conductor 'a', inner radius of outer conductor 'b' and its outer radius 'c'. The outer conductor carries current + I and inner conductor carries current – I. Determine and sketch variation of $\overline{H}$ against 'r' for : i) $r < a$ ii) $a < r < b$ iii) $b < r < c$ and iv) $r > c$.	10	L3	CO3
		Module – 4			
7	a.	In a certain region, the magnetic flux density in a magnetic material with $X_m = 6$ is given as $\overline{B} = 0.005y^2\overline{a}_x T$ at $y = 0.4m$, find $\overline{J}, \overline{J}_b$ and $\overline{J}_T$.	8	L3	CO4
	b.	Derive Lorentz force equation and explain.	5	L2	CO4
	c.	Derive an equation for the force between the two differential current elements.	7	L2	CO4
		OR			
8	a.	A square loop of wire in $z = 0$ plane carrying 2mA in the field of an infinite filament on the y-axis as shown in the Fig.Q8(a). Find the total force on the loop.	7	L3	CO4

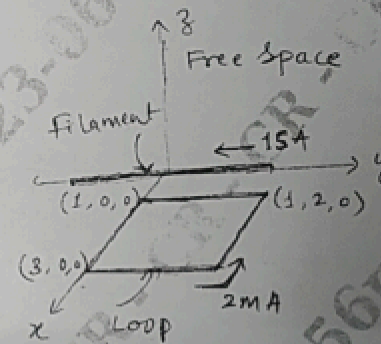

Fig.Q8(a)

	b.	Obtain the Tangential component of $\overline{B}$ and $\overline{H}$ is the boundary of two medium having the permeability of μ_1 and μ_2.	8	L2	CO4
	c.	Compare electric and magnetic circuits.	5	L2	CO4
		Module – 5			
9	a.	Explain inconsistency of current continuity equation in detail.	7	L2	CO5
	b.	Derive general wave equation of $\overline{E}$ and $\overline{H}$ for the media with parameters μ, ϵ and σ.	8	L2	CO5
	c.	A circular loop conductor lies in $z = 0$ plane and has a radius of 0.1 m and resistance of 5Ω. Given $\overline{B} = 0.2 \sin 10^3 t$ Tesla, determine the current in the loop.	5	L3	CO5
		OR			
10	a.	Derive Maxwell's equations in integral and point form for static electric and magnetic fields using Faraday's law, Ampere's circuital law and Coulomb's law.	8	L2	CO5
	b.	A 9375MHz uniform plane wave is propagating in polystyrene. If the amplitude of electric field intensity is 20 V/m and the material is assumed to be lossless, find Attenuation Constant (α), phase constant (β), Wavelength (λ), Velocity of propagation (v), intrinsic impedance (η), propagation constant (γ) and amplitude of the magnetic field. For polystyrene $\mu_r = 1$ and $\epsilon_r = 2.56$.	6	L3	CO5
	c.	State and explain Poynting theorem.	6	L2	CO5

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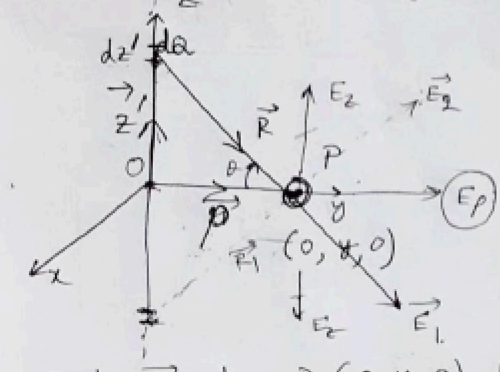
1.a)

Find the electric field intensity due to a line charge distribution of ρ_L C/m of infinite length where charge is uniformly distributed along the length.



$\frac{z'}{\rho} = \tan\theta$

$z' = \rho \tan\theta$

ρ_L C/m

Find $\vec{E}$ at $P(0, y, 0)$ because of the line charge along z-axis.

We consider incremental charge

$dQ = \rho_L dz'$ . . . (1) [charge on length dz']

$\vec{R} = \vec{\rho} - \vec{z}'$ $\quad (\vec{z}' + \vec{R} = \vec{\rho})$

$\vec{R} = (\rho \hat{a}_\rho - z'\hat{a}_z)$ . . (2) $\quad |\vec{R}| = \sqrt{\rho^2 + z'^2}$

∴ The field at P, because of dQ,

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 |\vec{R}|^2}\hat{a}_R = \frac{\rho_L dz'}{4\pi\epsilon_0(\rho^2 + z'^2)} \cdot \frac{(\rho\hat{a}_\rho - z'\hat{a}_z)}{(\rho^2+z'^2)^{1/2}}$$

$[\because |\vec{R}| = \sqrt{\rho^2 + z'^2}]$

$$= \frac{\rho_L dz' \cdot (\rho\hat{a}_\rho - z'\hat{a}_z)}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}$$

We know from the symmetry of the problem, – (3)

$$d\vec{E} = \frac{\rho_L dz' \rho \hat{a}_\rho}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

$\therefore$ The field at P,

$$\vec{E} = \int d\vec{E} = \int_{z' \to -\infty}^{\infty} \frac{\rho_L \, dz' \rho \, \hat{a}_\rho}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{dz'}{(\rho^2 + z'^2)^{3/2}} \hat{a}_\rho$$

Note: $\tan\theta = \frac{z'}{\rho}$

$z' = \rho \tan\theta$ | $z' \to \infty, \ \theta = \pi/2$

$dz' = \rho \sec^2\theta \, d\theta$ | $z' \to -\infty, \ \theta = -\pi/2$

$$\therefore \vec{E} = \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta \, d\theta}{(\rho^2 + \rho^2 \tan^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta \, d\theta}{(\rho^2)^{3/2} (1 + \tan^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \cancel{\rho}}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\cancel{\rho} \sec^2\theta \, d\theta}{\cancel{\rho^3}\rho \, (\sec^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta \, d\theta}{\sec^3\theta} \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta} d\theta \, \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0} \left[\sin\theta\right]_{-\pi/2}^{\pi/2} \hat{a}_\rho$$

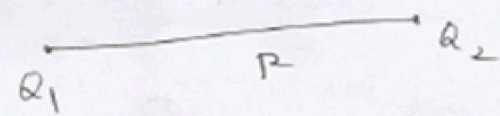
$$= \frac{\rho_L}{4\pi\epsilon_0} [1 + 1] \hat{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho$$

$\therefore$ The electric field intensity due to infinite static line charge along z-axis,

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho \ \text{V/m}}$$

1.(b)

Soln. The force b/w two very small charged objects separated in vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the dist. b/w them.



i.e. $F = \dfrac{k\, Q_1 Q_2}{R^2}$

Q_1 & Q_2 → +ve or −ve quantities of charge
unit coulomb (C)

R → separation in m.

k → constant of proportionality.

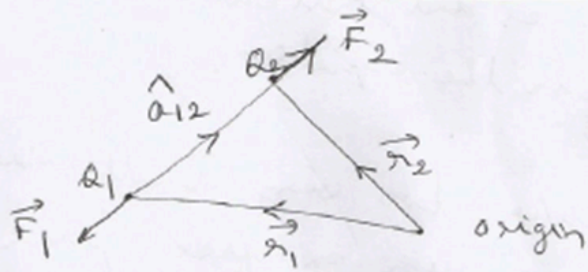
$k = \dfrac{1}{4\pi\epsilon_0}$

where, ϵ_0 → permittivity of free space.

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m

$= \dfrac{1}{36\pi} \times 10^{-9}$ F/m.

F → Force in Newton.



$\vec{r_1} \rightarrow$ locates Q_1

$\vec{r_2} \rightarrow$ locates Q_2

Q_1, Q_2 of same sign, F_2 in the direction as indicated.

$F_2 \rightarrow$ force ~~ext~~ exerted on Q_2 by Q_1.

$\hat{a_{12}} \rightarrow$ unit vector along $\vec{R_{12}}$.

