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CMR INSTITUTE OF TECHNOLOGY	USN ☐☐☐☐☐☐☐☐☐☐ Internal Assessment Test I May 2025							CMRIT
Sub:	Mathematics for EC						Code:	BMATE201
Date:	02/05/2025	Duration:	90 mins	Max Marks:	50	Sem: II	Branch:	EC

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE	
			CO	RBT
1	The area of a circle (A) corresponding to diameter (D) is given below.	[8]	CO4	L1,L2,L3
2	Use Lagrange's formula, find the interpolating polynomial that approximates to the function described by the following data: $f(5) = 12,\ f(6) = 13,\ f(9) = 14,\ f(11) = 16.$ Hence find y at x=10.	[7]	CO4	L1,L2,L3
3	Employ Taylor's series method to find y at $x = 4.1$ given that $\frac{dy}{dx} = \frac{1}{x^2+y}$ and $y(4) = 4.$	[7]	CO4	L1,L2,L3

Table for Question 1:

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

4	Find the cube root of 37 correct to 3 decimal places using the Newton-Raphson method.	[7]	CO4	L1,L2,L3
5	Evaluate $\int_{1}^{4} e^{1/x} dx$ by Simpson's 3/8th rule, taking 4 ordinates.	[7]	CO4	L1,L2,L3
6	Show that $F = (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2yz^2 + xy)\hat{k}$ is irrotational. Also check whether F is solenoidal.	[7]	CO1	L2,L
7	Verify Green's theorem for $\int_{C} (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.	[7]	CO1	L2,
8	Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(1, -1, 1)$.	[7]	CO1	L2,

1. * Since we need to find ~~a~~ Area when diameter is 105 and it is in the end of table.
* We use Newton Backward interpolation formula.
* The ~~Forward~~ Backward difference table is as follows:

x	y	$\nabla^1 y$	~~Δ~~ $\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
80	5026 (y_0)				
		648 (∇y_0)			
85	5674		40 $(\nabla^2 y_0)$		
		688		-2 $\nabla^3 y_0$	
90	6362		38		4 $\nabla^3 y_n$
		726		2 $\nabla^3 y_n$	
95	7088		40 $(\nabla^2 y_n)$		
		766 (∇y_n)			
100	7854 (y_n)				

* Substituting the values in the formula -

$$y_r = y_n + \frac{r}{1!}\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_n + \frac{r(r+1)(r+2)(r}{4!}\nabla$$

$$y(105) = 7854 + 766 + \frac{1(1+1)}{2}(40) + \frac{1(1+1)(1+2)(2)}{6} + \frac{1(1+1)(1+2)(1+3)(4)}{24}$$

where $r = \frac{x - x_n}{h}$

$r = \frac{105-100}{5}$

$r = \frac{5}{5} = 1$

$$= 7854 + 766 + 40 + 2 + 4$$

$$\boxed{y(105) = 8666}$$

$\therefore$ The Area when diameter is 105 is 8666 Sq.units

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formula : $\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$

$\times f(x_1)$ .... so on.

$$\Rightarrow \frac{(x-6)(x-9)(x-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(x-5)(x-9)(x-11)}{(6-5)(6-9)(6-11)} \times 13 +$$

$$\frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)(9-11)} \times 14 + \frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)} \times 16$$

$$\Rightarrow \frac{(x-6)(x-9)(x-11)(12)}{-24} + \frac{(x-5)(x-9)(x-11)(13)}{15}$$

$$+ \frac{(x-5)(x-6)(x-11)(14)}{-24} + \frac{(x-5)(x-6)(x-9)(16)}{60}$$

substitute $x = 10$;

$$\Rightarrow \cancel{(4)(1)}\left(\frac{(10-6)(10-9)(10-11)(12)}{-24} + \frac{(10-5)(10-9)(10-11)(13)}{15}\right.$$

$$+ \frac{(10-5)(10-6)(10-11)(14)}{-24} + \frac{(10-5)(10-6)(10-9)(16)}{60}$$

$$\Rightarrow \frac{(4)(1)(-1)(12)}{-24} + \frac{(5)(1)(-1)(13)}{15} + \frac{(5)(4)(-1)(14)}{-24}$$

$$+ \frac{(5)(4)(1)(16)}{60}$$

$$\Rightarrow 2 + \left(\frac{-13}{3}\right) + \frac{35}{3} + \frac{16}{3} \Rightarrow 12.66\cancel{7}7 + 2 \Rightarrow \underline{14.667}$$

