CBCS SCHEME

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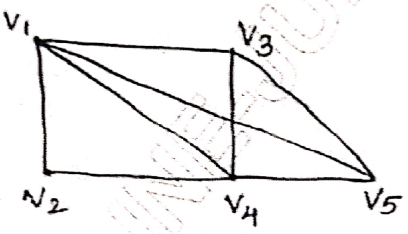
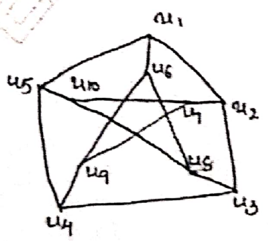
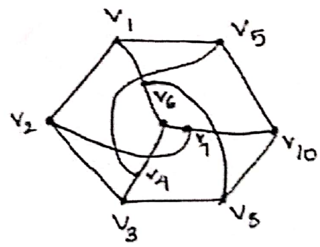
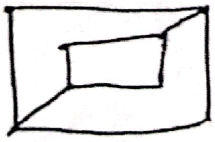
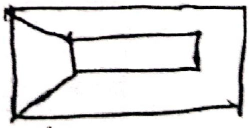
BCS405B

Fourth Semester B.E/B.Tech. Degree Examination, June/July 2025

Graph Theory

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, choosing ONE full question from each module.*
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
1	a.	Consider the following graph G Fig.Q1(a). Write : i) Open walk which is not a trail ii) Trial which is not a path iii) Closed walk which is a cycle iv) Closed walk which is a circuit but not a cycle v) Closed walk neither circuit nor cycle vi) Path of length 4.  Fig.Q1(a)	6	L3	CO1
	b.	Define bipartite graph and complete bipartite graph can a bipartite graph have odd length cycles. Explain.	7	L1	CO1
	c.	Is there a simple graph with 1, 1, 3, 3, 3, 4, 6, 7 as the degree of vertices? Explain.	7	L3	CO1
		OR			
2	a.	Define spanning subgraph and induced subgraph. Draw a complete graph G with 5 vertices and spanning subgraph and induced subgraph of G.	6	L1	CO1
	b.	Verify the following : i) Fig.Q2(b)(i) and Fig.Q2(b)(ii) are isomorphic.  Fig.Q2(b)(i)  Fig.Q2(b)(ii) ii) Fig.Q2(b)(iii) and Fig.Q2(b)(iv) are not isomorphic.  Fig.Q2(b)(iii)  Fig.Q2(b)(iv)	7	L2	CO2
	c.	A simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges.	7	L3	CO2

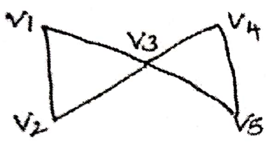
<table>
<tr><td colspan="6">Module – 2</td></tr>
<tr><td>3</td><td>a.</td><td>By specifying the walk draw two Euler graphs and unicursal graph.</td><td>6</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>b.</td><td>If all the vertices in a connected graph G are of even degree, then show that G is Eulerian.</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>c.</td><td>Define and find union, intersection and ring sum of $K_{2,3}$ and $K_{3,3}$.</td><td>7</td><td>L1</td><td>CO2</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>4</td><td>a.</td><td>i) Define reflexive relation, symmetric relation and transitive relation
ii) Draw a symmetric graph and complete asymmetric graph.</td><td>6</td><td>L1
L2</td><td>CO1
CO1</td></tr>
<tr><td></td><td>b.</td><td>Distinguish between Hamiltonian graph and Eulerian graph with two examples by specifying the walk.</td><td>7</td><td>L2</td><td>CO2</td></tr>
<tr><td></td><td>c.</td><td>Prove that a connected graph G has an Euler circuit if and only if G can be decomposed into edge-disjoint cycles.</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td colspan="6">Module – 3</td></tr>
<tr><td>5</td><td>a.</td><td>Prove that a tree with n vertices has n-1 edges.</td><td>6</td><td>L3</td><td>CO1</td></tr>
<tr><td></td><td>b.</td><td>i) Prove that a graph is connected if and only if it has a spanning tree
ii) Identify cut vertices if any in graph Fig.Q5(b)(i),Fig.Q5(b)(ii), Fig.Q5(b)(iii).