Then, the vector form of coulomb's law is

$$\vec{F_2} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \hat{a_{12}}$$

where, $$\hat{a_{12}} = \frac{\vec{R_{12}}}{|\vec{R_{12}}|} = \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - \vec{r_1}|}$$

$|\vec{R_{12}}| = R =$ Distance b/w the ~~tw~~ two charges.

Let, $\vec{F_1}$ be the force exerted by Q_1 on Q_2.

$$\vec{r_1} = \vec{r_2} + \vec{R_{21}}$$

$$\Rightarrow \vec{R_{21}} = \vec{r_1} - \vec{r_2} = -(\vec{r_2} - \vec{r_1})$$

$$\therefore \hat{a}_{12} = -\hat{a}_{21}$$

Important observations:

i) charges should be point charges and stationary in nature.

ii) Should consider the signs of the charges to decide wheather the force will be attractive or repulsive.

iii) Coulomb's law is linear.

i.e. if $\vec{F_2} = -\vec{F_1}$

then, $n\vec{F_2} = -n\vec{F_1}$

where n is a scalar.

iv) Force on a charge in the presence of several other charges is the sum of the forces on that charge due to each of the other charges acting alone.

1.(c)

Solution

(a) $F_\rho = \vec{F} \cdot \hat{a}_\rho = (10\hat{a}_x - 8\hat{a}_y + 6\hat{a}_z) \cdot \hat{a}_\rho$

$= 10\cos\phi - 8\sin\phi$

Now $P(10, -8, 6)$ is provided in cartesian co-ordinates

$\therefore$ corresponding, $\rho = \sqrt{100+64} = 12.80$

$\phi = \tan^{-1}\left(\frac{-8}{10}\right) = -38.65°$

$z = 6.$

$\therefore F_\rho = 10\cos(-38.65) - 8\sin(-38.65)$

$= 7.809 + 4.9964 = 12.80$

$F_\phi = F \cdot \hat{a}_\phi = (10\hat{a}_x - 8\hat{a}_y + 6\hat{a}_z) \cdot \hat{a}_\phi$

$= -10\sin\phi - 8\cos\phi$

$= -10\sin(-38.65) - 8\cos(-38.65)$

$= 6.24 - 6.24 = 0$

$F_z = 6.$

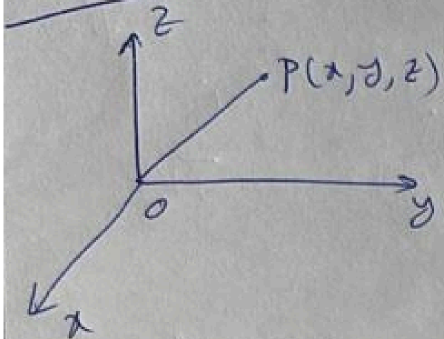
$\therefore \vec{F} = 12.80\hat{a}_\rho + 6\hat{a}_z$

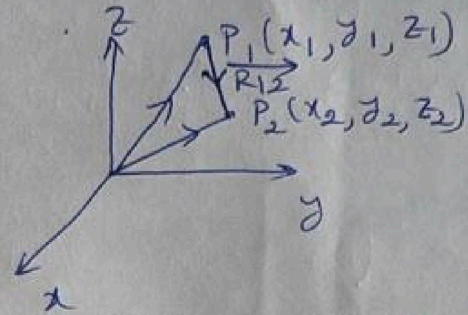
2.(a)

2.(a)

Position vector,

$\overline{OP} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$





$$\overline{R_{12}} = (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$

Distance vector.

2.(b)

2-(b)

$$\hat{A} = 2\hat{a}_x$$

$$\bar{B} = -2\hat{a}_x$$

$$\bar{C} = \hat{a}_x + 2\hat{a}_y + 2\hat{a}_z$$

$$\bar{R}_1 = (-\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z)$$

$$\bar{R}_2 = (3\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z)$$

$$\bar{E} = \bar{E}_1 + \bar{E}_2 = \frac{1}{4\pi\epsilon_0}\left\{\frac{Q_1(-\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z)}{27} + \frac{Q_2(3\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z)}{17\sqrt{17}}\right.$$

To remove y- component,

$$0 = \frac{1}{4\pi\epsilon_0}\left(\frac{Q_1 \times 2}{27} + \frac{Q_2 \times 2}{17\sqrt{17}}\right)$$

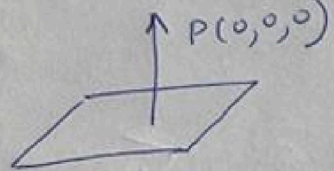
$$\therefore \boxed{Q_2 = -2.6\,\mu C}$$

2.(c)

2.(c) $\rho_s = 2(x^2+y^2+4)^{3/2}\ nC/m^2$

$dQ = \rho_s\, ds$ where $ds = dx\,dy$

$$\therefore d\bar{E} = \frac{2(x^2+y^2+4)^{3/2}\,dx\,dy\,(-x\hat{a}_x - y\hat{a}_y + 2\hat{a}_z)\times 10^{-9}}{4\pi\epsilon_0 (x^2+y^2+4)^{3/2}}$$



$$= \frac{dx\,dy\,\hat{a}_z}{\pi\epsilon_0}\times 10^{-9}$$

$$\therefore \bar{E} = \int d\bar{E} = \int_{y=-2}^{2}\int_{x=-2}^{2} \frac{1}{\pi\epsilon_0}\,dx\,dy\times 10^{-9}\,\hat{a}_z$$

$$= 575.21\,\hat{a}_z\ V/m$$

3.(a)

$y = 3x^2 + z, \quad z = x + 4, \quad Q = 6C$

$\bar{E} = -8xy\,\hat{a}_x - 4x^2\,\hat{a}_y + \hat{a}_z$ V/m.

$B(1,8,5)$ to $A(2,18,6)$

$$W = -Q\left[\int_{x=1}^{2} -8xy\,dx - \int_{y=8}^{18} 4x^2\,dy + \int_{z=5}^{6} dz\right]$$

$y = 3x^2 + x + 4, \quad dy = (6x+1)\,dx$

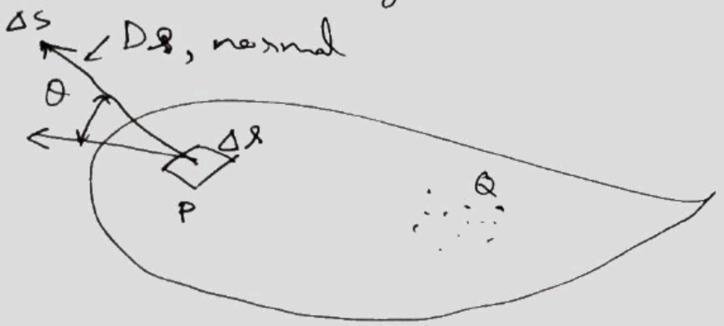
$$\therefore W = -Q\left\{-\int_{x=1}^{2} (24x^3 + 8x^2 + 32x)\,dx - \int_{x=1}^{2} 4x^2(6x+1)\,dx + \int_{z=5}^{6} dz\right\}$$

$\simeq 1530$ Joul,

3.(b)

Derive Mathematical form of Gauss's law and [illegible]

Gauss's law: The electric flux passing through any closed surface is equal to the total charge enclosed by the surface



Q → cloud of point charge

$D_s \rightarrow$ flux density at the surface

↓ varies from one point to another on the surface

$\Delta s \rightarrow$ a small portion of the surface.

$\vec{\Delta s} \rightarrow$ mag. and direction

direction normal at that point.

outward +ve for any surface.

At any pt. P consider an incremental surface Δs.

$\vec{D}_s$ makes an angle θ with $\vec{\Delta s}$.

Then flux crossing $\vec{\Delta s}$ is then,

$$\Delta \psi = \text{flux crossing } \Delta S$$

$$= \vec{D_s} \cdot \vec{\Delta s}.$$

$\therefore$ Total flux passing through entire closed surface,

$$\psi = \int d\psi = \oint_{\text{closed surface}} \vec{D_s} \cdot \vec{ds} = \text{charge enclosed} = Q$$

This type of closed surface is a // Gaussian surface.

