Sub:	Machine Learning					Sub Code:	BCS 602	Branch:	CSE
Date:	27/05/2025	Duration:	90 mins	Max Marks:	50	Sem / Sec:	6 A,B,C		

USN

Internal Assessment Test 2 – May 2025

Sub:	Machine Learning			Sub Code:	BCS602	Branch:	CSE
Date:	23.05.2025	Duration:	90 mins	Max Marks: 50	Sem / Sec:	VI/ A, B, C	OBE

Answer any FIVE FULL Questions

		MARKS	CO	RBT
1	Consider the following list which contains name, age, gender and class of sports. In the gender field, males are denoted by '0' and females are denoted by '1'. Using **K- Nearest Neighbor (KNN)** algorithm, find class of sports for a girl whose name is Angelina, whose k factor is 3 and age is 5.	[10]	CO3	L3

Name	Age	Gender	Class of Sports
Ajay	32	0	Football
Mark	40	0	Neither
Sara	16	1	Cricket
Zaira	34	1	Cricket
Sachin	55	0	Neither
Rahul	40	0	Cricket
Pooja	20	1	Neither
Smith	15	0	Cricket
Laxmi	55	1	Football
Michael	15	0	Football

		MARKS	CO	RBT
2	Construct Decision Tree using ID3 algorithm for the following data.	[10]	CO3	L3

Day	Outlook	Temp	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	yes
2	Sunny	Hot	High	Strong	no
3	Overcast	Hot	High	Weak	yes
4	Rain	Mild	High	Weak	no
5	Rain	Cool	Normal	Weak	yes

		MARKS	CO	RBT
3	Classify the test data {Red, SUV, Domestic} using Naïve Bayes classifier for the following dataset.	[10]	CO4	L3

Colour	Type	Origin	Stolen
Red	Sports	Domestic	yes
Red	Sports	Domestic	no
Yellow	Sports	Domestic	no
Yellow	Sports	Imported	yes
Yellow	SUV	Imported	no
Yellow	SUV	Domestic	no
Yellow	SUV	Imported	yes
Red	SUV	Imported	no
Red	Sports	Imported	yes
Red	Sports	Imported	no

		MARKS	CO	RBT
4 (a)	Explain different types of Artificial Neural Networks with neat diagram.	[5]	CO4	L2
(b)	Describe basic components of Reinforcement learning with neat diagram.	[5]	CO5	L2
5. (a)	Design a perceptron that performs Boolean function **OR** with the initial weights **w1=0.6** and **w2=0.6**, learning rate α **= 0.5** and θ **=1** and update the weights until the Boolean function gives the desired output.	[6]	CO4	L3
(b)	Explain Multi-Arm Bandit problem.	[4]	CO5	L2
6.	Consider the following dataset. Apply complete and average linkage. Draw the dendrograms.	[10]	CO5	L3

Sample No.	X	Y
S1	4	3
S2	1	4
S3	2	1
S4	3	8
S5	6	9
S6	5	1

Answer 1. Distance Calculation Mistake : 1 Mark for each distance, 2 marks if not in ascending order.

Age	Gender	class of sports	d (5,1)
32	0	F	27.01
40	0	N	35.01
16	1	C	11
34	1	C	29
55	0	N	50.009
40	0	C	35.01
20	1	N	15
15	0	C	10.04
55	1	F	50
15	0	F	10.04

$$d_1 = \sqrt{(32-5)^2 + (1-0)^2}$$

$$= \sqrt{(27)^2 + 1^2}$$

$$= 27.01$$

Arrange Distances in ascending order;

d	class of sports
10.04	C
10.04	F
11	C
15	N
27.01	F
29	C
35.01	N
35.01	C
50	F
50.009	N

According to K=3

d	class of sports
10.04	C
10.04	F
11	C

Hence, cricket is having majority, so class of sports for Angelina; age = 5 is cricket.

Answer

Answer 2: Entropy: 2 Marks, Entropy (A): 2 Marks, Information Gain: 2 Marks, Tree: 2 Marks, Calculation Mistake: 1 Marks.

Answer - 2.

Entropy → 1 M, E(A) → 1 M (each), gain = 1 M each, Tree → 1 M.

