CBCS SCHEME

USN 1 C R 2 2 E C 2 0 8

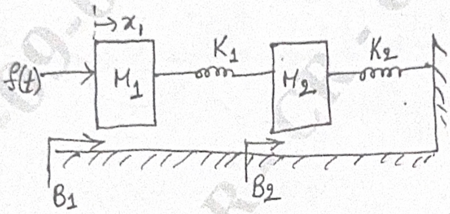
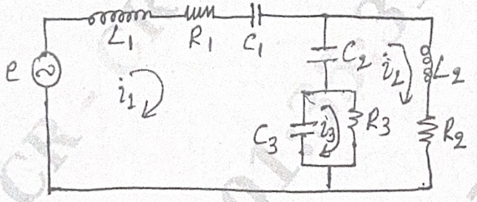
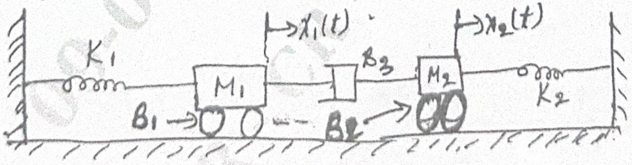
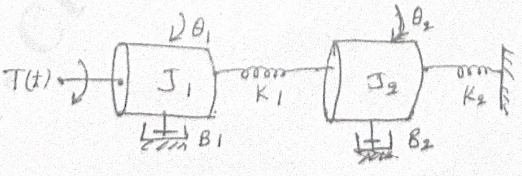
BEC403

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025

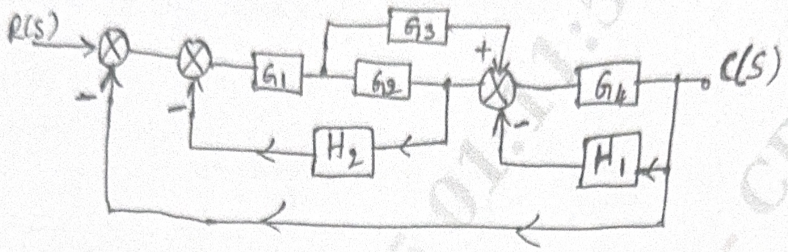
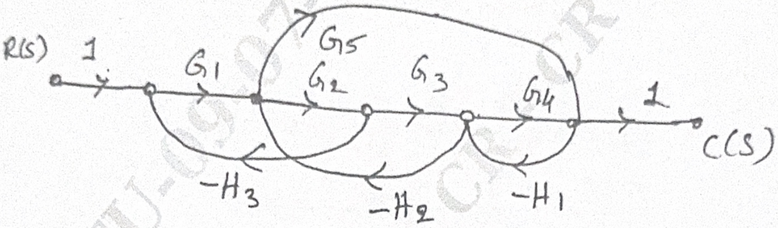
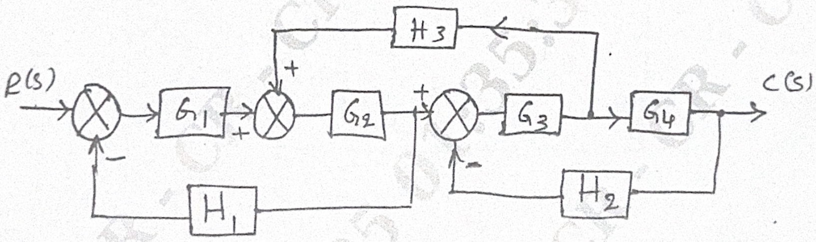
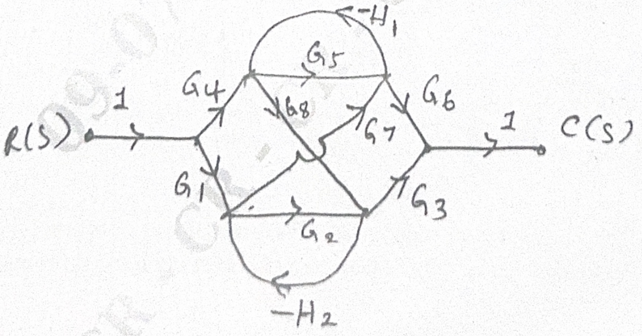
Control Systems

Time: 3 hrs. Max. Marks: 100

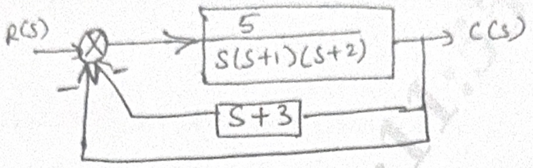
Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define control system with examples. Compare closed loop and open loop control systems.	06	L1 L2 L3	CO1
	b.	For the mechanical system shown in Fig.Q1(b), write the mechanical network, equilibrium equations and obtain the electrical network based on F-V analogy.  Fig.Q1(b)	08	L1 L2 L3	CO1
	c.	The force-voltage analogy of a mechanical system is shown in Fig.Q1(c). Obtain its analogous mechanical network.  Fig.Q1(c)	06	L1 L2 L3	CO1
		OR			
Q.2	a.	Explain the effect of feedback on control systems.	06	L1 L2 L3	CO1
	b.	Find the force-voltage analogous electrical network for the given mechanical system shown in Fig.Q2(b).  Fig.Q2(b)	06	L1 L2 L3	CO1
	c.	Derive the differential equation governing the mechanical rotational system shown in Fig.Q2(c). Draw the equivalent voltage and current analogy circuits.  Fig.Q2(c)	08	L1 L2 L3	CO1

Module – 2

Q.3	a.	Determine the transfer function C(S)/R(S) for the system shown in Fig.Q3(a), using block diagram reduction technique.  Fig.Q3(a)	10	L1 L2 L3	CO2
	b.	Determine the overall transfer function using Mason's gain formula for the signal flow graph shown in Fig.Q3(b).  Fig.Q3(b)	10	L1 L2 L3	CO2
		OR			
Q.4	a.	Find the transfer function by reducing the block diagram shown in Fig.Q4(a).  Fig.Q4(a)	10	L1 L2 L3	CO3
	b.	Find the transfer function by using Mason's gain formula for the signal flow graph shown in Fig.Q4(b).  Fig.Q4(b)	10	L1 L2 L3	CO2

Module – 3

Q.5	a.	For the system shown in Fig.Q5(a), find the (i) System type (ii) Static error constants K_p, K_v, K_a (iii) the steady state error for an input $r(t) = 3 + 2t$.  Fig.Q5(a)	08	L1 L2 L3	CO3
	b.	Find the step response c(t) for the system described by $\frac{C(s)}{R(s)} = \frac{4}{s+4}$ Also find time constant, rise time and settling time.	05	L1 L2 L3	CO3
	c.	Derive the equation steady state error of simple closed loop system.	07	L1 L2 L3	CO3

OR

Q.6	a.	Given a unity feedback system with $G(s) = \frac{20(1+s)}{s^2(2+s)(4+s)}$ (i) What is the type of system? (ii) Find static error coefficients. (iii) Find steady error if the input is $r(t) = 40 + 2t + 5t^2$	06	L1 L2 L3	CO3
	b.	Write the general block diagram of the following and explain : (i) PD type of controller (ii) PI type of controller	06	L1 L2 L3	CO3
	c.	Derive the response of an under damped second order system for unit step input.	08	L1 L2 L3	CO3

Module – 4

Q.7	a.	Mention limitations of Routh's criterion.	04	L1 L2 L3	CO4
	b.	Determine the range of K for which the system is stable such that a unity feedback system has $G(s) = \frac{K(s+13)}{s(s+3)(s+7)}$ using RH criterion. Also find closed loop, poles more negative than – 1.	08	L1 L2 L3	CO4
	c.	Check the stability of the given characteristic equation using Routh's method. $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$	08	L1 L2 L3	CO4

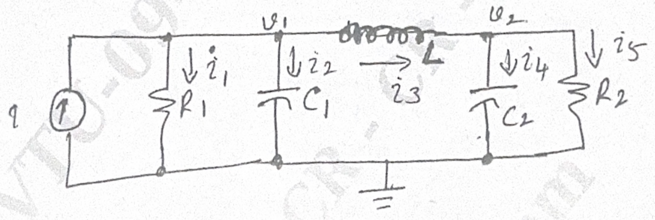
OR

Q.8	a.	Sketch the complete Root locus of system having $G(s)\,H(s) = \frac{K}{s(s+5)(s+10)}$	08	L1 L2 L3	CO4
	b.	Sketch the complete Root locus of system having $G(s)\,H(s) = \frac{K}{s(s+1)(s+2)(s+3)}$	12	L1 L2 L3	CO4

Module – 5

Q.9	a.	Draw the Bode plot for the open loop transfer function of a system is $G(s) = \dfrac{K(1+0.2s)(1+0.025s)}{s^3(1+0.001s)(1+0.005s)}$ Determine that the system is conditionally stable. Find the range of K for which the system is stable.	10	L1 L2 L3	CO5
	b.	The transfer function of a system is $G(s)\,H(s) = \dfrac{K}{s(s+2)(s+10)}$ Sketch the Nyquist plot and hence calculate the range of values of K for stability.	10	L1 L2 L3	CO5