The mathematical formulation of Gauss's law,

$$\psi = \oint_s \vec{D_s} \cdot \vec{ds} = \text{charge enclosed} = Q.$$

If several point charges,

$$Q = \Sigma Q_n.$$

Line charge, $Q = \int \rho_L \, dl$

surface charge, $Q = \int_s \rho_s \, ds$.

Volume charge, $Q = \int_{vol} \rho_v \, dv$.

Last form represents all the other forms

$$\oint_s \vec{D_s} \cdot \vec{ds} = \int_{vol} \rho_v \, dv .$$

flux   charge.

~~Proof~~ Verification of Gauss's law:

Sphere of radius r,   considering point charge at origin of sphere.

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a_r}$$

$D = \epsilon_0 E$

At the surface of the sphere,

$$D = \frac{Q}{4\pi a^2} \hat{a_r} .$$

$ds = r^2 \sin\theta \, d\theta \, d\phi = a^2 \sin\theta \, d\theta \, d\phi .$

$\therefore \vec{ds} = a^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$

$\therefore \vec{D_s} \cdot \vec{ds} = \frac{Q}{4\pi \cancel{a^2}} \cancel{a^2} \sin\theta \, d\theta \, d\phi \, \hat{a_r} \cdot \hat{a_r}$

$= \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi$

$\therefore$ Overall surface, $\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\theta=\pi} \vec{D_s} \cdot \vec{ds} = \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi$

$= \frac{Q}{4\pi} \left[-\cos\theta\right]_0^{\pi} \cdot 2\pi$

$= \frac{Q}{2} \cdot [1+1] = Q$.

$\therefore$ QC of electric flux are crossing the surface, as we should, since the enclosed charge is QC

3.(c)

Continuity of current → Applicable when we are considering closed surface.

Charges can be neither created nor destroyed

Equal amounts of +ve and −ve charge may be simultaneously created, obtained by separa destroyed or lost by recombination.

Current through the closed surface,

$$I = \oint_s \vec{J} \cdot \vec{ds}$$

outward flow of +ve charge balanced by a decrease of +ve charge within the closed surface.

Let, Q_i → be the charge inside the closed surface.

$$\therefore I = \oint_s \vec{J} \cdot \vec{ds} = -\frac{dQ_i}{dt}$$ → reduction in charge. ∴ negative sign.

Using Divergence theorem

$$\oint_s \vec{J} \cdot \vec{ds} = \int_{vol} (\vec{\nabla} \cdot \vec{J})\, dv$$

Now, $Q = \int_{vol} \rho_v \, dv$.

$$\therefore \int_{vol} (\vec{\nabla} \cdot \vec{J})\, dv = -\frac{d}{dt} \int_{vol} \rho_v \, dv$$

If the ~~de~~ surface is constant, derivative becomes a partial derivative.

$$\therefore \int_{vol} (\vec{\nabla} \cdot \vec{J})\, dv = \int_{vol} -\frac{\partial \rho_v}{\partial t}\, dv.$$

This expression is true for any volume, however small, it is true for an incremental volume,

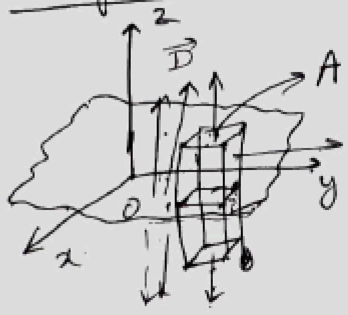
$$(\vec{\nabla} \cdot \vec{J})\, \Delta v = -\frac{\partial \rho_v}{\partial t}\, \Delta v.$$

$\therefore$ Point form of continuity equation,

$$\boxed{(\vec{\nabla} \cdot \vec{J}) = -\frac{\partial \rho_v}{\partial t}}$$

4.(a)

Infinite surface charge



$$Q_{en} = \oiint \vec{D} \cdot \vec{ds}$$

$$Q_{en} = \rho_s \cdot A \quad \text{.. (1)}$$

where, $\rho_s \rightarrow$ surface charge density (c/m^2)

$$\oiint \vec{D} \cdot \vec{ds} = \iint_{top} D_z \, ds + \iint_{bottom} D_z \, ds$$

$$= D_z (A + A) = 2A \cdot D_z$$

$$\rho_s \cdot \cancel{A} = 2\cancel{A} \, D_z$$

$$D_z = \frac{\rho_s}{2}$$

$$\vec{D} = D_z \hat{a}_z = \frac{\rho_s}{2} \hat{a}_z \ c/m^2$$

$$\therefore \vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{\rho_s}{2\epsilon_0} \hat{a}_z \ V/m$$

4.(b)

(i) Find charge density at $r = 0.06m$.

$\rho_v = (\vec{\nabla}.\vec{D})$, $\vec{D} = 5r^2 \hat{a}_r \; mC/m^2$

$\rho_v = \vec{\nabla}.\vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r)$

$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 . 5r^2) = \frac{1}{r^2}\frac{\partial}{\partial r}(5r^4)$

$= \frac{1}{r^2} . 20r^3 = 20r$

$\rho_v \Big|_{r=0.06m} = (20 \times 0.06) = 1.20 \; mc/m^3$

(ii) Find charge density at $r = 0.1m$.

$\vec{D} = \left(\frac{0.1}{r^2}\right)\hat{a}_r \; c/m^2$

$\vec{\nabla}.\vec{D} = \frac{1}{r^2}\left(\frac{\partial}{\partial r}(r^2 D_r)\right)$

$= \frac{1}{r^2}\left(\frac{\partial}{\partial r}\left(\cancel{r^2} . \frac{0.1}{\cancel{r^2}}\right)\right)$

$= 0$

$\vec{D} = D_r \hat{a}_r + D_\theta \hat{a}_\theta + D_\phi \hat{a}_\phi$

4. (c)

4.(c) $\bar{J} = \frac{2}{r}\cos\theta\,\hat{a}_r + 20e^{-2r}\sin\theta\,\hat{a}_\theta$

(i) $\bar{J} = \frac{2}{3}\cos(0)\,\hat{a}_r + 20e^{-2(3)}\sin(0)\,\hat{a}_\theta$

$= \frac{2}{3}\hat{a}_r = 0.666\,\hat{a}_r$

) $I = \oint\!\!\oint \bar{J}\cdot d\bar{s} = \iint \frac{2}{r}\cos\theta \cdot r^2\sin\theta\, d\theta\, d\phi$

$= \iint r\sin 2\theta\, d\theta\, d\phi$

at $r = 3m$, $I = 2.204\,A$.

5.(a)

Find $|\vec{E}|$ at $P(3,1,2)$ for the field of two coaxial conducting cylinders placed along z axis. $V = 50V$ at $\rho = 2m$ and $V = 20V$ at $\rho = 3m$.

Sol. Variation w.r.t ρ only.

$\therefore \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) = 0$

$\Rightarrow \frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) = 0$

$\Rightarrow \rho\frac{\partial V}{\partial\rho} = A$

$\Rightarrow \partial V = \frac{A\,\partial\rho}{\rho}$

$\Rightarrow V = A\ln\rho + B$

$V = 50$ at $\rho = 2 \Rightarrow 50 = A\ln 2 + B$

$V = 20$ at $\rho = 3 \Rightarrow 20 = A\ln 3 + B$

$30 = A\ln\left(\frac{2}{3}\right)$

$\Rightarrow 30 = -.405\,A$

$\Rightarrow A = -73.98$

$\therefore 50 = -73.98\ln 2 + B$

$\Rightarrow B = 50 + 73.98\ln 2 = 101.279$

$\therefore V = -73.98\ln\rho + 101.279$

$\therefore \vec{E} = -\vec{\nabla}V = -\frac{\partial V}{\partial\rho}\hat{a}_\rho = +\frac{73.98}{\rho}\hat{a}_\rho$

Here, $\rho = \sqrt{3^2+1^2} = \sqrt{10}$

$\therefore |\vec{E}| = \frac{73.98}{\sqrt{10}}$

$= 23.39\ V/m$.