ID3 →

$$E = -\sum_{i=1}^{n} P_i \log_2 P_i$$

$$E = -\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{3}{5}$$

$= -0.6 \log_2 0.6 - 0.4 \log_2 0.4$

$= 0.4421 + 0.52877$

$= 0.9708.$

$\left\{ \log_2 0.6 = \frac{\log_{10} 0.6}{\log_{10} 2} \right\}$

$$E(\text{outlook}) = \frac{2}{5}(1,1) + \frac{1}{5}(1,0) + \frac{2}{5}(1,1)$$

$= \frac{2}{5} + 0 + \frac{2}{5}$

$\left\{ E(1,0) = 0, \; E(1,1) = 1 \right\}$

$= \frac{2+2}{5}$

$= 4/5$

$= 0.8.$

$$E(\text{Temp}) = \frac{3}{5}(2,1) + \frac{1}{5}(0,1) + \frac{1}{5}(1,0)$$

$= \frac{3}{5}\left[-\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3}\right] + 0 + 0$

$= \frac{3}{5}\left[-0.66 \log_2 0.66 - 0.33 \log_2 0.33\right]$

$= 0.6\left[0.3956 + 0.5278\right] + 0$

$= 0.5540$

$$E(\text{Humidity}) = \frac{4}{5}(2,2) + \frac{1}{5}(1,0)$$

$= \frac{4}{5} + 0$

$= 0.8$

$$E(\text{wind}) = \frac{4}{5}[3,1] + \frac{1}{5}[0,1]$$

$= 0.8\left[-\frac{3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\frac{1}{4}\right]$

$= 0.8\left[-0.75 \log_2 0.75 - 0.25 \log_2 0.25\right]$

$= 0.8\left[0.3112 + 0.5\right]$

$= 0.6489.$

Information gain (IG) for outlook $= 0.9708 - 0.8.$

$= 0.1708.$

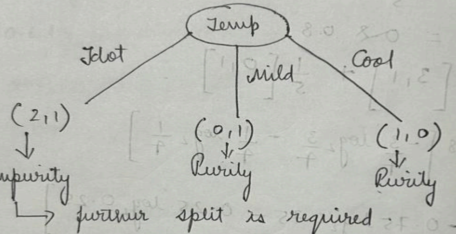
IG (Temp) $= 0.9708 - 0.5540$

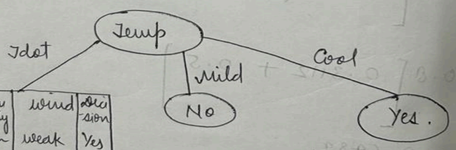
$= 0.4168.$

IG (Humidity) = 0.9708 − 0.8
= 0.1708

IG (wind) = 0.9708 − 0.6489
= 0.3219

Root node = Temp (Maximum IG)





Outlook	Humidity	wind	Decision
sunny	High	weak	Yes
overcast	High	weak	Yes
sunny	High	strong	No

$$E = -\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3}$$

= 0.3956 + 0.5278

= 0.9234

$$E(\text{outlook}) = +\frac{2}{3}[1,1] + \frac{1}{3}(1,0)$$

$$= \frac{2}{3} \times 1$$

= 0.66

$$E(\text{Humidity}) = \frac{3}{3}(2,1)$$

$$= \frac{3}{3}\left[-\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3}\right]$$