OR

Q.10	a.	Obtain the state model of the network shown in Fig.Q10(a) assuming $R_1 = R_2 = 1\ \Omega$, $C_1 = C_2 = 1F$, and $L = 1H$.  Fig.Q10(a)	10	L1 L2 L3	CO5
	b.	Obtain the state transition matrix for the state model whose A matrix is given by $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$	10	L1 L2 L3	CO5

* * * * *

Control Systems

BEC403

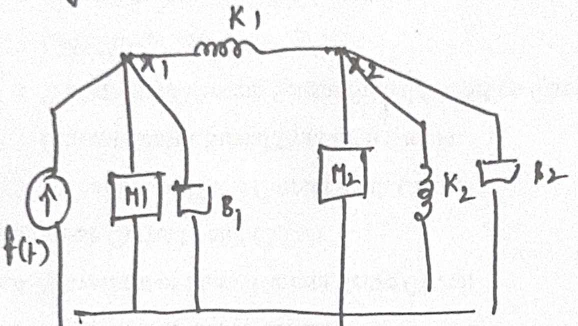
VTU — June / July 2025 Solutions

Q.1 a) Definition — 1 Mark

Example — 1 Mark

Any 4 Comparison between closed loop and open loop Control Systems — 4 marks

b)



$$F(s) = M_1 s^2 x_1(s) + B_1 s x_1(s) + K\,(x_1(s) - x_2(s))$$

$$0 = K_1\,[x_2 - x_1] + M_2 \cdot \frac{d^2 x_2(t)}{dt^2} + K_2\, x_2(t) + B_2 \cdot \frac{dx_2(t)}{dt}$$

$$0 = K_1\,[x_2 - x_1] + M_2 s^2 x_2(s) + K_2 X_2(s) + B_2 \cdot s\, x_2(s)$$

2M

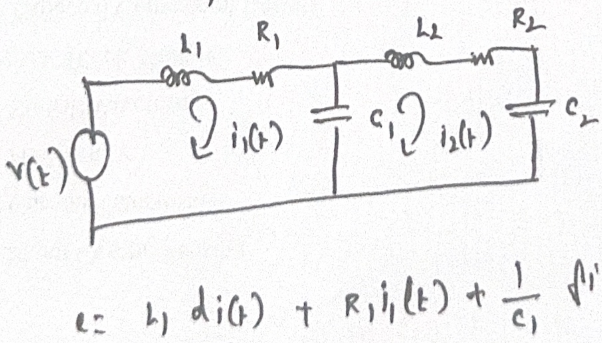
For force - voltage analogy

$F \rightarrow V \quad M \rightarrow L \quad B \rightarrow R \quad K - \frac{1}{C}$ ⟶ ① Mark

$$v(t) = L \cdot \frac{di_1(t)}{dt} + R_1\, i_1(t) + \frac{1}{C_1} \int (i_1 - i_2)\, dt$$

$$0 = \frac{1}{C_1} \int (i_2 - i_1)\, dt + L_2 \frac{di_2}{dt} + \frac{1}{C_2} \int i_2\, dt + R_2 i_2$$

2Marks



2M

c)

$$e = L_1 \frac{di(t)}{dt} + R_1 i_1(t) + \frac{1}{C_1} \int i_1\, dt + \frac{1}{C_2} \int (i_1 - i_2)\, dt + \frac{1}{C_3} \int (i_1 - i_3)\, dt$$

$$0 = R_3\,(i_3 - i_2) + \frac{1}{C_3} \int (i_3 - i_1)\, dt$$

$$0 = L_2 \cdot \frac{di_2(t)}{dt} + R_2 i_2 + R_3\,(i_2 - i_3) + \frac{1}{C_2} \int (i_2 - i_1)\, dt$$

②M

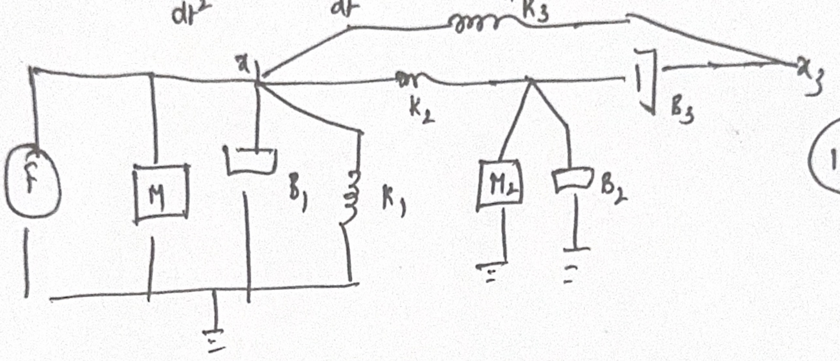
For F-V analogy $M \rightarrow L$, $B \rightarrow R$ $K \rightarrow \frac{1}{C}$ $x \rightarrow q$ $\frac{dq}{dt} \rightarrow i$

$$F = M_1 \frac{d^2 x_1(t)}{dt^2} + B_1 \frac{dx_1(t)}{dt} + K_1 x_1(t) + K_2 (x_1(t) - x_2(t)) + K_3 (x_1(t) - x_3(t))$$

$$0 = B_3 \cdot \frac{d}{dt} (x_3 - x_2) + K_3 (x_3 - x_1)$$

2 Marks

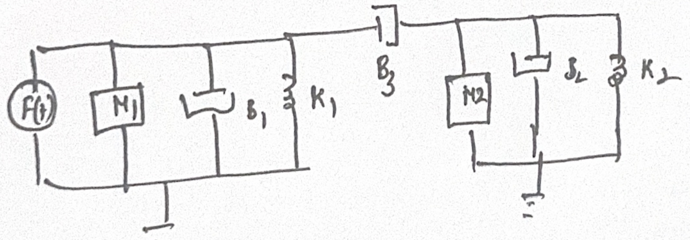
$$0 = M_2 \cdot \frac{d^2 x_2}{dt^2} + B_2 \cdot \frac{dx_2}{dt} + B_3 \cdot \frac{d}{dt} (x_2 - x_3) + k_3 (x_2 - x_1)$$



1 Mark

2. a) Need to write 2 points on stability, over all gain and on sensitivity ⟶ 2M each — 6M.

b) Mechanical network — 1M



$$F(t) = M_1 \frac{d^2 x_1(t)}{dt^2} + B_1 \frac{dx_1(t)}{dt} + K_1 x_1 + B_3 \left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt} \right)$$

$$F(s) = M_1 s^2 X_1(s) + B_1 s X_1(s) + K_1 X(s) + B_3 s [X_1(s) - X_2(s)]$$

$$0 = B_3 \left(\frac{d}{dt} x_2 - \frac{d}{dt} x_1 \right) + M_1 \frac{d^2 x_2}{dt^2} + K_2 x_2 + B_2 \frac{dx_2}{dt}$$

$$0 = M_2 s^2 X_2(s) + B_2 s X_2(s) + B_3 s [X_2(s) - X_1(s)] + K_2 X_2(s)$$

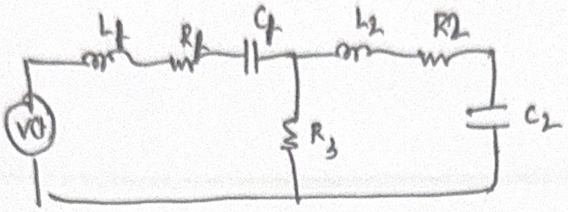
2Ⓜ

$F \rightarrow V$ $M \rightarrow L$ $B \rightarrow R$ $K \rightarrow \frac{1}{C}$

$$V(t) = L_1 \frac{di(t)}{dt} + R_1 i_1(t) + \frac{1}{C_1} \int i_1(t) + R_3 (i_1 - i_2)$$

$$0 = L_2 \cdot \frac{di_2(t)}{dt} + R\ i_2 + R_3 (i_2 - i_1) + \frac{1}{C} \int i_2(t) dt$$

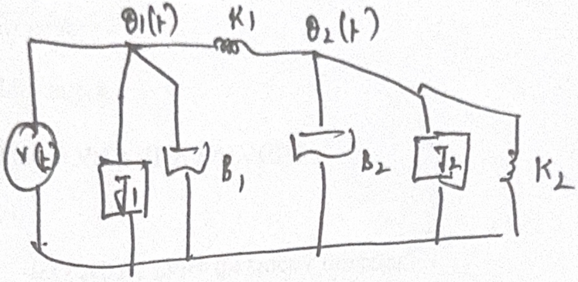
2M



(1M)

(2,1,3)

2c) Mechanical network



— 1 Mark

$$T(t) = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1(t)}{dt} + K_1(\theta_1 - \theta_2)$$

$$0 = J_2 \cdot \frac{d^2\theta_2}{dt^2} + B_2 \frac{d}{dt}\theta_2 + K_2\theta_2 + K_1(\theta_2 - \theta_1)$$