5. (b)

5.(b) $\bar{J} = \bar{\nabla} \times \bar{H}$

$\bar{H} = \frac{1}{\sin\theta} \hat{a}_\theta$

$$\therefore \bar{J} = \bar{\nabla} \times \bar{H} = \frac{1}{r^2 \sin\theta} \begin{bmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & rH_\theta & r\sin\theta\,H_\phi \end{bmatrix}$$

$$= \frac{1}{r^2 \sin\theta} \begin{bmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & \frac{r}{\sin\theta} & 0 \end{bmatrix}$$

$$= \frac{1}{r^2} \times \frac{r\cancel{\sin\theta}}{\cancel{\sin\theta}} \hat{a}_\phi \cdot \frac{1}{\sin\theta} = \frac{1}{r} \frac{\hat{a}_\phi}{\sin\theta}$$

$\therefore$ at $P(2, 30^\circ, 20^\circ)$, $\boxed{\vec{J} = 0.25\,\hat{a}_\phi}$

5.(c)

Derive Poisson's and Laplace's equation

$$\vec{\nabla}\cdot\vec{D} = \rho_v \quad \cdots (1)$$

$$\vec{D} = \epsilon_0 \vec{E} \quad \cdots (2)$$

$$\vec{E} = -\vec{\nabla} V \quad \cdots (3)$$

$$\vec{\nabla}\cdot\vec{D} = \vec{\nabla}\cdot(\epsilon_0\vec{E}) = \epsilon_0(\vec{\nabla}\cdot\vec{E}) = \rho_v$$

or $\epsilon_0\left(\vec{\nabla}\cdot(-\vec{\nabla}V)\right) = \rho_v$

or $-\epsilon_0\,\vec{\nabla}\cdot(\vec{\nabla}V) = \rho_v \quad \cdots (4)$

$\cancel{\vec{\nabla}\cdot(\vec{\nabla}V}$ $\vec{\nabla}\cdot(\vec{\nabla}V)$

$$= \left(\hat{a}_x\frac{\partial}{\partial x} + \hat{a}_y\frac{\partial}{\partial y} + \hat{a}_z\frac{\partial}{\partial z}\right)\cdot\left(\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right)$$

$$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \nabla^2 V$$

$\therefore$ eqn. (4) reduces to,

$$\epsilon_0 \nabla^2 V = -\rho_v$$

or $\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon_0}}$ → Poisson's equation.

Now if $\rho_v = 0$ → $\boxed{\nabla^2 V = 0}$ → Laplace's eqn.

→ zero volume charge density, but allowing point charges, line charge and surface density to exist at singular locations as sources of the field.

$\nabla^2 V$ → Laplacian of V.

In rectangular co-ordinate system,

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

In cylindrical co-ordinates,

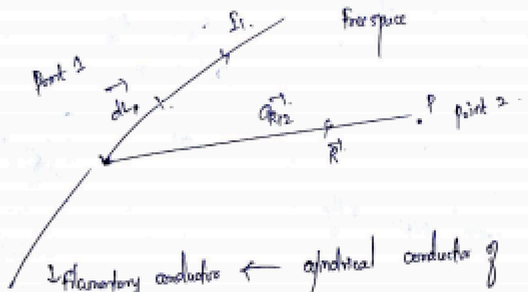
$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

In spherical co-ordinates,

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

6.a) Magnetic field intensity due to infinite long conductor using Biot Savart Law:

Differential current element



Filamentary conductor ← cylindrical conductor of circular cross section as $r \to 0$.

Biot - Savart law:

At any point P, magnitude of magnetic field intensity produced by the differential element is proportional to the product of current, the magnitude of differential length and the sine of the angle lying between the filament and the line connecting the filament & the point P at which the field is desired;

Also the magnitude of the magnetic field intensity is inversely proportional to the square of the distance from the differential element to the point P.

The direction of the magnetic field intensity is normal to the plane containing the filament and the line drawn from the filament to the point P.

$$d\vec{H} = \frac{I d\vec{L} \times \vec{a}_R}{4\pi |\vec{R}|^2}$$

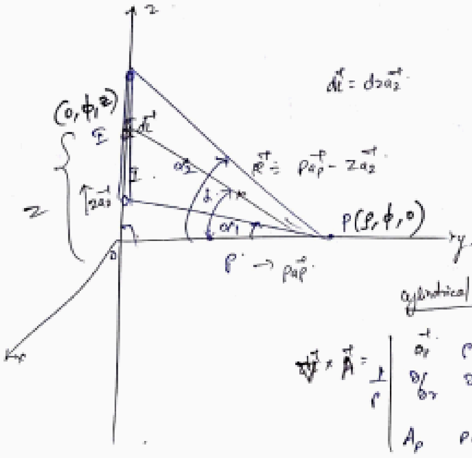
$$\boxed{d\vec{H} = \frac{I d\vec{L} \times \vec{R}}{4\pi |\vec{R}|^3}} \quad A/m$$

Current element at point 1 & point at which field is determined at point 2.

$$d\vec{H}_2 = \frac{I d\vec{L}_1 \times \vec{a}_{R_{12}}}{4\pi |\vec{R}_{12}|^2}$$

$$\boxed{d\vec{H}_2 = \frac{I d\vec{L}_1 \times \vec{R}_{12}}{4\pi |\vec{R}_{12}|^3}} \quad A/m$$

Infinitely long straight filament:



$$\vec{dl} = dz\,\vec{a_z}$$

$$\vec{H} = \int_L \frac{I\,\vec{dl} \times \vec{a_R}}{4\pi |\vec{R}|^2}$$

$$\vec{H} = \int_L \frac{I\,\vec{dl} \times \vec{R}}{4\pi |\vec{R}|^3}$$

cylindrical

$$\vec{\nabla} \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \vec{a_\rho} & \rho\vec{a_\phi} & \vec{a_z} \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

In spherical

$$\nabla \times \vec{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \vec{a_r} & r\vec{a_\theta} & r\sin\theta\,\vec{a_\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

$$\vec{dl} \times \vec{R} = \frac{1}{\rho}\begin{vmatrix} \vec{a_\rho} & \rho\vec{a_\phi} & \vec{a_z} \\ 0 & 0 & dz \\ \rho & 0 & -z \end{vmatrix}$$

$$= \frac{1}{\rho}\left[-\rho\,\vec{a_\phi}\,(0 - \rho\,dz)\right]$$

$$= \frac{\rho^2}{\rho}\,dz\,\vec{a_\phi}$$

$$\boxed{\vec{dl} \times \vec{R} = \rho\,dz\,\vec{a_\phi}}$$

$$\vec{H} = \int_{z=z_1}^{z_2} \frac{I\,\rho\,dz\,\vec{a_\phi}}{4\pi\left[\rho^2 + z^2\right]^{3/2}}$$

$$\tan\alpha = \frac{z}{\rho}$$

$$z = \rho\tan\alpha$$

z	z_1	z_2
α	α_1	α_2

$$dz = \rho\sec^2\alpha\,d\alpha$$

$$\vec{H} = \frac{I}{4\pi}\int_{\alpha=\alpha_1}^{\alpha_2} \frac{\rho^2\sec^2\alpha\,d\alpha\,\vec{a_\phi}}{\left[\rho^2 + \rho^2\tan^2\alpha\right]^{3/2}}$$

$$= \frac{I}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2\sec^2\alpha\,d\alpha\,\vec{a_\phi}}{(\rho^2\sec^2\alpha)^{3/2}}$$

$$= \frac{I}{4\pi\rho}\int_{\alpha_1}^{\alpha_2} \cos\alpha\,d\alpha\,\vec{a_\phi}$$

Finite conductor. $\boxed{\vec{H} = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right]\vec{a_\phi}}$ A/m

$\therefore z_1 \to -\infty$ & $z_2 \to \infty$.