= 0.9234

$$E(\text{wind}) = \frac{2}{3}(2,0) + \frac{1}{3}(0,1)$$

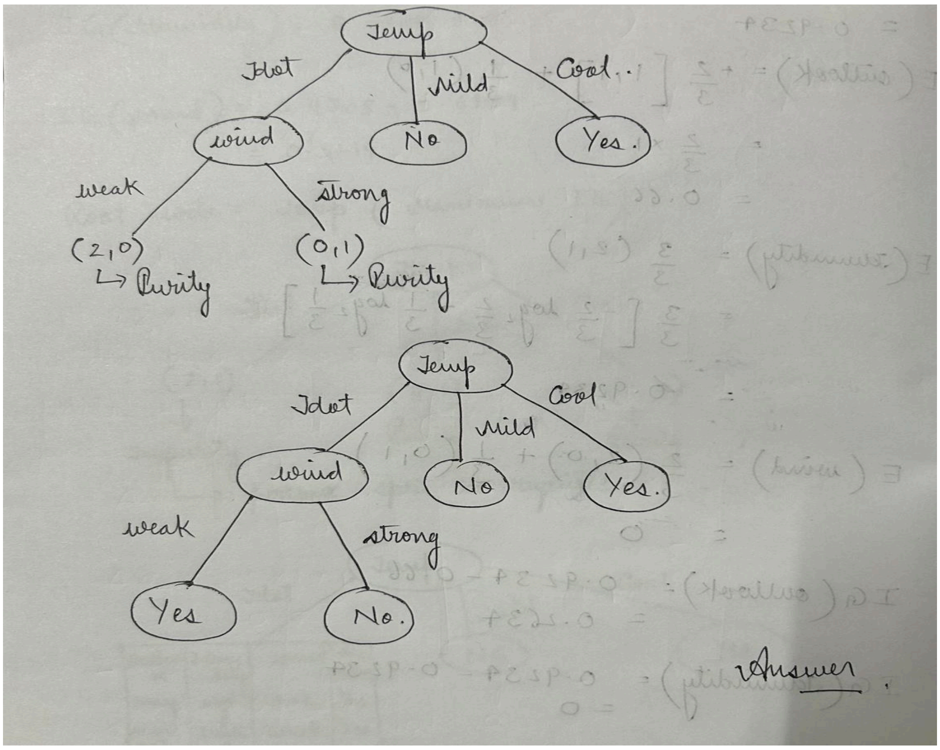
= 0

IG (outlook) = 0.9234 − 0.66
= 0.2634

IG (Humidity) = 0.9234 − 0.9234
= 0

IG (wind) = 0.9234 − 0
= 0.9234

root node is wind (Maximum IG)



Answer 3 : Prior Probability: 2 Marks, Likelihood Probability: 6 Marks, Calculation Mistake: 2 Marks.

Answer - 3:

P(~~Stolen~~ Yes) = 4/10 = 0.4

↳ Prior Probability

$P(No) = 6/10$
$= 0.6$

$P\{Red, SUV, Domestic\}/Yes = P(Red/Yes) \cdot P(SUV/Yes) \cdot P(Domestic/Yes)$

$= \frac{2}{4} \times \frac{1}{4} \times \frac{1}{4}$

$= 2/64$

$P(Y/Test\ Data) = \frac{2}{64} \times 0.4$

$= 0.0125$

$P\{Red, SUV, Domestic\}/No = P(Red/No) \times P(SUV/No) \cdot P(Domestic/No)$

$= \frac{3}{6} \times \frac{3}{6} \times \frac{3}{6}$

$= 0.125$

$P(N/Test\ Data) = 0.125 \times 0.6$

$= 0.075$

Hence, No is having Maximum Probability, so as per the MAP concept, the class will be No.

Answer 4 a Different Types of Neural Network: Each type and figure are 1 mark.

10.5.1 Feed Forward Neural Network

This is the simplest neural network that consists of neurons which are arranged in layers and the information is propagated only in the forward direction. This model may or may not contain a hidden layer and there is no back propagation. Based on the number of hidden layers they are further classified into single-layered and multi-layered feed forward networks. These ANNs are simple to design and easy to maintain. They are fast but cannot be used for complex learning. They are used for simple classification and simple image processing, etc. The model of a Feed Forward Neural Network is shown in Figure 10.7.

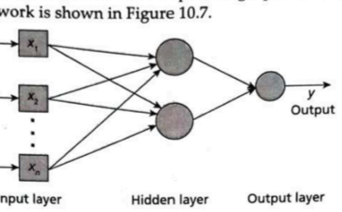


Figure 10.7: Model of a Feed Forward Neural Network

10.5.2 Fully Connected Neural Network

Fully connected neural networks are the ones in which all the neurons in a layer are connected to all other neurons in the next layer. The model of a fully connected neural network is shown in Figure 10.8.

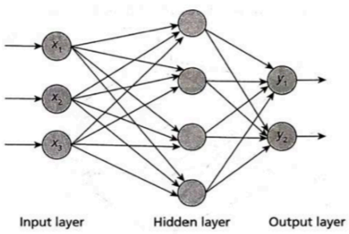


Figure 10.8: Model of a Fully Connected Neural Network

10.5.3 Multi-Layer Perceptron (MLP)

This ANN consists of multiple layers with one input layer, one output layer and one or more hidden layers. Every neuron in a layer is connected to all neurons in the next layer and thus they are fully connected. The information flows in both the directions. In the forward direction, the inputs are multiplied by weights of neurons and forwarded to the activation function of the

neuron and output is passed to the next layer. If the output is incorrect, then in the backward direction, error is back propagated to adjust the weights and biases to get correct output. Thus, the network learns with the training data. This type of ANN is used in deep learning for complex classification, speech recognition, medical diagnosis, forecasting, etc. They are comparatively complex and slow. The model of an MLP is shown in Figure 10.9.