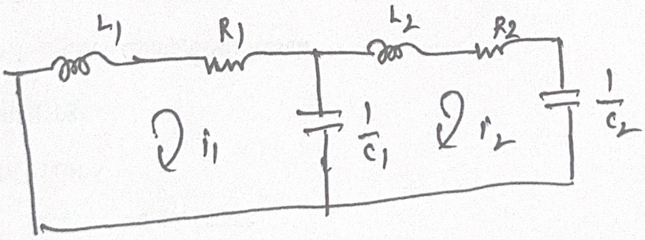
1 Mark

Voltage analogy

$$V(t) = L_1 \frac{di_1(t)}{dt} + R_1 i_1 + \frac{1}{C_1}\int (i_1 - i_2)\,dt$$

$$0 = L_2 \cdot \frac{di_2(t)}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + \frac{1}{C_1}\int (i_2 - i_1)\,dt$$

(2M)

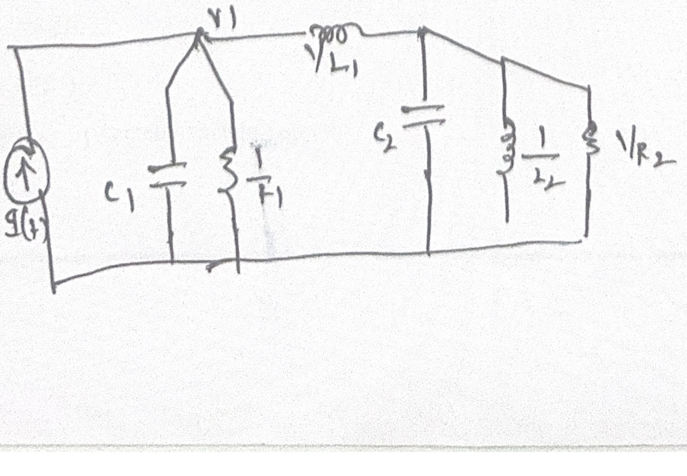


→ (1M)

Current analogy

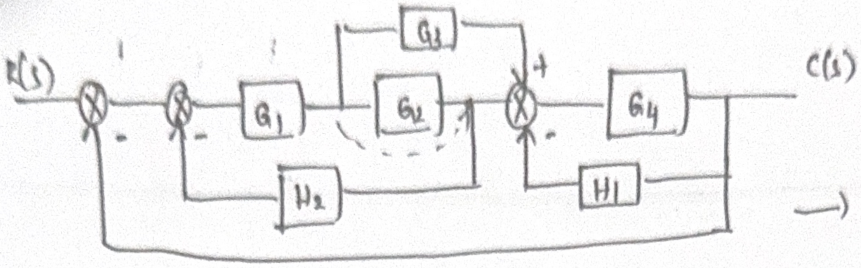
$$I(t) = C_1 \frac{d}{dt} v_1 + \frac{1}{R_1} v_1 + \frac{1}{L_1}\int (v_1 - v_2)\,dt$$

$$0 = C_2 \cdot \frac{d}{dt} v_2 + \frac{1}{L_2}\int v_2\,dt + \frac{1}{R_2} v_2 + \frac{1}{L_1}\int (v_2 - v_1)\,dt$$



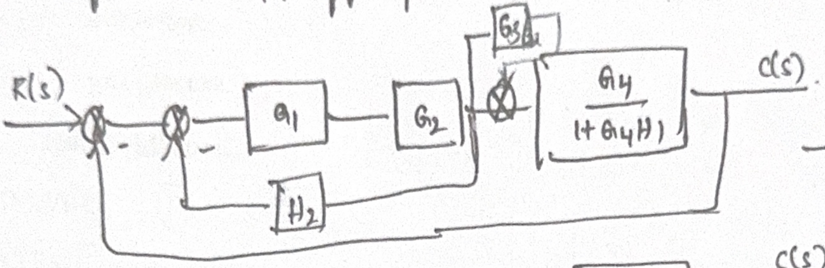
— (1M)

3 ⓐ

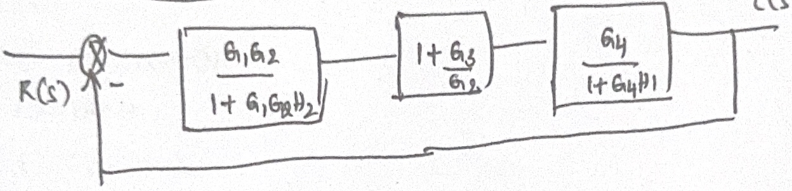


⟶ 2M

Shift the take off point after G_2 .



⟶ 2M



⟶ 2M

⟶ 4M

$$\frac{C(s)}{R(s)} = \frac{G_1G_4\left(G_2+G_3\right)}{1+G_1G_2H_2+G_4H_1+G_1G_2+H_1H_2+G_1G_4G_2+G_1G_4G_3}$$

3 b)

$K=2$ $\quad T_1 = G_1G_2G_3G_4$

$T_2 = G_1G_5$ $\Big\}$ 1M

Loops

$L_1 = -G_1G_2H_3 \quad L_2 = -G_2G_3H_2 \quad L_3 = -G_4H_1$

$L_4 = G_5H_1H_2$ $\Big\}$ 2M

Non touching loops — L_1 & L_3

$\Delta = 1-\left(L_1+L_2+L_3+L_4\right)+L_1L_3$ — 2M

$\Delta_1 = 1 \quad \Delta_2 = 1$ $\Big\}$ 2M

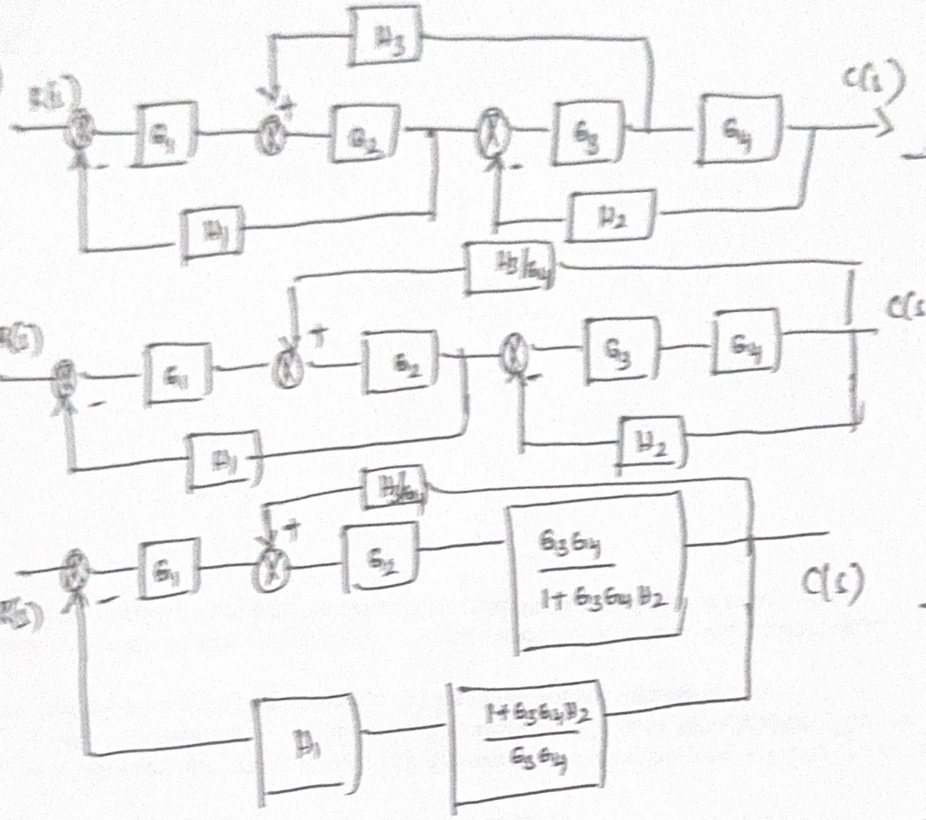
$$T.F = \frac{\Sigma T_K \Delta_K}{\Delta}$$

$$= \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta}$$

$$\boxed{T.F = \frac{G_1G_2G_3G_4 + G_1G_5}{1+G_1G_2H_8+G_2G_3H_2+G_4H_1-G_5H_1H_2+G_1G_2G_4H_1H_3}}$$

⟶ 3M

4.a)



→ 2M

→ 2M

→ 2M

→ 2M

$$\frac{C(s)}{R(s)} : \frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)+G_2G_3H_3}$$