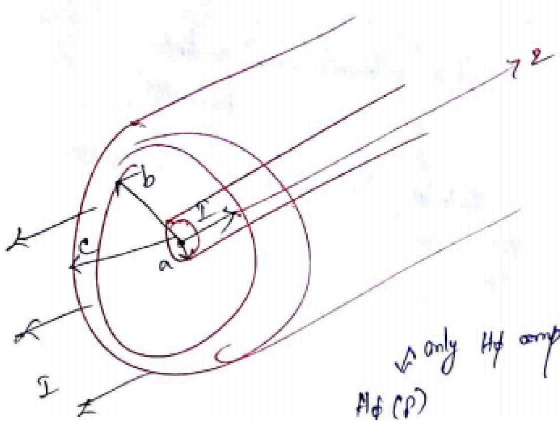
$\alpha_1 = -\pi/2$ & $\alpha_2 = \pi/2$.

$$\vec{H} = \frac{I}{4\pi\rho}\left[1 - (-1)\right]\vec{a_\phi}$$

Infinite conductor. $\boxed{\vec{H} = \frac{I}{2\pi\rho}\vec{a_\phi}}$ A/m

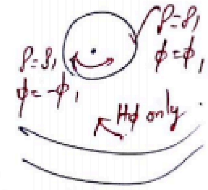
6.b) H due to coaxial cable:

(2) Infinitely long coaxial transmission line
carrying total current I in center conductor &
$-I$ in outer conductor



$H \rightarrow$ does not vary with z or ϕ due to symmetry.

Radial symmetry
$\downarrow$
a_ρ component cancel.



only H_ϕ component which varies with ρ.

$H_\phi(\rho)$

(i) $a < \rho < b$ larger than a, smaller than b

$I_{enc} = I$.

$\oint \vec{H} \cdot d\vec{l} = I$.

$H_\phi \int_0^{2\pi} \rho \, d\phi = I$

$$\boxed{H_\phi = \frac{I}{2\pi\rho}} \; ; \; a < \rho < b.$$

(ii) $\rho < a$ smaller than a

$\frac{I_{enc}}{I} = \frac{\pi\rho^2}{\pi a^2}$.

$\pi a^2 \rightarrow I$.

$\pi\rho^2 \rightarrow \; ? \; I_{enc}$

$$\boxed{I_{enc} = \frac{\rho^2 I}{a^2}}$$

$\int_0^{2\pi} H_\phi \, \rho \, d\phi = \frac{\rho^2 I}{a^2}$

$2\pi\rho \, H_\phi = \frac{\rho^2 I}{a^2}$

$$\boxed{H_\phi = \frac{I\rho}{2\pi a^2}} \; ; \; \rho < a$$

(iii) $\rho > c$ larger than c

$I_{enc} = I + (-I)$

$I_{enc} = 0$

$$\boxed{H_\phi = 0} \; ; \; \rho > c.$$

(iv) $\underline{b < \rho < c}$

$\pi(c^2-b^2) \rightarrow -I.$

$\pi(\rho^2-b^2) \rightarrow I_x = ?$ $\quad \dfrac{I_x}{-I} = \dfrac{\pi(\rho^2-b^2)}{\pi(c^2-b^2)}$

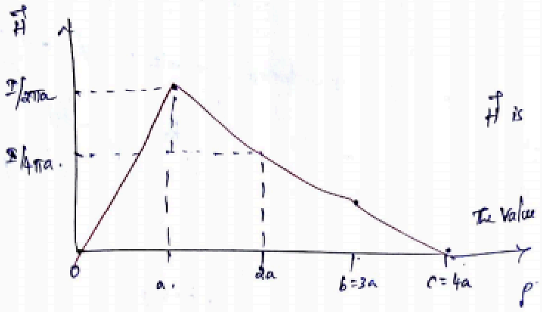
$I_{enc} = I - I_x.$ $\quad \boxed{I_{enc} = I - \dfrac{I(\rho^2-b^2)}{(c^2-b^2)}}$

$2\pi\rho H_\phi = I - \dfrac{I(\rho^2-b^2)}{(c^2-b^2)}$

$H_\phi = \dfrac{I}{2\pi\rho}\left[\dfrac{c^2-b^2-\rho^2+b^2}{c^2-b^2}\right]$

$\boxed{H_\phi = \dfrac{I}{2\pi\rho}\left[\dfrac{c^2-\rho^2}{c^2-b^2}\right]}$; $\underline{b<\rho<c}$.

Magnetic field strength variation with radius for coaxial cable with $b=3a$, $c=4a$.



$\vec{H}$ is continuous at all conductor boundaries.

The value of H_ϕ shows no sudden jumps.

External field is zero.

7.a) Problem:

7) a) $\vec{B} = 0.005y^2\hat{a}_x$ T at $y = 0.4$ m

$\chi_m = 68$

$\mu_0 = 4\pi\times10^{-7}$ $\quad \vec{H} = \dfrac{\vec{B}}{\mu} = \dfrac{\vec{B}}{\mu_0(1+\chi_m)}$ $\quad \vec{J} = \dfrac{1}{\mu_0(1+\chi_m)}(\vec{\nabla}\times\vec{B})$

1) $\vec{J} = \vec{\nabla}\times\vec{H}$

$$\vec{\nabla}\times\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0.005y^2 & 0 & 0 \end{vmatrix} = -0.01y\,\hat{a}_z$$

At $y = 0.4$m

$\therefore \boxed{\vec{J} = -454.72\,\hat{a}_z}$ A/m^2

2) $\vec{J}_b = \vec{\nabla}\times\dfrac{\chi_m}{\mu_0(1+\chi_m)}\vec{B}$ $\quad$ At $y = 0.4$m $\quad \boxed{\vec{J}_b = -2728.37\,\hat{a}_z}$ A/m^2

3) $\vec{J}_T = \dfrac{\vec{\nabla}\times\vec{B}}{\mu_0} = ?$ $\quad$ At $y = 0.4$m $\quad \boxed{\vec{J}_T = -3183.09\,\hat{a}_z}$ A/m^2

7.b) Lorentz force equation:

Force on a moving charge:

Electric force on a charged particle, $\boxed{\vec{F} = Q\vec{E}}$ $\longrightarrow (1)$

$\downarrow$ Same dirn as $\vec{E}$ for a positive charge.

If the charge is in motion, the above equation gives the force at any point in its trajectory.

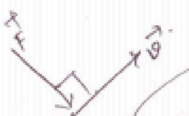
Force on a charged particle is in motion in a magnetic field of flux density, $\vec{B}$

$\downarrow$

$\boxed{\vec{F} = Q\,\vec{v} \times \vec{B}}$ direction of force is $\perp^r$ to both $\vec{v}$ and $\vec{B}$ $\longrightarrow (2)$

$\vec{F}$ applied $\perp^r$ to the dirn in which charge is moving.

$\downarrow$



can never change its velocity.

Acceleration vector is always normal to velocity vector

Kinetic energy of particle remains unchanged.

Steady magnetic field is incapable of transferring energy to a moving charge

Electric field $\rightarrow$ exerts force on particle which is independent of the dirn of progressing charge

& effects an energy transfer between field and particle in general.

Force on a moving particle arising from combined electric & magnetic field.
(by superposition)

$\boxed{\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})}$ $\longrightarrow (3)$ $\rightarrow$ Lorentz force equation.