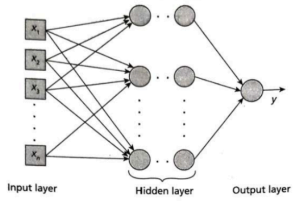


Figure 10.9: Model of a Multi-Layer Perceptron

10.5.4 Feedback Neural Network

Feedback neural networks have feedback connections between neurons that allow information flow in both directions in the network. The output signals can be sent back to the neurons in the same layer or to the neurons in the preceding layers. Hence, this network is more dynamic during training. The model of a feedback neural network is shown in Figure 10.10.

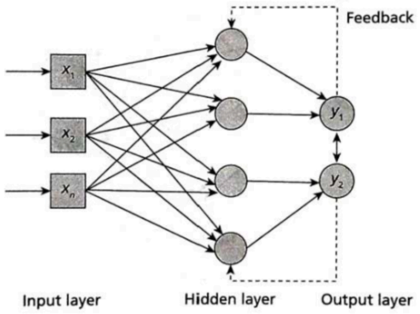


Figure 10.10: Model of a Feedback Neural Network

Answer 4-b Reinforcement Learning Components: All components 1 mark, Mathematical Expression 1 mark and figure is 1 mark.

Components of Reinforcement Learning

- Environment and Agent
- States and Actions
- Policies
- Rewards:
  1. Immediate Award

$$G_t = r_{t+1} + r_{t+2} + r_{t+3} + \dots + r_T$$

  2. Long term Award

$$G_t = \gamma_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3\dots}$$
$$= \sum_{j=0}^{\infty} \gamma^j r_{t+j+1}$$

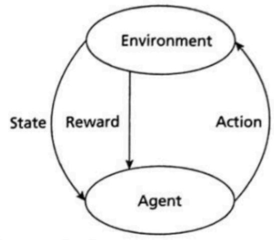


Figure 14.4: Basic Components of RL

Key components of reinforcement learning:

1.Agent: The learner or decision-maker that interacts with the environment. It observes the state of the environment, takes actions, and receives feedback.

2.Environment: The external system or world with which the agent interacts. It responds to the actions taken by the agent and transitions between different states.

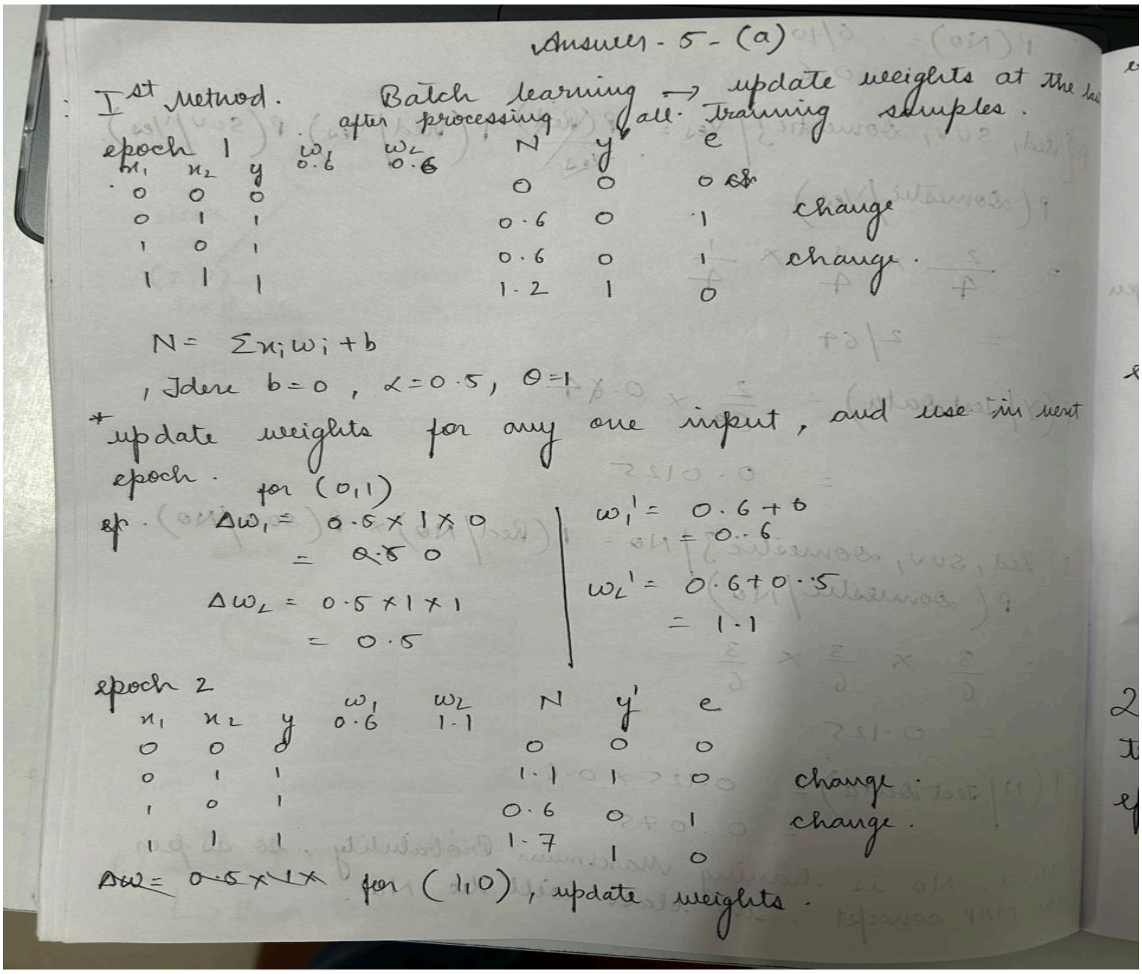
3.Actions: Choices made by the agent at each time step, affecting the state of the environment.