4b) $P_1 : G_1G_2G_3$ $P_2 = G_4G_5G_6$ $P_3 = G_1G_7G_6$ $P_4 = G_4G_8G_3$

$P_5 : G_4G_8(-H_2)G_7G_6$ $P_6 = G_1G_7(-H_1)G_8G_3$ } 2M

Loops $L_1 = -G_5H_1$ $L_2 = -G_2H_2$ $L_3 = G_7H_1G_8H_2$ → 1M

Two non touching loops L_1L_2 : $G_2G_5H_1H_2$

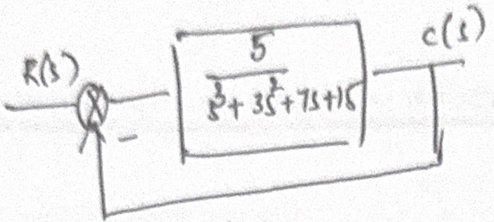
$\Delta = 1-(L_1+L_2+L_3)+L_1L_2$

$\Delta_1 = 1+G_5H_1$ $\Delta_2 = 1+G_2H_2$ $\Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$ } 4M

$$T.F : \frac{P_1\Delta_1+P_2\Delta_2+P_3\Delta_3+P_4\Delta_4+P_5\Delta_5+P_6\Delta_6}{\Delta.}$$ → 3M.

$$= \frac{G_1G_2G_3(1+G_5H_1)+G_4G_5G_6(1+G_2H_2)+G_1G_7G_6+G_4G_8G_3-G_4G_8G_7G_6H_2-G_1G_3G_7G_8H_1)}{\Delta.}$$

5a)



} 3M

i) type 0 system.

ii): $K_p = \underset{s\to 0}{Lt}\ G(s)H(s) = \underset{s\to 0}{Lt}\ \frac{5}{s^3+3s^2+7s+15} = \frac{5}{15} = \frac{1}{3}$ } 3M

$K_v = 0$

$K_a = 0$

iii) $e_{ss1} = \frac{3}{1+K_p} = \frac{3}{1+\frac{1}{3}} = 9/4 = 2.25$

$e_{ss2} = \frac{2}{0} = \infty$

$\boxed{e_{ss} = e_{ss1} + e_{ss2} = \infty}$ ⟶ 2M.

b) $\frac{C(s)}{R(s)} = \frac{4}{s+4}$ $R(s) = \frac{1}{s}$

$C(s) = \frac{4}{s+4} \cdot R(s)$

$= \frac{4}{s(s+4)} = \frac{1}{s} - \frac{1}{s+4}$ } 2M,

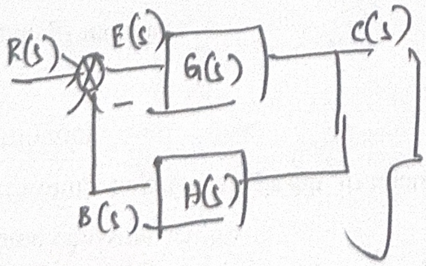
$c(t) = 1 - e^{-4t}$

i) Time Constant $T = \frac{1}{a} = \frac{1}{4} = 0.25$ sec } 3M

ii) Rise time $t_r = \frac{2.2}{a} = 0.55$ sec

iii) settling time $t_s = \frac{4}{a} = 1$ sec

c)



} 2M

$E(s) = R(s) - B(s)$

$= R(s) - H(s) \cdot C(s)$

$= R(s) - H(s) \cdot E(s) \cdot G(s)$ } 2M

$E(s) + E(s)H(s)G(s) = R(s)$

$E(s) = \frac{R(s)}{1+G(s)H(s)}$ } 2M

$\boxed{e_{ss}(t) = \underset{s\to 0}{Lt}\ \frac{s \cdot R(s)}{1+G(s)H(s)}}$ 1M

6a	Given a unity feedback system with $G(s) = \frac{20(1+s)}{s^2(2+s)(4+s)}$ (i) What is the type of system? (ii) Find static error coefficients. (iii) Find steady error if the input is $r(t) = 40 + 2t + 5t^2$
Sol:	[6-a] $G(s) = \frac{20(1+s)}{s^2(s+2)(4+s)}$ $= \frac{20(1+s)}{s^2(1+0.5s)(1+0.25s)}$. Hence the type of System is '2'. $K_p = \lim_{s\to 0} \frac{20(1+s)}{s^2(2+s)(4+s)} = \infty$ Ily $K_v = \lim_{s\to 0} \frac{20(1+s)}{s^2(2+s)(4+s)} = \infty$ & $K_a = \lim_{s\to 0} \frac{20(1+s)}{(2+s)(4+s)} = \frac{20}{2\cdot 4} = 2.5$ $r(t) = 40 + 2t + 5t^2 \equiv A_1 + A_2 t + \frac{A_3}{2} t^2.$ $\Rightarrow A_1 = 40,\ A_2 = 2,\ A_3 = 10$ $\therefore e_{ss} = e_{ss1} + e_{ss2} + e_{ss3}$ $= 0 + 0 + 4 = 4.$
6b	Write the general block diagram of the following and explain : (i) PD type of controller (ii) PI type of controller
Sol:	**Proportional + Derivative Controller (P-D Controller) :**

We have observed that the proportional controller is slow, as increasing K_p would reduce the damping and thereby produce overshoots. It could also tend to make the system unstable.

A derivative action could help increase the speed of the response.

The derivative action gives an output which is proportional to the rate of change of error.

$$m_d(t) = K_d \frac{d}{dt} e(t)$$

As the error initially is large, the derivative action gives a large value (A large current to the valve). As the error eventually becomes constant (offset), the derivative action becomes zero as $\frac{d}{dt}$ (constant) = 0.

When combined with a proportional controller, the derivative controller increases the proportional action initially (as the error is high) and decreases the proportional action to its normal value as error reduces and becomes constant.

The P-D controller hence increases the speed of the controller output. The mathematical expression for a P-D controller is given below,

$$m(t) = K_p e(t) + K_d \frac{d}{dt} e(t) + m_0$$

Computing the Laplace transform, we get .

$$M(s) = K_p E(s) + K_d s E(s) + M_0$$

It is important to realize that as the error becomes constant (offset), $K_d \frac{d}{dt} e(t) = 0$ and the above equation reduces to a simple proportional controller. Hence the P-D controller does not eliminate OFFSET. A block diagram of the P-D controller is shown in Fig. 12.2.11.

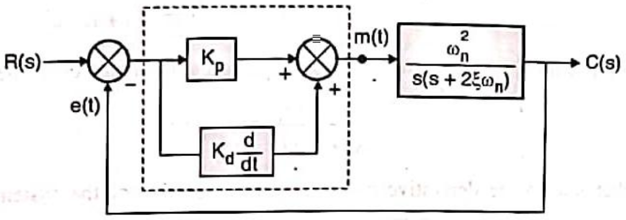


Fig. 12.2.11

A derivative action is equivalent to adding a zero to the transfer function in the Laplace domain (Since $L\left[\frac{d}{dt}\right] = s$). Therefore Fig. 12.2.11 can be redrawn as shown in Fig. 12.2.12.

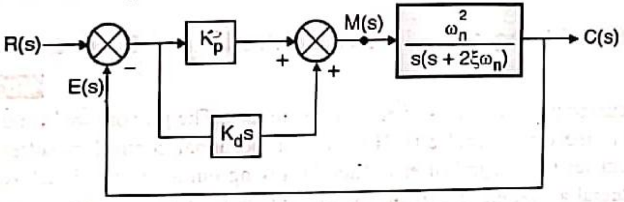


Fig. 12.2.12

Here $\frac{\omega_n^2}{s(s+2\xi\omega_n)}$ is a standard second order system. We now solve this to obtain the P-D transfer function.

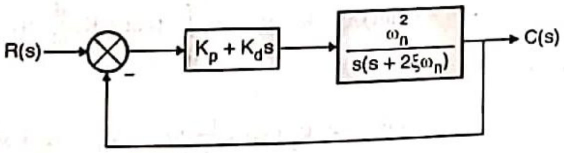


Fig. 12.2.13

We know $$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

$$= \frac{\dfrac{(K_p + K_d s)\,\omega_n^2}{s\,(s + 2\xi\,\omega_n)}}{1 + \dfrac{(K_p + K_d s)\,\omega_n^2}{s\,(s + 2\xi\,\omega_n)}}$$

$$\therefore \quad \frac{C(s)}{R(S)} = \frac{(K_p + K_d s)\,\omega_n^2}{s(s + 2\xi\omega_n) + (K_p + K_d s)\,\omega_n^2}$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{(K_p + K_d s)\,\omega_n^2}{s^2 + s\,(2\xi\omega_n + K_d\,\omega_n^2) + K_p\,\omega_n^2}$$

Hence the transfer function of a P-D controller is

$$\frac{C(s)}{R(s)} = \frac{(K_p + sK_d)\,\omega_n^2}{s^2 + 2\omega_n\left(\xi + \dfrac{K_d\,\omega_n}{2}\right) s + K_p\,\omega_n^2}$$

Proportional + Integral Controller (P-I Controller) :

Proportional controllers suffer from a drawback known as offset. One way of eliminating the offset is by incorporating an integral controller along with the proportional controller.

The integral action is given by the formula

$$m_i\,(t) = K_i \int e(t)\,dt + m(0)$$

m(0) is the controller output when the integral action starts.

As we know, the integral of a function basically calculates the area under the curve. Hence an integral controller gives out a signal which is the integral of the error signal e(t).