3. Given, $y' = \frac{1}{x^2+y}$ $\Rightarrow y'(x_0) = \frac{1}{x_0^2 + y_0}$

$x_0 = 4$

$y_0 = 4$

$$= \frac{1}{4^2 + 4}$$

$$= \frac{1}{16 + 4}$$

$$y'(x_0) = \frac{1}{20} = 0.05$$

$$y' = \frac{1}{x^2+y}$$

$$(x^2+y)\,y' = 1$$

Diff. wrt x,

$$(x^2+y)\,y'' + (2x + y')y' = 0$$

$$(x^2+y)y'' + 2xy' + y'y' = 0$$

$$y'' = \frac{-2xy' - yy'}{(x^2+y)} \Rightarrow y''(x_0) = \frac{-2(x_0)(y_0') - y'(x \\ y'(x}{x_0^2 + y_0}$$

$$\Rightarrow y''(x_0) = \frac{-2(4)\left(\frac{1}{20}\right) - \left(\frac{1}{20}\right)\left(\frac{1}{20}\right.}{(4)^2 + 4}$$

$$\Rightarrow y''(x_0) = \frac{-0.4 - 0.0025}{16+4}$$

$$y''(x_0) = -0.020125$$

Taylor's series expansion:

$$y = f(x) = y_0 + \frac{(x-x_0)^1}{1!}y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0)$$

$$y(4.1) = 4 + (4.1-4)0.05 + \frac{(4.1-4)^2}{2}(-0.020125)$$

$= 4 + 0.005 - 0.000100625$

$\boxed{y(4.1) = 4.0049}$

1. Given, $x = \sqrt[3]{37}$

$x^3 = 37$

$x^3 - 37 = 0$

Let $\underline{x_0 = 3}$

$\therefore f(x_0) = (3)^3 - 37$

$\underline{f(x_0) = -10}$

$f'(x) = 3x^2$

$f'(x_0) = 3(3)^2$

$\underline{f'(x_0) = 27}$

1st iteration

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{(-10)}{27}$$

$$= 3 + 0.37037$$

$$x_1 = 3.37037$$

2nd iteration Now, $(x_0 = 3.37037)$

$$x_2 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3.37037 - \frac{(3.37037)^3 - 37}{3(3.37037)^2}$$

$$= 3.37037 - \frac{1.2854}{34.078}$$

$$= 3.37037 - 0.0377$$

$$x_2 = 3.3327$$

3rd iteration   Now, ($x_0 = 3.3327$).

$$x_3 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_3 = 3.3327 - \frac{(3.3327)^3 - 37}{3(3.3327)^2}$$

$$= 3.3327 - \frac{0.01592}{33.320}$$

$$= 3.3327 - 0.0004778$$

$$\boxed{x_3 = 3.3322}$$

$\therefore$ Cube root of 37 is 3.3322

5. $\int_1^4 e^{1/x}\,dx$

By taking 4 ordinates indicates that we have to divide the interva [1,4] in 3 equal parts.

x	y
1	$e = 2.7182$ (y_0)
2	$e^{0.5} = 1.6487$ (y_1)
3	$e^{1/3} = 1.3956$ (y_2)
4	$e^{1/4} = 1.2840$ (y_3)

<u>simpson's $\frac{3}{8}$th rule</u>

⇒ $\int \cancel{ydx}$ $\frac{3h}{8}\left[[y_0+y_n] + 2[y_3+y_6 \ldots] + 3[y_1+y_2 \ldots y_n\right.$

here, the formula becomes

$$\frac{3\times1}{8}\left[(y_0+y_3) + \cancel{2[0]} + 3[y_1+y_2]\right]$$

$$\Rightarrow \frac{3}{8}\left[[2.7182+1.2840] + 3[1.6487+1.3956]\right]$$

$$\Rightarrow \frac{3}{8}\left[4.0022 + 9.1329\right]$$

$$\Rightarrow \frac{3}{8}(13.1351)$$

$$\Rightarrow \underline{4.9256}$$

6. $\vec{F} = (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2yz^2 + xy)\hat{k}$