4.State: represents the current situation or configuration of the environment, which influences the next state based on the agent's actions.

5.Rewards: Numeric values provided by the environment to indicate the desirability of the actions taken by the agent. The goal of the agent is to maximize the cumulative reward over time.

6.Policy: The strategy or set of rules that the agent uses to determine its actions based on the observed states. It maps states to actions and guides the decision-making process.

Answer 5-a: Epoch: 2 Marks, Calculated Output Calcu;ation: 2 Marks, Calculation: 2 Marks



Answer - 5 - (a)

I^{st} Method. Batch learning → update weights at the last after processing all. Training samples.

epoch 1

x_1	x_2	y	w_1 0.6	w_2 0.6	N	y'	e	
0	0	0			0	0	0	
0	1	1			0.6	0	1	change
1	0	1			0.6	0	1	change.
1	1	1			1.2	1	0	

$N = \sum x_i w_i + b$

, Here $b = 0$, $\alpha = 0.5$, $\theta = 1$

* update weights for any one input, and use in next epoch.

for (0,1)

$\Delta w_1 = 0.5 \times 1 \times 0 = 0$

$\Delta w_2 = 0.5 \times 1 \times 1 = 0.5$

$w_1' = 0.6 + 0 = 0.6$

$w_2' = 0.6 + 0.5 = 1.1$

epoch 2

x_1	x_2	y	w_1 0.6	w_2 1.1	N	y'	e	
0	0	0			0	0	0	
0	1	1			1.1	1	0	change.
1	0	1			0.6	0	1	change.
1	1	1			1.7	1	0	

$\Delta w = 0.5 \times 1 \times$ for (1,0), update weights.

$\Delta w_1 = 0.5 \times 1 \times 1$
$= 0.5$

$\Delta w_2 = 0.5 \times 1 \times 0$
$= 0$

$w_1' = 0.6 + 0.5$
$= 1.1$

$w_2' = 1.1 + 0$
$= 1.1$

epoch 3

x_1	x_2	y	w_1 1.1	w_2 1.1	N	y'	e
0	0	0			0	0	0
0	1	1			1.1	1	0
1	0	1			1.1	1	0
1	1	1			2.2	1	0

Hence, the actual output and calculated output is same, so it is convergence.

2^{nd} Method → stochastic learning → update weights immediately after training sample and use the updated weights for the calⁿ of N.

epoch 1

x_1	x_2	y	w_1 0.6	w_2 0.6	N	y'	e	Δw_1	Δw_2	w_1'	w_2'
0	0	0			0	0	0	—	—	—	—
0	1	1			0.6	0	1	0	0.5	0.6	1.1
1	0	1			$0.6\times1+1.1\times0 = 0.6$	0	1	0.5	0	1.1	1.1
1	1	1			2.2	1	0	—	—	1.1	1.1

epoch 2

x_1	x_2	y	w_1 1.1	w_2 1.1	N	y'	e	Δw_1	Δw_2	w_1'	w_2'
0	0	0			0	0	0	—	—	—	—
0	1	1			1.1	1	0	—	—	—	—
1	0	1			1.1	1	0	—	—	—	—
1	1	1			2.2	1	0	—	—	—	—

Hence, Actual output is same as predicted (calculated) output, so it is convergence step.