In the P-I controller, the proportional unit tries to correct the error by giving a signal proportional to the error but does not succeed completely. The integral unit of the P-I controller continues to give a control signal as long as the error (offset, in this case) exists. Larger the error, faster is the increase in the controller output. Hence as long as the offset exists, the integral action provides extra current to the control valve.

The integral action eventually removes the offset. Due to this property , the integral control action is also known as automatic reset.

The mathematical expression for a P-I controller is given below,

$$m(t) = K_p\,e(t) + K_i \int_0^t e(t)\,dt + m(0)$$

A block diagram of a P-I controller is shown in Fig. 12.2.7.

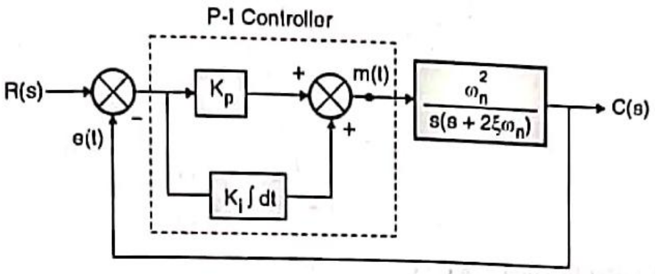


Fig. 12.2.7

Here, $\frac{\omega_n^2}{s\,(s\;+2\xi\omega_n)}$ is a standard second order system.

Comparing this with the block diagram of a proportional controller, we realize that an integral action is added.

An integral action is equivalent to adding a pole to the transfer function in the Laplace domain. (Since $L[\int dt] = \frac{1}{s}$).

∴ The block diagram can be redrawn as shown in Fig. 12.2.8.

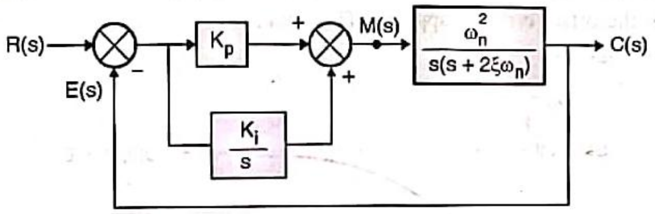


Fig. 12.2.8

$$m(t) \;=\; K_p\, e(t) + K_i \int e(t)\, dt + m_0$$

Computing the Laplace transform we get,

$$M(s) \;=\; K_p\, E(s) + \frac{K_i}{s} E(s) + M_0$$

$$\therefore \quad M(s) \;=\; \left[K_p + \frac{K_i}{s}\right] E(s) + M_0$$

We use block diagram reduction techniques on Fig. 12.2.8 to obtain Fig. 12.2.9.

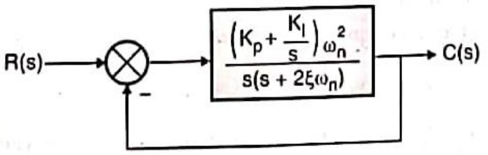


Fig. 12.2.9

We know

$$\frac{C(s)}{R(s)} \;=\; \frac{G(s)}{1+G(s)\,H(s)}$$

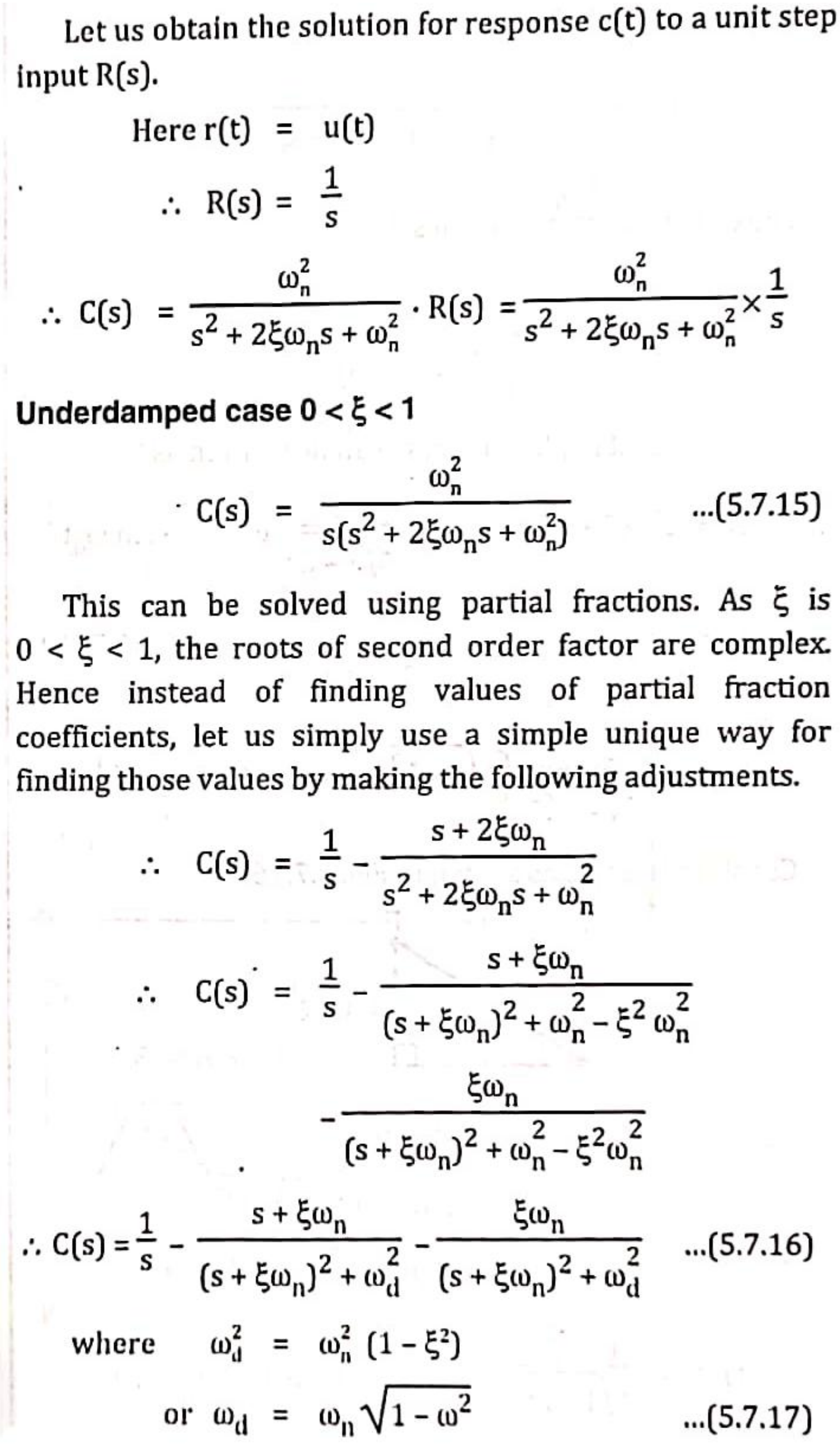
$$= \frac{\dfrac{\left(K_p + \frac{K_i}{s}\right)\omega_n^2}{s\,(s+2\xi\omega_n)}}{1 + \dfrac{\left(K_p + \frac{K_i}{s}\right)\omega_n^2}{s\,(s+2\xi\omega_n)}}$$

Hence the transfer function of a P-I controller is

$$\frac{C(s)}{R(s)} \;=\; \frac{(K_p\, s + K_i)\,\omega_n^2}{s^3 + 2\xi\omega_n s^2 + K_p\,\omega_n^2\, s + K_i\,\omega_n^2}$$

<table>
<tr><td></td><td></td></tr>
<tr><td>6c</td><td>Derive the response of an under damped second order system for unit step input.</td></tr>
<tr><td>Sol
:</td><td>

Derivation of Unit Step Response of a 2nd Order Underdamped System :



Let us obtain the solution for response c(t) to a unit step input R(s).

$$\text{Here } r(t) = u(t)$$

$$\therefore R(s) = \frac{1}{s}$$

$$\therefore C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \cdot R(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \times \frac{1}{s}$$

Underdamped case 0 < ξ < 1

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} \qquad \text{...(5.7.15)}$$

This can be solved using partial fractions. As ξ is $0 < \xi < 1$, the roots of second order factor are complex. Hence instead of finding values of partial fraction coefficients, let us simply use a simple unique way for finding those values by making the following adjustments.

$$\therefore C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\therefore C(s) = \frac{1}{s} - \frac{s + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_n^2 - \xi^2\omega_n^2} - \frac{\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_n^2 - \xi^2\omega_n^2}$$

$$\therefore C(s) = \frac{1}{s} - \frac{s + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} - \frac{\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} \qquad \text{...(5.7.16)}$$

$$\text{where } \omega_d^2 = \omega_n^2 (1 - \xi^2)$$

$$\text{or } \omega_d = \omega_n \sqrt{1 - \omega^2} \qquad \text{...(5.7.17)}$$

</td></tr>
</table>

ω_d is called the damped natural frequency and ω_n is called the undamped natural frequency.