$\downarrow$

Solution is required in determining

1) electron orbits in magnetron
2) Proton paths in cyclotron
3) plasma characteristics in a magneto hydro dynamic (MHD) generator

In general charged particle motion in combined electric and magnetic fields

7.c)Force between two differential current elements:

Consider two current elements, to find force b/w two current elements

W.K.T: Magnetic field at point 2 due to current element at point 1.

$$d\vec{H_2} = \frac{I_1\, d\vec{L_1} \times \vec{a_{R_{12}}}}{4\pi\, |\vec{R_{12}}|^2}$$

Differential force on a differential current element

$$d\vec{F} = \vec{I}\, d\vec{L} \times \vec{B}$$

Differential Flux density (dB_2) at point 2 caused by current element 1 ($I\, d\vec{L_1}$)

Differential amount of differential force on element 2,

$$d(d\vec{F_2}) = I_2\, d\vec{L_2} \times d\vec{B_2}$$

where

$$d\vec{B_2} = \mu_0\, d\vec{H_2} = \frac{\mu_0\, I_1\, d\vec{L_1} \times \vec{a_{R_{12}}}}{4\pi\, |\vec{R_{12}}|^2}.$$

$$\therefore \boxed{d(d\vec{F_2}) = \mu_0 \frac{I_1 I_2}{4\pi\, |\vec{R_{12}}|^2} \cdot d\vec{L_2} \times (d\vec{L_1} \times \vec{a_{R_{12}}})} \rightarrow (13)$$

$d(d\vec{F_1}) \neq d(d\vec{F_2})$ because of the non physical nature of the current element.

From the differential force, we can get

Total force between two filamentary circuits

$$\boxed{\vec{F_2} = \frac{\mu_0 I_1 I_2}{4\pi} \oint \left[d\vec{L_2} \times \oint \frac{d\vec{L_1} \times \vec{a_{R_{12}}}}{|\vec{R_{12}}|^2} \right]}$$

(or)

$$\boxed{\vec{F_2} = \frac{\mu_0 I_1 I_2}{4\pi} \oint \left[\oint \frac{\vec{a_{R_{12}}} \times d\vec{L_1}}{|\vec{R_{12}}|^2} \right] \times d\vec{L_2}}$$

8.a) Probem:

8	a.	A square loop of wire in z = 0 plane carrying 2mA in the field of an infinite filament on the y-axis as shown in the Fig.Q8(a). Find the total force on the loop.

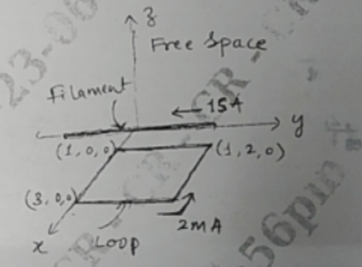


Fig.Q8(a)

(3,2,0)

Magnetic field due to infinite current element along y-axis

$$\vec{H} = \frac{I}{2\pi x}\vec{a_z} \qquad \boxed{\vec{H} = \frac{15}{2\pi x}\vec{a_z}} \text{ A/m}$$

∴ Flux density $\vec{B} = \mu_0 \vec{H} = \frac{4\pi\times10^{-7}\times 15}{2\pi x}\vec{a_z}$

$$\boxed{\vec{B} = \frac{3\times10^{-6}}{x}\vec{a_z}} \text{ T}$$

Force on the loop, $\boxed{\vec{F} = -I\oint_L \vec{B}\times d\vec{L}}$

(124)

Total force = sum of the forces on four sides

$$\vec{F} = -2\times10^{-3}\left[\oint_L \vec{B}\times d\vec{L}\right]$$

$$= -2\times10^{-3}\times 3\times10^{-6}\left[\int_{x=1}^{3}\frac{\vec{a_z}}{x}\times dx\,\vec{a_x} + \int_{y=0}^{2}\frac{\vec{a_z}}{x}\times dy\,\vec{a_y}\Big|_{x=3} + \int_{x=3}^{1}\frac{\vec{a_z}}{x}\times dx\,\vec{a_x} + \int_{y=2}^{0}\frac{\vec{a_z}}{x}\times dy\,\vec{a_y}\Big|_{x=1}\right]$$

$$= -2\times10^{-3}\times 3\times10^{-6}\left[-\vec{a_x}\left(\frac{1}{3}\right)(2-0) - \vec{a_x}\left(\frac{1}{1}\right)(0-2)\right]$$

$$= -2\times10^{-3}\times 3\times10^{-6}\times\frac{4}{3}\vec{a_x}$$

$\boxed{\vec{F} = -8\,\vec{a_x}}$ n N → The net force on the loop is in $-\vec{a_x}$ direction

8.b) Magnetic boundary conditions:

Magnetic boundary conditions:

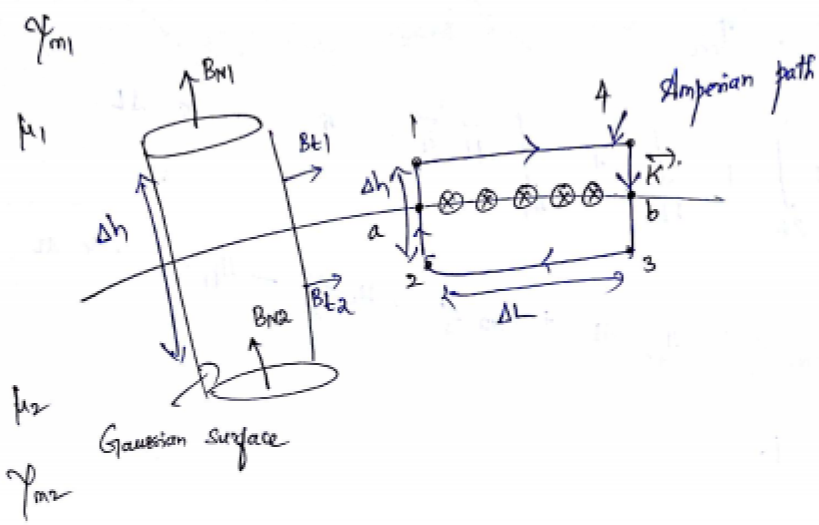
$\vec{B_1} = \vec{B_{t1}} + \vec{B_{N1}}$

$\vec{B_2} = \vec{B_{t2}} + \vec{B_{N2}}$

$$\boxed{\begin{gathered}\vec{B} = \mu \vec{H} \\ \vec{M} = \chi_m \vec{H}\end{gathered}}$$

$\vec{H_1} = \vec{H_{t1}} + \vec{H_{N1}}$

$\vec{H_2} = \vec{H_{t2}} + \vec{H_{N2}}$



① Gauss's law for Magnetic fields:

$$\phi = \oint_S \vec{B} \cdot \vec{ds} = 0$$

$$\Rightarrow \int_{top} + \int_{bottom} + \int_{lateral} \vec{B} \cdot \vec{ds} = 0$$

$$\Rightarrow B_{N1} \cdot \Delta S - B_{N2} \cdot \Delta S + B_{t_1} \cancelto{0}{\frac{\Delta h}{2}} 2\pi r + B_{t2} \cancelto{0}{\frac{\Delta h}{2}} 2\pi r = 0$$

We are obtaining conditions at the boundary.

$\therefore \Delta h \longrightarrow 0.$

$$B_{N1} \cdot \Delta S - B_{N2} \Delta S = 0$$

$$\boxed{B_{N1} = B_{N2}} \Rightarrow \mu_1 H_{N1} = \mu_2 H_{N2}$$

$$\Rightarrow \boxed{H_{N1} = \frac{\mu_2}{\mu_1} H_{N2}}$$

$$\frac{M_{N1}}{\chi_{m1}} = \frac{\mu_2}{\mu_1} \frac{M_{N2}}{\chi_{m2}} \Rightarrow \boxed{M_{N1} = \frac{\chi_{m1}}{\chi_{m2}} \cdot \frac{\mu_2}{\mu_1} M_{N2}}$$

② Ampere's circuital Law:

$$\oint_L \vec{H} \cdot d\vec{L} = I_{enc}$$

$$\int_{14} + \int_{4b} + \int_{b3} + \int_{32} + \int_{2a} + \int_{a1} \vec{H} \cdot d\vec{L} = K \cdot \Delta L$$

$$H_{t1} \cdot \Delta L + \left(-H_{N1} \cancelto{0}{\frac{\Delta h}{2}}\right) + \left(-H_{N2} \cdot \cancelto{0}{\frac{\Delta h}{2}}\right) - H_{t2}\, \Delta L + H_{N2} \cancelto{0}{\frac{\Delta h}{2}} + H_{t1} \cancelto{0}{\frac{\Delta h}{2}} = K \cdot \Delta L$$