$$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy^2 + yz & 2x^2y + xz + 2yz^2 & 2yz^2 + xy \end{vmatrix}$$

$$i\left[\frac{\partial}{\partial y}(2yz^2 + xy) - \frac{\partial}{\partial z}(2x^2y + xz + 2yz^2)\right] - j\left[\frac{\partial}{\partial x}(2yz^2 + xy) - \frac{\partial}{\partial z}(2xy^2 +$$

$$+ k\left[\frac{\partial}{\partial x}(2x^2y + xz + 2yz^2) - \frac{\partial}{\partial y}(2xy^2 + yz)\right]$$

$$\Rightarrow i\left[(2z^2 + x) - (\cancel{x} + 4yz)\right] - j\left[\overset{0}{\cancel{y - y}}\right] + \left[(4xy \overset{0}{\cancel{+ z)}} - (4xy + z$$

$$\Rightarrow i(2z^2 + x - x - 4yz)$$

$$\Rightarrow \underline{i\,(2z^2 - 4yz)}$$

since curl $\vec{F} \neq 0$, $\vec{F}$ is not irrotational.

<u>solenoidal</u>

div.$\vec{F}$ must be 0.

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left((2xy^2 + yz)i + (2x^2y + xz + 2yz^2)j + (2yz^2 + xy)\hat{k}\right)$$

$$\Rightarrow (2xy^2+yz)\frac{\partial}{\partial x} + \frac{\partial}{\partial y}(2x^2y + xz + 2yz^2) + \frac{\partial}{\partial z}(2yz^2 + xy)$$

$$\Rightarrow 2y^2 + 2x^2 + 2z^2 + 4yz$$

$$\Rightarrow \underline{2y^2 + 2x^2 + 2z^2 + 4yz}$$

$$\text{div}\vec{F} = 2\left[x^2+y^2+z^2+2yz\right]$$

since $\text{div}\vec{F} \neq 0$; $\vec{F}$ is not solenoidal.

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7. Green theorem

$$\int_C (xy+y^2)\,dx + x^2dy \text{ bounded by } y=x \text{ and } y=x^2$$

we know that,

$$\int_C Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dxdy$$

$M = xy+y^2$; $N = x^2$

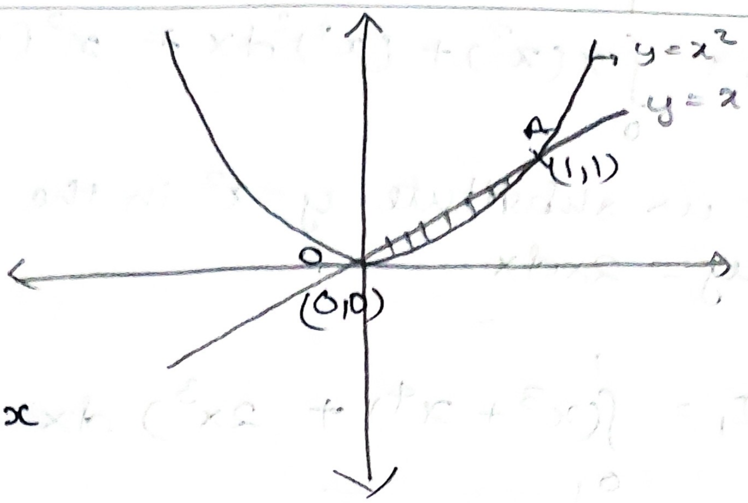
$\frac{\partial M}{\partial y} = x+2y$; $\frac{\partial N}{\partial x} = 2x$

$$\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = 2x - x - 2y$$

$$\Rightarrow \underline{\underline{x-2y}}$$

The intersecting points of the curves are (0,0) and (1,1)

Solving RHS



$$\iint_R (x-2y)\,dx\,dy$$

changing limits,

$$\Rightarrow \int_0^1\int_{x^2}^{x} (x-2y)\,dy\,dx$$

$$\Rightarrow \int_0^1 \left. xy - \cancel{2}\frac{y^2}{\cancel{2}}\right]_{x^2}^{x} dx$$

$$\Rightarrow \int_0^1 (x^2-x^3)-(x^2-x^4)\,dx$$

$$\Rightarrow \int_0^1 \frac{\cancel{x^3}}{\cancel{3}} - \frac{x^4}{4} - \frac{\cancel{x^3}}{\cancel{3}} + \frac{x^5}{5}$$

$$\Rightarrow \left[\frac{x^5}{5}-\frac{x^4}{4}\right]_0^1 \Rightarrow \frac{1}{5}-\frac{1}{4} = \frac{-1}{20}$$