From Laplace transforms we know that,

$$L^{-1}\left\{\frac{s+a}{(s+a)^2+d^2}\right\} = e^{-at}\cos dt$$

$$L^{-1}\left\{\frac{a}{(s+a)^2+d^2}\right\} = e^{-at}\sin dt \quad \text{and} \quad L^{-1}\left\{\frac{1}{s}\right\} = 1$$

Hence in our case we have,

$$L^{-1}\left\{\frac{s+\xi\omega_n}{\left(s+\xi\omega_n\right)^2+\omega_d^2}\right\} = e^{-\xi\omega_n t}\cos\omega_d t$$

$$L^{-1}\left\{\frac{\xi\omega_n}{\left(s+\xi\omega_n\right)^2+\omega_d^2}\right\} = \frac{\xi\omega_n}{\omega_d}L^{-1}\left\{\frac{\omega_d}{\left(s+\xi\omega_n\right)^2+\omega_d}\right\}$$

$$= \frac{\xi\omega_n}{\omega_d}\cdot e^{-\xi\omega_n t}\sin\omega_d t$$

Since, $\omega_d = \omega_n\sqrt{1-\xi^2}$

Thus the term $\frac{\xi\omega_n}{\omega_d}$ becomes,

$$\frac{\xi\omega_n}{\omega_d} = \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} = \frac{\xi}{\sqrt{1-\xi^2}}$$

Thus the total Laplace inverse can be written as,-

$$\therefore \; c(t) = 1-e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}}\,e^{-\xi\,\omega_n t}\cdot\sin\omega_d t$$

$$c(t) = 1-e^{-\xi\omega_n t}\left\{\cos\omega_d t+\frac{\xi}{\sqrt{1+\xi^2}}\sin\omega_d t\right\} \quad \text{...(5.7.18)}$$

$$c(t) = 1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left\{\sqrt{1-\xi^2}\cos\omega_d t+(\sin\omega_d t)\,\xi\right\}$$

Consider triangle as shown in Fig. 5.7.16.

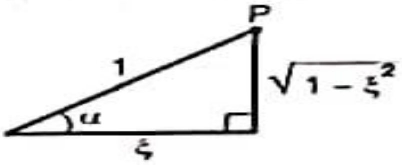


Fig. 5.7.16

$$\cos\alpha = \xi \qquad \sin\alpha = \sqrt{1-\xi^2}$$

The important of this triangle is discussed later.

$$c(t) = 1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sin\alpha\cos\omega_d t+\cos\alpha\sin\omega_d t\right)$$

We have, $\sin A \cos B + \cos A \sin B = \sin (A + B)$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin (\omega_d t + \alpha) \quad ...(5.7.19)$$

But we have,

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\therefore \quad \alpha = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}$$

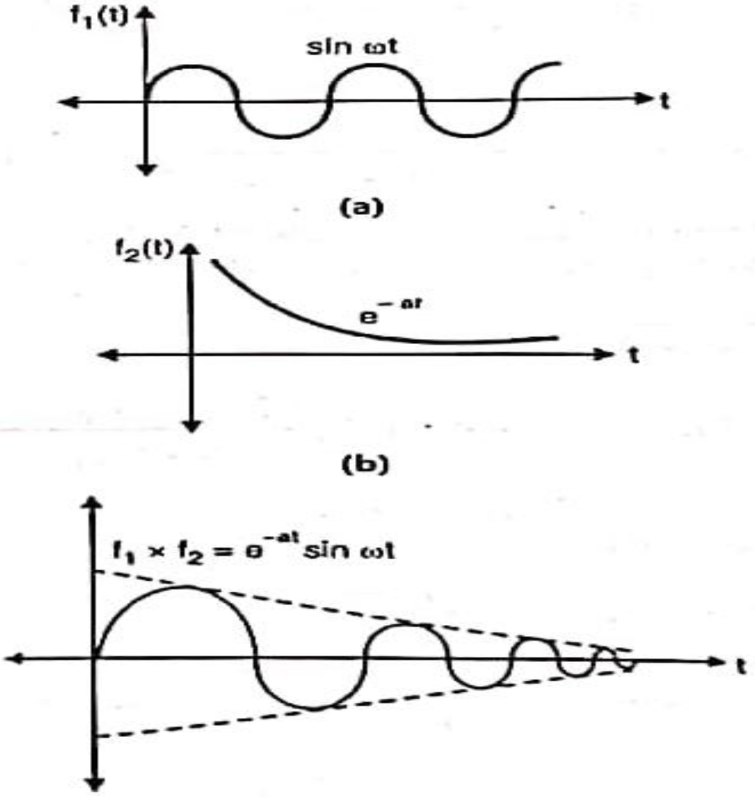
Putting this value in Equation (5.7.19) we get,

$$\therefore c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} . \sin \left(\omega_d t + \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi} \right) \quad ...(5.7.20)$$

Equations (5.7.19) and (5.7.20) represent the solution for $0 < \xi < 1$ for the underdamped case.

We notice that the response contains a sine term multiplied by an exponential decaying term. The result will be sinusoidal decaying on an exponential envelope.

See Figs. 5.7.17(a), (b), (c), (d).

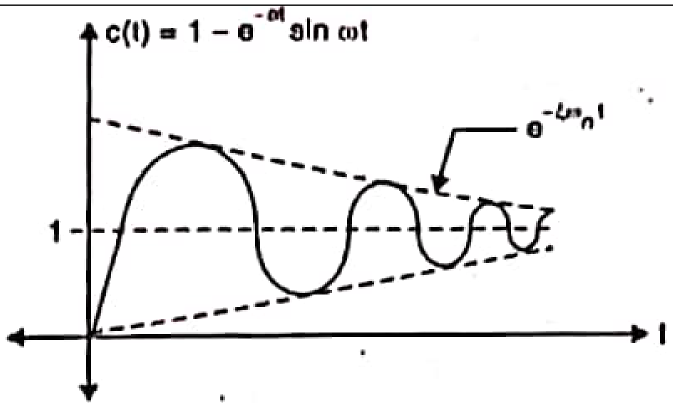
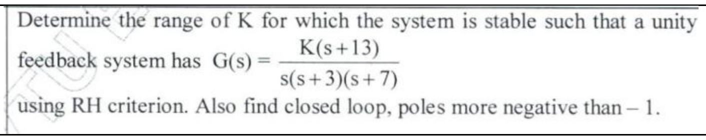


(a)

(b)

(c)

Fig. 5.7.17 (Cont...)

	 (d) Fig. 5.7.17 : Underdamped response 1) Notice that the response overshoots the reference and oscillates before settling to the final value. Underdamped systems, thus and exhibit overshoots and undershoots. 2) The pole-locations are in the 2nd and 3rd quadrant for this response. It is important to note that the complex poles are at, $s_{1,2} = -\xi\omega_n \pm j\,\omega_n\sqrt{1-\xi^2}$ The real part of poles generate the decaying exponential, while the imaginary part produces the pure sinusoidal function.
7a	Mention Limitations of Routh's Criterion
Sol:	a. Applies Only to Linear Time-Invariant Systems b. Does Not Provide Exact Root Locations (location of poles) c. Valid for only real coefficients d. Does not suggest methods for stabilizing
7b	Determine the range of K for which the system is stable such that a unity feedback system has $G(s) = \frac{K(s+13)}{s(s+3)(s+7)}$ using RH criterion. Also find closed loop, poles more negative than – 1.

Sol:

[7-b] Given: $G(s) = \frac{K(s+13)}{s(s+3)(s+7)}$

$H(s) = 1.$

$1 + G(s)H(s) = 0$ is the char. equation

$\Longrightarrow 1 + \frac{K(s+13)}{s(s+3)(s+3)} = 0.$

$\Longrightarrow s^3 + 10s^2 + (21+K)s + 13K = 0.$

Forming the Routh's array we have:

s^3	1	$(21+K)$
s^2	10	$13K.$
s^1	$(210+10K-13K)$	0
s^0	$13K.$	

Hence for stability:

From $s^0 \Longrightarrow 13K > 0$

$\Longrightarrow K > 0$

From $s^1 \Longrightarrow \frac{210+10K-13K}{10} > 0$

$\Longrightarrow K < 70.$

Hence $\boxed{0 < K < 70}$ is the Range of 'K' for stability

(ii) For its closed loop poles more -ve than -1 $\Longrightarrow$

$s = s'-1.$

$\therefore (s'-1)^3 + 10(s'-1)^2 + (s'-1)(K+21) + 13K = 0$

$\Longrightarrow (s')^3 + 7(s')^2 + s'(4+K) + (12K-12) = 0$

∴ Rouths arrg ⇒

$$\begin{array}{c|cc}
(s')^3 & 1 & 4+k. \\
(s')^2 & 7 & 12k-12 \\
(s')^1 & \frac{28+7k-12k+12}{7} & 0 \\
(s')^0 & 12k-12. &
\end{array}$$

$12k-12>0$

$\Rightarrow \therefore k > 01$

Also,

$40-5k>0$

$\Rightarrow 40>5k$

$\therefore k<8$

∴ the range of K to get closed loop poles more negative (−ve) than −1 is

$$\boxed{1 < k < 8}$$

7c	Check the stability of the given characteristic equation using Routh's method. $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$

Sol:

7-c] :→ Char.eq: $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$

Rouths Array:

$$\begin{array}{c|cccc}
s^6 & 1 & 8 & 20 & 16 \\
s^5 & 2 & 12 & 16 & 0 \\
s^4 & 2 & 12 & 16 & 0 \\
s^3 & 0 & 0 & 0 & 0
\end{array}$$

Hence Special case due to all zeros in (s^3) row.