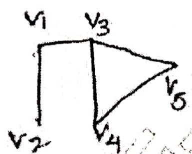
Fig.Q5(b)(i)


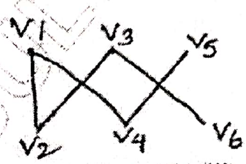
Fig.Q5(b)(ii)


Fig.Q5(b)(iii)</td><td>7</td><td>L3
L2</td><td>CO2
CO2</td></tr>
<tr><td></td><td>c.</td><td>Show that for any graph G, the vertex connectivity cannot exceed the edge connectivity and edge connectivity cannot exceed the degree of the vertex with the smallest degree in G.</td><td>7</td><td>L3</td><td>CO3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>6</td><td>a.</td><td>Prove that a connected graph G is a tree if and only if there is one and only one path between every pair of vertices.</td><td>6</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>b.</td><td>Define a tree and forest. Prove that with two or more vertices in a tree, there are at last two pendent vertices.</td><td>7</td><td>L1</td><td>CO1</td></tr>
<tr><td></td><td>c.</td><td>Show that a Hamiltonian path is a spanning tree. Draw all the spanning trees of the graph Fig.Q6(c).

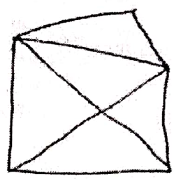



Fig.Q6(c)</td><td>7</td><td>L2</td><td>CO2</td></tr>
<tr><td colspan="6">Module – 4</td></tr>
<tr><td>7</td><td>a.</td><td>i) State Kuratowski's theorem and draw Kuratowski's two graphs
ii) Draw planar graphs of : i) Order 5 and size 8 ii) Order 6 and size 12.</td><td>6</td><td>L1
L3</td><td>CO2
CO2</td></tr>
<tr><td></td><td>b.</td><td>Show that a connected planar graph with n vertices and e-edges has e-n+2 regions.</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>c.</td><td>Draw the geometric dual of graphs Fig.Q7(c)(i) and Fig.Q7(c)(ii).

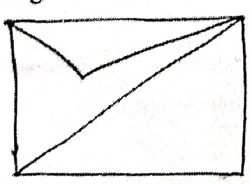



Fig.Q7(c)(i)

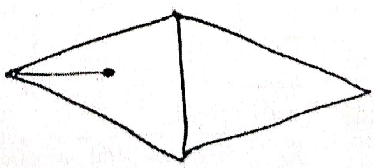
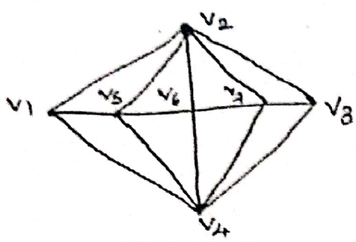
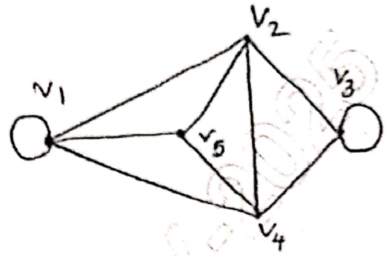
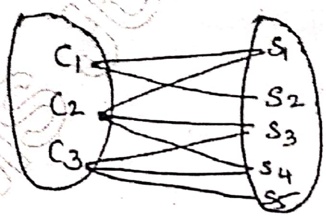
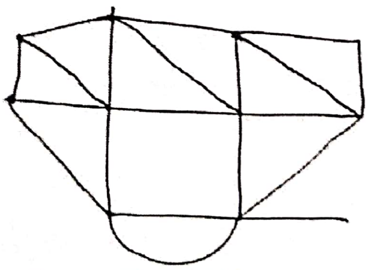



Fig.Q7(c)(ii)</td><td>7</td><td>L2</td><td>CO3</td></tr>
</table>

		OR			
8	a.	i) Show that Kuratowski's first graph K_5 isnon planar ii) Show that every connected simple graph G contains a vertex of degree less than 6.	6	L2 L2	CO2 CO2
	b.	If G is a simple planar graph with at least three vertices then show that : (i) $e \le 3n - 6$ ii) $e \le 2n - 4$ if G is triangle free.	7	L3	CO2
	c.	Write down adjacency matrix, path matrix and circuit matrix for the given graphs Fig.Q8(c)(i) and Fig.Q8(c)(ii).  Fig.Q8(c)(i)  Fig.Q8(c)(ii)	7	L2	CO3
		Module – 5			
9	a.	Prove that a graph with at least one edge is 2-chromatic if and only if it has no circuits of odd length.	6	L3	CO2
	b.	Define chromatic number. Find chromatic polynomial of C_4 of length 4.	7	L2	CO3
	c.	State and prove 5 colour problems.	7	L3	CO2
		OR			
10	a.	Prove that every connected simple planner graph is 6-colourable.	6	L3	CO3
	b.	Define matching and complete matching. Find the two complete matching of :  Fig.Q10(b)	7	L1	CO2
	c.	Define covering and minimal covering of a graph. Obtain two minimal covering from the given graph.  Fig.Q10(c)	7	L2	CO3