Answer 5-b MultiArm Bandit Problem: Steps: 2 Marks, Selection Policies : 2 Marks.

MULTI-ARM BANDIT PROBLEM

❖Imagine you're in front of several slot machines (arms).

❖Each machine gives a random reward, but some are better than others.

❖You don't know which one is best, so you have to try different machines, observe their rewards, and figure out where to keep playing to maximize total reward over time.

Key Methods Used in Multi Arm Bandit Problem:

Exploration

- Trying out different arms to gather information.
- Helps discover which machines give higher rewards.
- Prevents getting stuck with a suboptimal choice too early.

Exploitation

- Playing the arm with the highest known average reward.
- Focuses on maximizing immediate reward using current knowledge.
- Might miss out on better options if not explored enough.

Selection Policies (How to choose an arm)

- Rules/strategies used to balance exploration and exploitation.
- Aim: maximize long-term reward.
- Examples: Greedy, Epsilon-Greedy

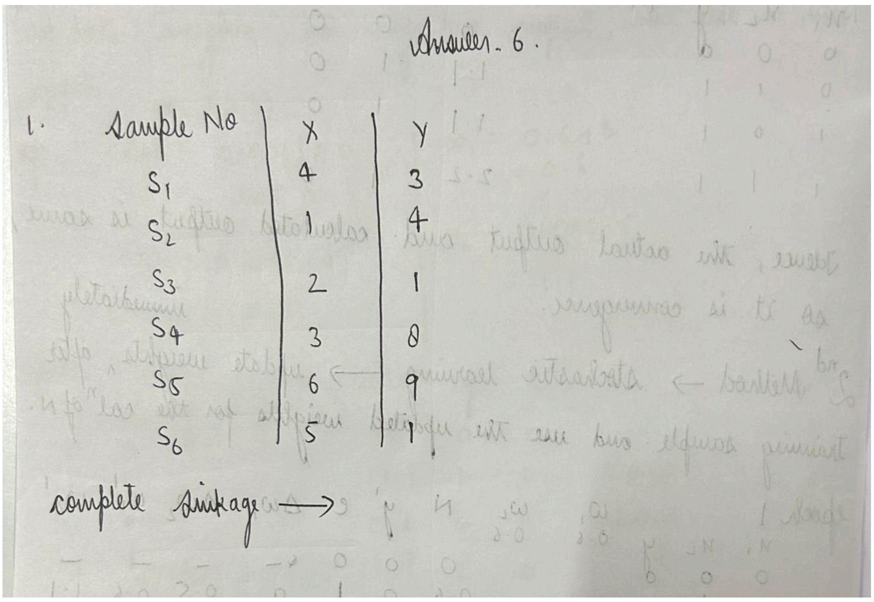
A. Greedy Method

- Always selects the arm with the highest average reward so far.
- Exploitation-only: no exploration.
- Simple, but can get stuck in a local optimum.

B. Epsilon-Greedy Method

- Mostly greedy, but with a small chance (ε) to explore.
- With probability 1−ε, choose the best-known arm (exploit).
- With probability ε, choose a random arm (explore).
- Balances exploration and exploitation better than greedy.

Answer 6: Complete Linkage Calculation 3 marks, Dendrogram 2 marks and Average Linkage Calculation 3 Marks, Dendrograms: 2 Marks


Answer. 6.

1.