Hence,

$A(s) = 0$

$\Rightarrow 2s^4 + 12s^2 + 16 = 0$

$\frac{dA(s)}{ds} = 8s^3 + 24s = 0$

$\Rightarrow$ ~~$8s^2 = -24$~~

~~$s^2 = -3$~~

~~$s = \sqrt{-3}$~~ $= j\sqrt{3}$

Hence the Rouths array is

$$\begin{array}{c|cccc}
s^6 & 1 & 8 & 20 & 16 \\
s^5 & 2 & 12 & 16 & 0 \\
s^4 & 2 & 12 & 16 & 0 \\
s^3 & 8 & 24 & 0 & 0 \\
s^2 & 6 & 16 & 0 & 0 \\
s^1 & 2.69 & 0 & & \\
s^0 & 1.6 & & &
\end{array}$$

Hence solving

$A(s) = 0$

$\Rightarrow$

$2s^4 + 12s^2 + 16 = 0$

Putting $s^2 = y$

$\Rightarrow$

$2y^2 + 12y + 16 = 0$

$y = \frac{-12 \pm \sqrt{144 - 128}}{4}$

$= -3 \pm 1$

$y = -2, -4$

$\Rightarrow s^2 = -2, -4$

$\therefore s = \pm j\sqrt{2}, \pm j2.$

Since we have non repeated roots on the imaginary axis, hence the system is marginally stable.//..

8a Sketch the complete Root locus of system having

$$G(s)\,H(s) = \frac{K}{s(s+5)(s+10)}$$

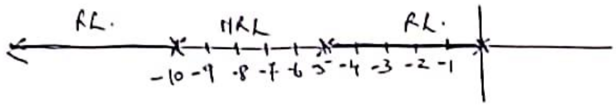
Sol:

[8-a] Poles = 3, Zeros = 0. ∴ $N = P = 3$ branches

Branches approaching $\infty \equiv P - Z = 3 - 0 = 3$.

Starting points $\Rightarrow$ 0, −5, −10. $\Longrightarrow$ Poles.

Terminating point $\Rightarrow$ ∞, ∞, ∞. $\Longrightarrow$ Zeros.



· One breakaway point between 0 & −5

Step 3: Angles asymptotes $\Rightarrow \theta_q = \frac{(2q+1)180}{p-z}$; $q = 0, 1, 2$

$\theta_1 = 60^\circ, \ \theta_2 = 180^\circ, \ \theta_3 = 300^\circ$

Step 4: Centroid

$$\sigma_c = \frac{\Sigma \text{ Real parts of poles } - \Sigma \text{ Real parts of zeros}}{p - z}.$$

$$= -5.$$

Step 5: Breakaway point:

$1 + G(s) H(s) = 0. \Longrightarrow 1 + \frac{K.}{s(s+5)s+10)} = 0$

$\Longrightarrow s(s+5)(s+10) + K = 0. \Longrightarrow (s^2+5s)(s+10) + K = 0$

$\Rightarrow s^3 + 10s^2 + 5s^2 + 50s + K = 0 \Longrightarrow s^3 + 15s^2 + 50s + K = 0$

$\Rightarrow K = -[s^3 + 15s^2 + 50s]$ Now $\frac{dK}{ds} = 0$

$\Longrightarrow 0 = 3s^2 + 30s + 50$ Solving we get

$s = -2.113 \longrightarrow$ is the valid breakaway point

& $s = -7.88$

step-6: Intersection points with imaginary axis using Routh array :

char-eq : $1 + G(s) H(s) = 0$

$$\Rightarrow 1 + \frac{k}{s(s+5)(s+10)} = 0 \Rightarrow s(s+5)(s+10) + k = 0$$

$$(s^2+5s)(s+10)+k=0 \Rightarrow s^3+10s^2+5s^2+50s+k=0$$

$$\Rightarrow s^3+15s^2+50s+k=0.$$

s^3	1	50
s^2	15	K.
s^1	$\frac{750-K}{15}$	0
s^0	K.	

For stability column 1 should be +ve.

$\therefore K > 0$ &

$$\frac{750-K}{15} > 0$$

$$\Rightarrow 750 - k > 0$$

$\therefore 750 > k.$

i.e. $K < 750.$

$\therefore K_{max} = 750$

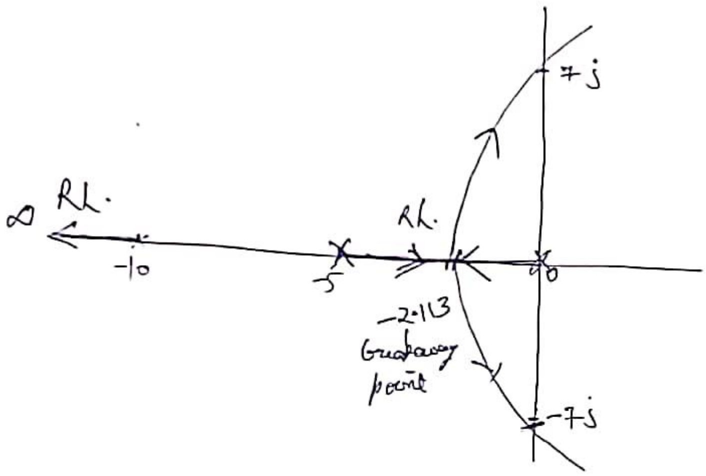
at K_{max} all element of s^1 is zero

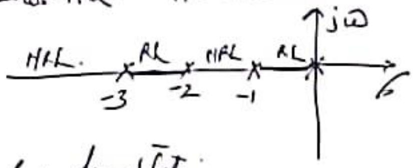
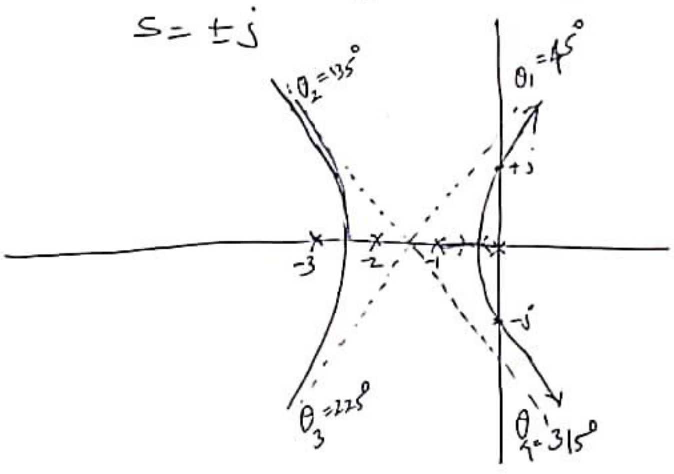
$$\therefore A(s) = 15s^2 + K_{max} = 0.$$

$$\Rightarrow 15s^2 = -K_{max} = -750$$

$$\Rightarrow s^2 = -\frac{750}{15} = -50$$

$$s = \sqrt{-50} = j\sqrt{50} = \underline{\underline{\pm j\,7.071.}}$$

	Step 7: No complex poles, Hence no angle of departure is there.  Hence for $0 < K < 750$, system is stable since the entire root locus is in left half of s-plane. For $K = 750$ the system is marginally stable For $750 < K < \infty$, the system is unstable.
8b	Sketch the complete Root locus of system having $$G(s)\,H(s) = \frac{K}{s(s+1)(s+2)(s+3)}$$

Sol :	[8-b] ⇒ Step 1: Poles at $s=0, s=-1, s=-2, s=-3$. Hence $P=4$. Zeros-Nil. Hence $Z=0$. Step 2:  Step 3: Angles Asymptotes: $\theta_1 = 45^\circ, \theta_2 = 135^\circ, \theta_3 = 225^\circ, \theta_4 = 315^\circ$ Step 4: $\sigma_c = -1.5$ Step 5: Breakaway point: after solving $\frac{dK}{ds} = 0$. $s = -0.381$, $s = -2.616$ } Both are valid. Step 6: Intersection with imaginary axis: using Routh's array $s = \pm j$ 
9a	Draw the Bode plot for the open loop transfer function of a system is $G(s) = \frac{K(1+0.2s)(1+0.025s)}{s^3(1+0.001s)(1+0.005s)}$ Determine that the system is conditionally stable. Find the range of K for which the system is stable.

Sol :

1) 3 poles at origin, straight line of slope − 60dB/dec passing through intersection of $\omega = 1$ and 0dB.