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Scheme & Solutions

Signature of Scrutinizer 29/7/24

Subject Title : Graph Theory **Subject Code :** BCS405B

Question Number	Solution	Marks Allocated
1.	a Illustrating appropriate walks	1+1+1+1 +1+1
	b. Explaining the bipartities $V_0, V_1 \ldots V_m$	2M
	Forming the cycle using V_1 & V_2 of the bipartities	2M
	Finally showing no cycle can be formed of odd cycle	3M
	c. Taking 8 vertices and naming them	1M
	Explaining how to construct graph with given degrees	5M
	final graph & conclusion	1M
2	a. Defination : Spanning Subgraph, Induced Subgraph	1½+1½M
	writing K_5 and	1M
	Drawing Spanning Subgraph, induced Subgraph of G	1M+1M
	b.(i) Showing the graphs are isomorphic: u_i & v_i have 1-1 correspondence	3M
	(ii) Proving non isomorphic by comparing adjacencies	4M
	c. Numbering the vertices and arriving till $n_1+n_2+\ldots n_k = n \rightarrow (1)$	½M
	$(n_1-1)^2+(n_2-1)^2+\ldots+(n_k-1)^2+S=(n-k)^2 \rightarrow (2)$	1M
	$S=2(n_i-1)(n_j-1)$, $i \neq j$, taking $n_1 \geq 1 .\ n_2 \geq 1 .\ . n_k \geq 1$, $S \geq 0$	1M
	Expansion of (2)	
	$n_1^2+n_2^2+\ldots+n_k^2 \leq (n-k)^2+2n-k$	1½M
	$N = \frac{1}{2}\sum_{i=1}^{k} n_i(n_i-1)$	1M
	final equation $N = \frac{1}{2}(n-k)(n-k+1)$	2M
3	a. writing 2 graphs with appropriate Euler graph and unicursal graph	3+3M
	b Explaining construction of a circuit of having v as the initial and final vertex by tracing exactly once in G	3M
	if it is not Eulerian, explaining construction of Subgraph H by removing edges of G	3M
	Final Conclusion	1M

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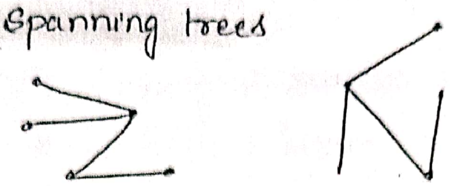
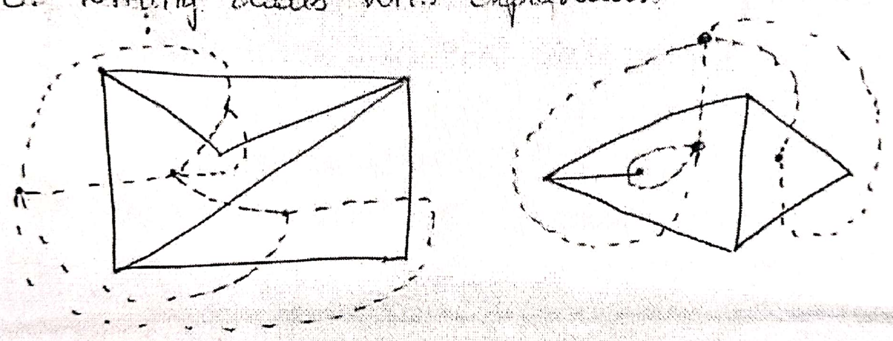
Question Number	Solution	Marks Allocated
	c. Defination: Union, intersection, ring Sum. Drawing $K_{2,3}$ & $K_{3,3}$ constructing Union, Intersection, ring sum of $K_{2,3}$ + $K_{3,3}$	1+1+1+1 1+1+1 M
4	a. Defination: reflexive, symmetric, transitive relation Constructing symmetric digraph & complete asymmetric digraph b. Defination: Hamiltonian graph and Eulerian graph constructing 2 graphs & distinguish between them c. Proving by Induction Showing result true for $n=2$ assuming result true for $n=k$ Explaining and showing the result true for $n=k+1$	1+1+1 M 1½+1½ M 1½+1½ M 2+2 1M 1M 1M 4M
5	a. Proving by Induction Showing result true for $n=1$, $n=2$ & $n=3$ Assuming the result true for all trees with fewer than K vertices Explaining and Showing result true for $n=K$ by taking 2 components T_1 & T_2 b. (i) Explaining neccessary condition & proving (ii) proving sufficiency (iii) Identifying cut vertices as V_3, V_3, V_2 c. Drawing a Graph G finding edge connectivity, Vertex connectivity, Smallest degree conclusion	1M. 1M 1M 4M 2M. 2M 1+1+1 1M 2+2 M 2M 1M
6.	a. proving necessary condition Proving Sufficiency condition b. Defination: Tree and forest Taking degrees of n vertices $d_1, d_2 \dots d_n$ $d_1+d_2+\dots+d_n = 2(n-1)$ Explaing & arriving to $d_2+d_3+\dots+d_n = (2n-2)-1 = 2n-3$ conclusion	3M 3M 1+1 M 1M. 1M. 2M 1M