At the boundary $\Delta h \longrightarrow 0$

$$(H_{t1} - H_{t2}) \cdot \Delta L = K \cdot \Delta L$$

$$\boxed{H_{t1} - H_{t2} = K} \Rightarrow \boxed{\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = K}$$

$$\Downarrow$$

$$\frac{M_{t1}}{\chi_{m1}} - \frac{M_{t2}}{\chi_{m2}} = K \Rightarrow \boxed{M_{t2} = \frac{M_{t1}}{\chi_{m1}} \cdot \chi_{m2} - K \chi_{m2}}$$

The directions are specified more exactly by using the cross product to identify the tangential components,

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{N12} = \mathbf{K}$$

where $\mathbf{a}_{N12}$ is the unit normal at the boundary directed from region 1 to region 2. An equivalent formulation in terms of the vector tangential components may be more convenient for $\mathbf{H}$:

$$\mathbf{H}_{t1} - \mathbf{H}_{t2} = \mathbf{a}_{N12} \times \mathbf{K}$$

8.c) Compare electric and magnetic circuits:

Electric Circuit	Magnetic Circuit
(Electric circuit)	
$\vec{E} = -\vec{\nabla} V$	$\vec{H} = -\vec{\nabla} V_m$
$V_{a,b} = -\int_a^b \vec{E} \cdot \vec{dL}$	$V_{m\,a,b} = \int_a^b \vec{H} \cdot \vec{dL}$
$\vec{D} = \varepsilon \vec{E}$, $\vec{J} = \sigma \vec{E}$	$\vec{B} = \mu \vec{H}$
$I = \int_S \vec{J} \cdot \vec{ds}$	$\phi = \int_S \vec{B} \cdot \vec{ds}$
Resistance	Reluctance
$V = I R$	$V_m = \phi \mathcal{R}$
Conductance	Permeance
$G = \frac{1}{R}$	$\mathcal{P} = \frac{1}{\mathcal{R}}$
$R = \frac{l}{\sigma S}$	$\boxed{\mathcal{R} = \frac{l}{\mu S}}$

9.a) Inconsistency of current continuity equation and Modified Ampere's law:

Point form of Ampere's circuital law for steady magnetic fields

$$\boxed{\nabla \times \vec{H} = \vec{J}} \quad (1) \leftarrow \text{inadequate for time varying conditions.}$$

Taking divergence,

$$\nabla \cdot \nabla \times \vec{H} = \nabla \cdot \vec{J} = 0.$$

But from continuity equation

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \neq 0} \quad \text{for time varying conditions}$$

Suppose we add unknown $\vec{G}$ to (1)

$$\Rightarrow \boxed{\nabla \times \vec{H} = \vec{J} + \vec{G}}$$

Taking divergence,

$$\nabla \cdot \nabla \times \vec{H} = \nabla \cdot \vec{J} + \nabla \cdot \vec{G}.$$

$$0 = \nabla \cdot \vec{J} + \nabla \cdot \vec{G}$$

$$\Rightarrow \boxed{\nabla \cdot \vec{G} = \frac{\partial \rho_v}{\partial t}}$$

w.k.t $\nabla \cdot \vec{D} = \rho_v$.

$$\nabla \cdot \vec{G} = \frac{\partial \, \nabla \cdot \vec{D}}{\partial t}$$

The solution for $\vec{G}$ is obtained as

$$\vec{G} = \frac{\partial \vec{D}}{\partial t}$$

$\therefore$ Ampere's circuital law in point form.

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

$\frac{\partial \vec{D}}{\partial t} \rightarrow$ has the dimensions of current density, A/m^2.

it results from time varying displacement flux density

Maxwell termed it as displacement current density

$$\boxed{\nabla \times \vec{H} = \vec{J} + \vec{J_d}}$$

$$\boxed{\vec{J_d} = \frac{\partial \vec{D}}{\partial t}}$$

We have already met other two current densities

Conduction current density (motion of charge in the region of zero net charge density) $\boxed{\vec{J} = \sigma \vec{E}}$

Convection current density (motion of volume charge density) $\boxed{\vec{J} = \rho_v \vec{v}}$

In a non conducting medium, where $\rho_v = 0 \Rightarrow \vec{J} = 0$,

$$\boxed{\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}} \quad \text{if } \vec{J} = 0$$

Total displacement current crossing any given surface,

$$\boxed{I_d = \int_S \vec{J_d} \cdot \vec{ds} = \int_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}}$$

A.C.L. for Time varying conditions:

$$\int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_S \vec{J} \cdot \vec{ds} + \int_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$$

Apply stokes' theorem

$$\oint_L \vec{H} \cdot \vec{dL} = I + I_d = I + \int_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$$

9.b) General wave equation:

General wave equation (for the media considered) of UPW

Maxwell's equations :

$$\vec{\nabla}\cdot\vec{D}=0 \Longrightarrow \vec{\nabla}\cdot\vec{E}=0 \rightarrow (1)$$

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} \Longrightarrow \vec{\nabla}\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} \rightarrow (2)$$

$$\vec{\nabla}\cdot\vec{B}=0 \Longrightarrow \vec{\nabla}\cdot\vec{H}=0 \rightarrow (3)$$

$$\vec{\nabla}\times\vec{H} = \sigma\vec{E}+\varepsilon\frac{\partial\vec{E}}{\partial t} \Longrightarrow \vec{\nabla}\times\vec{H} = \sigma\vec{E}+\varepsilon\frac{\partial\vec{E}}{\partial t} \rightarrow (4)$$

Take curl on both sides of (2)

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\vec{\nabla}\times\left(\mu\frac{\partial\vec{H}}{\partial t}\right).$$

Vector identity $\rightarrow$ $\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E}$

$$\vec{\nabla}(\cancel{\vec{\nabla}\cdot\vec{E}}) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H}).$$

Apply equation (1) Apply equation (4)

$$-\nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}\left[\sigma\vec{E}+\varepsilon\frac{\partial\vec{E}}{\partial t}\right]$$

$$-\nabla^2\vec{E} = -\mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\nabla^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

For UPW $\vec{E}(z,t)$ and $\vec{H}(z,t)$

$\nabla^2\vec{E} = \frac{\partial^2\vec{E}}{\partial z^2}$

$$\boxed{\frac{\partial^2\vec{E}}{\partial z^2} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}} \rightarrow (5)$$

General wave equation for $\vec{E}$ field of UPW.

Similarly,

Take curl on both sides of (4)

$$\vec{\nabla} \times \vec{\nabla} \times \vec{H} = \vec{\nabla} \times \left(\sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{H}) - \nabla^2 \vec{H} = \sigma (\vec{\nabla} \times \vec{E}) + \varepsilon \frac{\partial (\vec{\nabla} \times \vec{E})}{\partial t}$$

applying equation (3) applying equation (2)

$$-\nabla^2 \vec{H} = -\sigma \cdot \mu \frac{\partial \vec{H}}{\partial t} - \varepsilon \mu \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{H} = \mu \sigma \frac{\partial \vec{H}}{\partial t} + \mu \varepsilon \frac{\partial^2 \vec{H}}{\partial t^2}}$$

and $\vec{H}(z,t)$

$$\nabla^2 \vec{H} = \frac{\partial^2 \vec{H}}{\partial z^2}$$

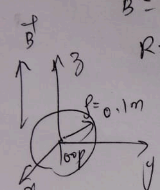
$$\boxed{\frac{\partial^2 \vec{H}}{\partial z^2} = \mu \sigma \frac{\partial \vec{H}}{\partial t} + \mu \varepsilon \frac{\partial^2 \vec{H}}{\partial t^2}} \rightarrow (6)$$

$\Rightarrow$ General wave equation for $\vec{H}$ field of UPW.