Sample No	X	Y
S_1	4	3
S_2	1	4
S_3	2	1
S_4	3	8
S_5	6	9
S_6	5	1

complete linkage →

$d_{12} = \sqrt{(4-1)^2 + (3-4)^2}$

$= \sqrt{3^2 + (-1)^2}$

$= \sqrt{9 + 1}$

$= \sqrt{10}$

$d_{13} = \sqrt{(4-2)^2 + (3-1)^2}$

$= \sqrt{2^2 + 2^2}$

$= \sqrt{4+4}$

$= \sqrt{8} = 2.82$

$d_{14} = \sqrt{(4-3)^2 + (3-8)^2}$

$= \sqrt{1^2 + 5^2}$

$= \sqrt{26}$

$= 5.09$

$d_{15} = \sqrt{(4-6)^2 + (3-9)^2}$

$= \sqrt{2^2 + 6^2}$

$d_{16} = \sqrt{(4-5)^2 + (3-1)^2}$

$= \sqrt{1^2 + 2^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$

$d_{24} = \sqrt{(1-3)^2 + (4-8)^2}$

$= \sqrt{2^2 + 4^2}$

$= 4.47$

$d_{25} = \sqrt{(1-6)^2 + (4-9)^2}$

$= \sqrt{5^2 + 5^2} = 7.07$

$d_{23} = \sqrt{(1-2)^2 + (4-1)^2}$

$= \sqrt{1^2 + 3^2}$

$= \sqrt{10}$

$= ~~2.23~~\ 3.16.$

$d_{26} = \sqrt{(1-5)^2 + (4-1)^2}$

$= \sqrt{25}$

$= 5$

$d_{34} = \sqrt{(2-3)^2 + (1-8)^2}$

$= \sqrt{1^2 + 7^2}$

$= 7.07$

$d_{35} = \sqrt{(2-6)^2 + (1-9)^2}$

$= \sqrt{4^2 + 8^2}$

$= \sqrt{80}$

$= 8.94$

$d_{36} = \sqrt{(2-5)^2 + (1-1)^2}$

$= \sqrt{3^2 + 0^2} = 3.$

$d_{45} = \sqrt{(3-6)^2 + (8-9)^2}$

$= \sqrt{3^2 + 1^2}$

$= \sqrt{10}$

$d_{46} = \sqrt{(3-5)^2 + (8-1)^2}$

$= \sqrt{2^2 + 7^2}$

$= 7.28.$

$d_{56} = \sqrt{(6-5)^2 + (9-1)^2}$

$= \sqrt{1^2 + 8^2}$

$= 8.06$

Complete Linkage

	S_1	S_2	S_3	S_4	S_5	S_6
S_1	0	3.16	2.82	5.09	6.32	(2.23)
S_2		0	3.16	4.47	7.07	5
S_3			0	7.07	8.94	3
S_4				0	3.16	7.28
S_5					0	8.06
S_6						0

Merge (S_1, S_6) as new cluster due to having min. Distance.

	(S_1,S_6)	S_2	S_3	S_4	S_5
(S_1,S_6)	0	5	(3)	7.28	8.06
S_2		0	3.16	4.47	7.07
S_3			0	7.07	8.94
S_4				0	3.16
S_5					0

Merge (S_1, S_6, S_3) as new cluster, due to having low distance.

	(S_1,S_6,S_3)	S_2	S_4	S_5	
(S_1,S_6,S_3)	0	5	7.28	8.94	X
S_2		0	4.47	7.07	X
S_4			0	(3.16)	X
S_5				0	X
	X	X	X	X	X

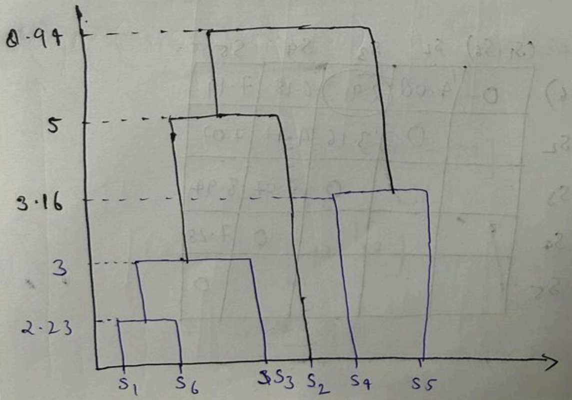
Merge (S_4, S_5) as new cluster, due to having low distance.

	(S_4, S_5)	(S_1, S_3, S_6)	S_2
(S_4, S_5)	0	8.94	7.07
(S_1, S_3, S_6)		0	5
S_2			0

Merge (S_1, S_2, S_3, S_6) as new cluster, due to low dist-ance.

	(S_1, S_2, S_3, S_6)	(S_4, S_5)
(S_1, S_2, S_3, S_6)	0	8.94
(S_4, S_5)		0

→ Merge $(S_1, S_2, S_3, S_4, S_5, S_6)$ as new cluster.



Average linkage.

	S_1	S_2	S_3	S_4	S_5	S_6
S_1	0	3.16	2.82	5.09	6.32	2.23
S_2		0	3.16	4.47	7.07	5
S_3			0	7.07	8.94	3
S_4				0	3.16	7.28
S_5					0	8.06
S_6						0

Merge (S_1, S_6) as new cluster, due to minimum distance.