Subject Title : Graph Theory Subject Code : BCS405B

Question Number	Solution	Marks Allocated
	c. Recalling Hamiltonian path has n vertices and n-1 edges	1M.
	Showing path p is a tree & a spanning tree	$1\frac{1}{2}+1\frac{1}{2}$
	writing spanning trees 	$1\frac{1}{2}+1\frac{1}{2}$.
7	a(i) Statement of theorem	2M.
	writing K_5 and $K_{3,3}$	1M.
	(ii) Appropriate planar graphs with (a) order 5 and size 8	$1\frac{1}{2}$M
	(b) order 6 and size 12	$1\frac{1}{2}$M.
	b. ~~Definition~~ proving by induction	
	Showing result holding good for $m=0$ and $n=1$	2M
	Assuming result true for $m=k$	1M
	Explaining and showing result true for $m=k+1$	4M.
	c. Writing duals with explanation 	$\frac{3}{2}+\frac{3M}{2}$.
8	a.(i) constructing K_5 writing Euler's formula, conclusion	1+1+1M
	(ii) Taking degrees of n vertices ≥ 6	1M
	$d_1+d_2+\ldots+d_n \geq 6M$	½M
	Applying corolory of euler's theorem	1M
	final conclusion	1M
	b. Showing $e \leq 3n-6$ for simple planar graph.	3M
	for triangle free simple planar graph $e \leq 2n-4$	4M.
	c. writing adjacency matrix for both graphs	2M.
	Identifying paths for both graphs + writing path matrix	1+1M

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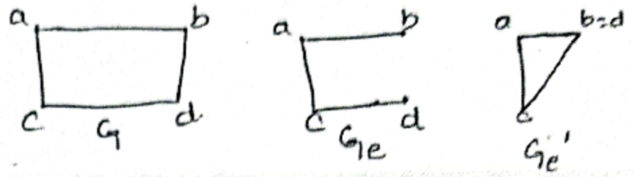
Question Number	Solution	Marks Allocated
	Identifying circuits and writting Circuit Matrix	$1\frac{1}{2}$M + $1\frac{1}{2}$M
9	a. proving neccessary condition	3M
	Proving Sufficiency	3M.
	b. Defination: Chromatic number	2M.
	Drawing 	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$
	finding $P(G_e,\lambda) = \lambda(\lambda-1)^3$, $P(G_e',\lambda) = \lambda(\lambda-1)(\lambda-2)$	1+1
	writting $P(C_4,\lambda) = P(G_e-\lambda) - P(G_e',\lambda) = \lambda^4 - 4\lambda^3 + 6\lambda^2 - 3\lambda$	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$.
	c. Theorem Statement	2M.
	explaining by taking 5 vertices and conclusion	5M
10	a. proving by induction	
	Showing result is true for $n=6$ vertices	1M
	Assuming result true for $n=k$	1M.
	Explaining and showing the result true for $n=k+1$ & writting conclusion	4M.
	b. Defination: Matching and complete matching	2+2M.
	writting appropriate two complete matching	$1\frac{1}{2}+1\frac{1}{2}$M
	c. Defination: covering and minimal covering	2+2M.
	writting two appropriate minimal covering	$1\frac{1}{2}+1\frac{1}{2}$M