Solving LHS

$$\oint_C (xy+y^2)\,dx + x^2dy$$ (limit being 0 to 1)

$$I_1 = \int_{OA} (xy+y^2)\,dx + x^2dy.$$

$$I_2 = \int_{AO} (xy+y^2)\,dx + x^2dy$$

$$I_1 = \int_0^1 x(x^2) + (x^2)^2 dx + x^2(2x dx)$$

let us substitute $y = x^2$ in the given eq. then

$dy = 2x dx$

$$\Rightarrow I_1 = \int_0^1 (x^3 + x^4) + 2x^3) dx$$

$$\Rightarrow \left[\frac{3x^4}{4} + \frac{x^5}{5}\right]_0^1 \Rightarrow 3\left[\frac{1}{4} + \frac{1}{5}\right] = \frac{19}{20}$$

$\Rightarrow$

$$I_2 = \int_1^0 (y^2 + y^2) dy + y^2 dy$$

Let us substitute $y = x$ in the given eq. as it is a closed path.

$$I_2 = \int_1^0 (2y^2 + y^2) dy$$

$$\int_1^0 3y^2 dy \Rightarrow \left[\frac{\cancel{3}y^3}{\cancel{3}}\right]_1^0 \Rightarrow (-1)^3 = -1$$

$\therefore$ ~~B~~ $I = I_1 + I_2$

$$\Rightarrow \frac{19}{20} - 1 = \frac{-1}{20}$$

hence green's theorem proved

LHS = RHS

8. $x^2+y^2+z^2=9$ —(I) $\Rightarrow$ $x^2+y^2+z^2-9=0$

$x^2+y^2-3=z$ —(II) $\Rightarrow$ $x^2+y^2-3-z=0$

Angle between 2 surfaces ·

$$\Rightarrow \cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1| \cdot |\nabla\phi_2|}$$

$$\nabla\phi_1 = \frac{\partial}{\partial x}(x^2+y^2+z^2-9)\hat{i} + \frac{\partial}{\partial y}(x^2+y^2+z^2-9)\hat{j} + \frac{\partial}{\partial z}(x^2+y^2+z^2-9)\hat{k}$$

$\Rightarrow$ $2x\hat{i}+2y\hat{j}+2z\hat{k}$

$\nabla\phi_1$ at $(1,-1,1)$

$\Rightarrow$ $\cancel{2-2+2}$ $2\hat{i} -2\hat{j}+2\hat{k}$.

$$|\nabla\phi_1| = \frac{2\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{4+4+4}} \Rightarrow \frac{2\hat{i}-2j+2k}{\sqrt{12}}$$

$$\nabla\phi_2 = \frac{\partial}{\partial x}(x^2+y^2-z-3)\hat{i} + \frac{\partial}{\partial y}(x^2+y^2-z-3)\hat{j} + \frac{\partial}{\partial z}(x^2+y^2$$

$\Rightarrow$ $2x\hat{i}+2y\hat{j}-\hat{k}$

$\nabla\phi_2$ at $(1,-1,1)$

$\Rightarrow$ $2\hat{i}-2\hat{j}-\hat{k}$

$$|\nabla\phi_2| = \frac{2\hat{i}-2\hat{j}-\hat{k}}{\sqrt{2^2+2^2+1}} \Rightarrow \frac{2\hat{i}-2\hat{j}-\hat{k}}{\sqrt{9}} = \frac{2\hat{i}-2\hat{j}-\hat{k}}{3}$$

$$\cos\theta = \frac{2\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{12}} \cdot \frac{2\hat{i} - 2\hat{j} - \hat{k}}{\cancel{6}\ 3}$$

$$\cos\theta = \frac{4+4-2}{3\sqrt{12}} \Rightarrow \frac{\cancel{6}\,2}{\cancel{3}\sqrt{12}} \Rightarrow \frac{\cancel{2}}{\cancel{2}\sqrt{3}} =$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \underline{\underline{54.7356}}$$

hence the angle between 2 surfaces is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ or $\underline{\underline{54.7356^{\circ}}}$.