2) Simple zero, $(1+0.2S)$, $T_1 = 0.2$, $\omega_{c1} = \frac{1}{T_1} = 5$

St. line of slope + 20dB/dec for $\omega \geq 5$

3) zero, $(1+0.025S)$, $T_2 = 0.025$, $\omega_{c2} = \frac{1}{T_2} = 40$

St. line of slope +20dB/dec for $\omega \geq 40$

4) Simple pole, $\frac{1}{(1+0.005S)}$, $T_3 = 0.005$, $\omega_{c3} = \frac{1}{T_3} = 200$

St. line of slope − 20dB/dec for $\omega \geq 200$

5) Simple pole, $\frac{1}{(1+0.001S)}$, $T_4 = 0.001$, $\omega_{c4} = \frac{1}{T_4} = 1000$

$$G(j\omega) = \frac{K(1+0.2j\omega)(1+0.025j\omega)}{(j\omega^3)(1+0.001j\omega)(1+0.005j\omega)}$$

Phase angle table

ω	ϕ_R
0.5	−263.75°
5	−219.56°
40	−155.79°
200	−159.12°
1000	−216.19°
∞	−270°

From phase angle plot intersects −180° plot) axis twice i.e.,

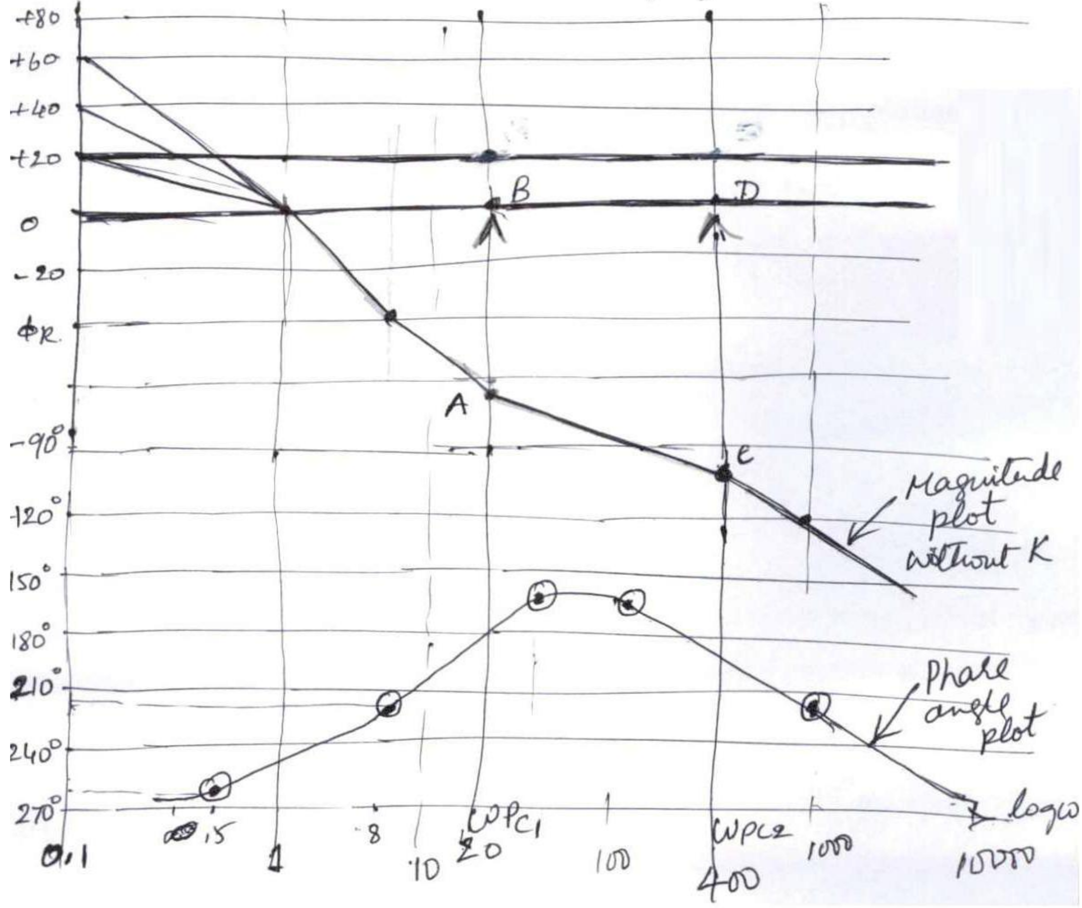
$\omega_{pc1} = 20$ and $\omega_{pc2} = 400$

System is stable only for values of K corresponding to ω_{gc} between B and D as shown.

This shows that system is conditionally stable.

At $\omega_{pc1} = 20$, $AB = +64dB$

$\therefore 20 \log K = 64 \Rightarrow K = 1585$

	At ω_{pc2} = 400, CD = 100dB, ∴ 20 log K = 100 K = 1,00,000 ∴ Range of K is 1585 < K 1,00,000 
9b	The transfer function of a system is $G(s)\,H(s) = \dfrac{K}{s(s+2)(s+10)}$ Sketch the Nyquist plot and hence calculate the range of values of K for stability.

Sol :

$P=0$, $N=-P=0$, critical point $-1+j0$

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(2+j\omega)(10+j\omega)}$$

$$M = |G(j\omega)H(j\omega)| = \frac{K}{\omega\sqrt{4+\omega^2}\sqrt{100+\omega^2}}$$

$$\phi = -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

Section-I ; $s=+j\infty$ to $s=+j0$ Starting pt. $\omega\to\infty$ $0\angle -270^\circ$

$\omega\to 0$ to $\omega\to 0$ Terminating pt. $\omega\to 0$ $0\angle -90^\circ$

$-90^\circ-(-270^\circ)$

$=+180^\circ$

anticlockwise

Section-II ; $s=j0$ to $s=-j0$; 180°

Section III ; mirror image about real axis

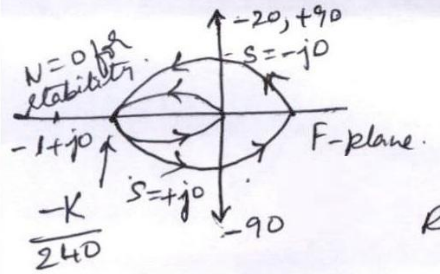
Section IV ; origin

$$G(j\omega)H(j\omega) = \frac{-Kj\omega[20-12j\omega-\omega^2]}{\omega^2(4+\omega^2)(100+\omega^2)} = \frac{-12K\omega^2}{P} - \frac{Kj\omega(20\omega^2)}{D}$$

$$D = \omega^2(4+\omega^2)(100+\omega^2)$$

imaginary part ; $\omega(20-\omega^2)=0$, $\omega_{pc}=\sqrt{20}$

Real part ; point $Q = \dfrac{-12K\times 20}{20(20+4)(100+20)} = \dfrac{-K}{240}$



For absolute stability, $N=0$

$$\left|\frac{-K}{240}\right| < 1$$

$\therefore K < 240$

Range of K for stability if

$0 < K < 240$.

10 a

Obtain the state model of the network shown in Fig.Q10(a) assuming $R_1 = R_2 = 1\ \Omega$, $C_1 = C_2 = 1$F, and L = 1H.

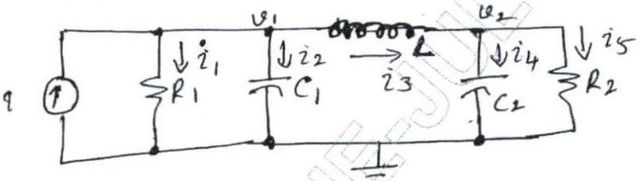


Fig.Q10(a)

Sol :

[10 - a]

After analysis & solving we get the state model as:

$$\begin{bmatrix} \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \\ \frac{di_3}{dt} \end{bmatrix} \begin{bmatrix} -1/R_1C_1 & 0 & -1/C_1 \\ 0 & -1/R_2C_2 & 1/C_2 \\ \frac{1}{L} & -1/L & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix} + \begin{bmatrix} 1/C_1 \\ 0 \\ 0 \end{bmatrix} \cdot i$$

$$\Rightarrow \begin{bmatrix} v_1' \\ v_2' \\ i_3' \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} i$$

$$\Rightarrow \begin{bmatrix} i_5 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 1/R_2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix}$$

$$\begin{bmatrix} i_5 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix} \quad //$$

10 b

Obtain the state transition matrix for the state model whose A matrix is given by

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Sol:

[10-b]

$$[s\underline{I}-A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

$$\phi(s) = [s\underline{I}-A]^{-1} = \frac{Adj[s\underline{I}-A]}{[s\underline{I}-A]}$$

$$= \frac{\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}}{s^2+3s+2}$$

$$\therefore STM = \phi(t) = L^{-1}\phi(s) = L^{-1}[s\underline{I}-A]^{-1}$$

$$= L^{-1}\begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & s/(s+1)(s+2 \end{bmatrix}$$

On simplification we get

$$STM = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$