	(S_1, S_6)	S_2	S_3	S_4	S_5
(S_1, S_6)	0	4.08	2.91	6.18	7.19
S_2		0	3.16	4.47	7.07
S_3			0	7.07	8.94
S_4				0	7.28
S_5					0

$d\{(S_1, S_6), \{S_2\}\} = \frac{1}{2 \cdot 1} \{d(S_1, S_2) + d(S_2, S_6)\}$

$= \frac{1}{2}[3.16 + 5]$

$= 4.08$

$d\{(S_1, S_6), (S_3, S_6)\} = \frac{1}{2 \cdot 1}[~~2.82 +~~ d(S_1, S_3) + d(S_3, S_6)]$

$= \frac{1}{2}[2.82 + 3]$

$= 2.91$

$d(\{S_1, S_6\}, \{S_4\}) = \frac{1}{2 \cdot 1}[d(S_1, S_4), (S_4, S_6)]$

$= \frac{1}{2}[5.09 + 7.28]$

$= \frac{1}{2}[12.37]$

$= 6.18$

$d(\{S_1, S_6\}, \{S_5, S_6\}) = \frac{1}{2 \cdot 1}[d(S_1, S_5), d(S_5, S_6)]$

$= \frac{1}{2}[~~2.23~~ 6.32 + 0.06]$

$= 7.19$

Merge (S_1, S_3, S_6) as new cluster, due to minimum distance.

	(S_1, S_3, S_6)	S_2	S_4	S_5
(S_1, S_3, S_6)	0	3.77	6.47	7.77
S_2		0	4.47	7.07
S_4			0	7.28
S_5				0

$d((S_1, S_3, S_6) \cdot \{S_2\}) = \frac{1}{3 \cdot 1}[d(S_1, S_2) + d(S_2, S_3) + d(S_2, S_6)]$

$= \frac{1}{3}[3.16 + 3.16 + 5]$

$= 3.77$

$$d(\{s_1,s_3,s_6\},\{s_4\}) = \frac{1}{3\cdot 1}\left[d(s_1,s_4) + d(s_3,s_4) + d(s_4,s_6)\right]$$

$$= \frac{1}{3}\left[5.09 + 7.04 + 7.28\right]$$

$$= 6.47$$

$$d(\{s_1,s_3,s_6\},\{s_5\}) = \frac{1}{3\cdot 1}\left[d(s_1,s_5) + d(s_3,s_5) + d(s_5,s_6)\right]$$

$$= \frac{1}{3}\left[6.32 + 8.94 + 8.06\right]$$

$$= 7.77$$

Merge (s_1, s_3, s_2, s_6) as new cluster, due to low Distance.

	(s_1,s_3,s_2,s_6)	s_4	s_5
(s_1,s_3,s_2,s_6)	0	5.97	7.59
s_4		0	7.20
s_5			0

$$d(\{s_1,s_2,s_3,s_6\},(s_4)) = \frac{1}{4\cdot 1}\left[d(s_1,s_4) + d(s_2,s_4) + d(s_3,s_4) + d(s_4,s_6)\right]$$

$$= \frac{1}{4}\left[5.09 + 4.47 + 7.07 + 7.20\right]$$

$$= 5.97$$

$$d(\{s_1,s_2,s_3,s_6\},\{s_5\}) = \frac{1}{4\cdot 1}\left[d(s_1,s_5) + d(s_2,s_5) + d(s_3,s_5) + d(s_5,s_6)\right]$$

$$= \frac{1}{4}\left[6.32 + 7.07 + 8.94 + 8.06\right]$$

$$= 7.59$$

Merge $(s_1, s_2, s_3, s_4, s_6)$ as new cluster, due to minimum Distance.

	(s_1,s_2,s_3,s_4,s_6)	s_5
(s_1,s_2,s_3,s_4,s_6)	0	6.71
s_5		0

$$d\{\{S_1, S_2, S_3, S_4, S_6\}, \{S_5\}\} = \frac{1}{5 \cdot 1}\Big[d(S_1, S_5) + d(S_2, S_5)$$

$$+ d(S_3, S_5) + d(S_4, S_5) + d(S_5, S_6)\Big]$$

$$= \frac{1}{5 \cdot 1}\Big[6.32 + 7.07 + 8.94 + 3.16 + 8.06\Big]$$

$$= 6.71$$