Similar equations can be written for **D** and **B**

9.c) Problem:

9)c) Problem:



$$\vec{B} = 0.2 \sin 10^3 t \ \hat{a}_z \ T$$

$R = 5\Omega$

$$d\vec{s} = \rho \, d\rho \, d\phi \ \hat{a}_z$$

$$\vec{B} \cdot d\vec{s} = 0.2 \sin 10^3 t \ \rho \, d\rho \, d\phi$$

$$\phi = \iint_s \vec{B} \cdot d\vec{s} = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{0.1} \vec{B} \cdot d\vec{s}$$

$$\phi = 6.283 \sin 10^3 t \quad mWb$$

$$emf = -\frac{d\phi}{dt} = -6.283 \cos 10^3 t$$

$$i = \frac{emf}{R} = \frac{-6.283 \cos 10^3 t}{5} = -1.256 \cos 10^3 t \ A$$

$$\boxed{i = -1.256 \cos 10^3 t \ A}$$

10.a) Maxwell's equations for static fields:

Static Fields:

List the Maxwell's Equations in Integral form and differential (point) form: Four Equations

	Integral form	Point form
Faraday's law	$\oint_L \vec{E}\cdot d\vec{l} = 0$	$\vec{\nabla}\times\vec{E} = 0$
Ampere's circuital Law	$\oint_L \vec{H}\cdot d\vec{l} = \iint_S \sigma\vec{E}\cdot d\vec{s}$ $\oint_L \vec{H}\cdot d\vec{l} = \iint_S \vec{J}\cdot d\vec{s}$	$\vec{\nabla}\times\vec{H} = \sigma\vec{E}$ $\vec{\nabla}\times\vec{H} = \vec{J}$
Gauss's law ($\vec{E}$- field)	$\oint_S \vec{D}\cdot d\vec{s} = Q_{enc} = \iiint_V \rho_v\, dv$	$\vec{\nabla}\cdot\vec{D} = \rho_v$
Gauss's law ($\vec{H}$- field)	$\oint_S \vec{B}\cdot d\vec{s} = 0$	$\vec{\nabla}\cdot\vec{B} = 0$

10.b) Problem:

10) b) $f = 9375\ MHz$

$E_x = 20\ V/m$

$\mu_r = 1$

$\epsilon_r = 2.56$

$\mu = \mu_0\mu_r = 4\pi\times10^{-7}\times 1$

$\epsilon = \epsilon_0\epsilon_r = 8.854\times10^{-12}\times 2.56$

$\omega = 2\pi f$

1) $\alpha = 0$

2) $\beta = \omega\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r} = 314.37\ rad/m$

3) $\lambda = \frac{2\pi}{\beta} = 0.0199\ m$

4) $v = f\lambda = 1.873\times10^{8}\ m/s$

5) $\eta = \sqrt{\mu/\epsilon} = 235.45\ \Omega$

6) $\gamma = \alpha + j\beta = j\,314.37\ /m$

7) $H_y = \frac{E_x}{\eta} = \frac{20}{235.45} = 0.08495\ A/m$

10.c) Poynting's theorem:

Poynting's theorem and Wave Power:

Poynting's theorem states that the net power flowing out of a given volume is equal to the time rate of decrease in stored energy within the volume minus conduction losses (Ohmic losses)

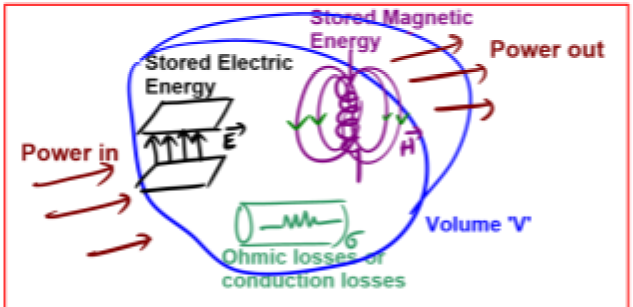


Fig. Illustration of Power Balance for EM fields

Proof:

Maxwell's Equations:

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \rightarrow (1) \qquad \nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \rightarrow (2) \qquad \nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon} \rightarrow (3) \qquad \nabla \cdot \vec{H} = 0 \rightarrow (4)$$

Vector Identity:

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H})$$

From (1) From (2)

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot \left(-\mu \frac{\partial \vec{H}}{\partial t}\right) - \vec{E} \cdot \left(\sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla \cdot (\vec{E} \times \vec{H}) = -\mu \, \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} - \sigma \vec{E} \cdot \vec{E} - \varepsilon \, \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

We can also write, $\frac{\partial |\vec{H}|^2}{\partial t} = 2 \vec{H} \cdot \frac{\partial \vec{H}}{\partial t}$; $\frac{\partial |\vec{E}|^2}{\partial t} = 2 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$ Using these expressions, the above equation can be written as follows.

$$\nabla \cdot (\vec{E} \times \vec{H}) = -\mu \left(\frac{1}{2} \frac{\partial H^2}{\partial t}\right) - \sigma E^2 - \varepsilon \left(\frac{1}{2} \frac{\partial E^2}{\partial t}\right)$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2 + \frac{1}{2}\epsilon E^2\right] - \sigma E^2$$

Differential form or Point form of Poynting's theorem

Integrating the above expression over the given volume "V" as depicted in the figure, provides the Integral form of Poynting's theorem.

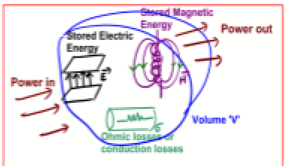


Fig. Illustration of Power Balance for EM fields

$$\iiint_V \vec{\nabla}\cdot(\vec{E}\times\vec{H})\, dv = -\frac{d}{dt}\left[\iiint_V \left(\frac{1}{2}\mu H^2 + \frac{1}{2}\epsilon E^2\right) dv\right] - \iiint_V \sigma E^2\, dv$$

Using Divergence Theorem for the L.H.S.,

$$\iiint_V \vec{\nabla}\cdot\vec{D}\, dv = \oint_S \vec{D}\cdot\vec{ds}$$

L.H.S. can be wrtitten as follows:

$$\iiint_V \vec{\nabla}\cdot(\vec{E}\times\vec{H})\, dv = \oint_S (\vec{E}\times\vec{H})\cdot\vec{ds}$$

$$\oint_S (\vec{E}\times\vec{H})\cdot\vec{ds} = -\frac{d}{dt}\left[\frac{1}{2}\iiint_V \mu H^2\, dv + \frac{1}{2}\iiint_V \epsilon E^2\, dv\right] - \iiint_V \sigma E^2\, dv$$

Integral form of Poynting's theorem

Stored Magnetic Energy within the volume 'V'

Stored Electric Energy within the volume 'V'

Net power flowing out of the given Volme 'V'

Rate of decrease in stored energy within the Volume 'V'

Conduction losses or Ohmic Losses

Poynting's theorem and Wave Power:

Poynting's theorem states that the net power flowing out of a given volume is equal to the time rate of decrease in stored energy within the volume minus conduction losses (Ohmic losses)

Using Phasor Forms:

$$\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}(\mathbf{E}_s \times \mathbf{H}_s^*) \quad \mathrm{W/m^2}$$

Power of an EM wave, $P = \oint_S (\vec{E}\times\vec{H})\cdot\vec{ds}$ W

Power Density vector = Poynting's Vector
(Power per unit Area) $\vec{S} = \vec{P} = \vec{E}\times\vec{H}$ (W/m^2)

Instantaneous Power Density Vector, $\vec{P} = \vec{E}\times\vec{H}$ W/m^2

Active Power Density Vector or Real Power Density Vector, $\vec{P}_{avg} = \vec{P}_{real} = \frac{1}{2}\mathrm{Re}\left\{\vec{E}\times\vec{H}^*\right\}$ W/m^2

Reactive Power Density Vector, $\vec{P}_{reactive} = \frac{1}{2}\mathrm{Im}\left\{\vec{E}\times\vec{H}^*\right\}$ W/m^2

Instantaneous Power Density = Real Power Density + Reactive Power Density