USN

Internal Assessment Test II – JUNE 2025

Sub:	Mathematics – II for CSE Stream					Sub Code:	BMATS201	
Date:	17/06/2025	Duration:	90 mins	Max Marks:	50	Sem / Sec:	II / A-H	OBE

	Question 1 is compulsory. Answer any SIX from the remaining questions.	MARKS	CO	RBT
1.	Apply Milne's predictor-corrector method to compute $y(1.4)$ for the differential equation $\frac{dy}{dx}=x^2+\frac{y}{2}$, given $y(1)=2, y(1.1)=2.2156, y(1.2)=2.4649, y(1.3)=2.7514$	[8]	CO5	L3
2.	Determine whether the matrix $\begin{pmatrix}-1 & 7\\ 8 & -1\end{pmatrix}$ is a linear combination of $\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix},\begin{pmatrix}2 & -3\\ 0 & 2\end{pmatrix}$ and $\begin{pmatrix}0 & 1\\ 2 & 0\end{pmatrix}$ in the vector space M_{22} of 2×2 matrices.	[7]	CO3	L3
3.	Prove that the subset $W=\{(x,y,z)\ /\ x-3y+4z=0)\}$ of the vector space R^3 is a subspace of R^3.	[7]	CO3	L3
4.	Using Newton-Raphson method, find the root that lies near $x=4.5$ of the equation $tanx=x$. Correct to four decimal places.	[7]	CO4	L3
5.	Use Newton's appropriate formulae to find f (8) and f (22) given that	[7]	CO4	L3

x	0	5	10	15	20	25
y=$f(x)$	7	11	14	18	24	32

6.	Use Simpson's 1/3rd rule to find $\int_0^{0.6} e^{-x^2}dx$ by taking 6 sub-intervals.	[7]	CO4	L3
7.	Employ Taylor's series method to solve the initial value problem $\frac{dy}{dx}=x-y^2\,;y(0)=1$ at the point $x=0.1$ by considering up to 4th degree terms.	[7]	CO5	L3
8.	Apply Runge-Kutta method of forth order to find an approximate solution at $x=0.1$ given $\frac{dy}{dx}=3x+\frac{y}{2}, y(0)=1\ taking\ h=0.1.$	[7]	CO5	L3

Internal Assesment - II

1.

x	y	$\frac{dy}{dx} = x^2 + \frac{y}{2}$
1	2 (y_0)	2
1.1	2.2156 (y_1)	2.3~~88~~178 (y_1')
1.2	2.4649 (y_2)	2.67245 (y_2')
1.3	2.7514 (y_3)	3.0657 (y_3')

$$y_4^p = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$

$$y_4^c = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4')$$

$h = 0.1$, $y_0 = 2$

$$y_4^{(p)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$

$$= 2 + \frac{4 \times 0.1}{3}\left(2 \times 2.3178 - 2.67245 + 2(3.0657)\right)$$

$$= 2 + 01{,}333'(4.6356 - 2.67245 + 6.1314)$$

$= ~~[illegible]~~$

$$= 2 + 1.07900$$

$$= 3.079003515$$

$~~f(1.4~~ \; f(1.4, 3.079) = (1.4)^2 + \frac{(3.079)}{2}$

$$\boxed{y'^{p}_{p_4} = 3.4995}$$

$$y_4^c = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4')$$

$$y_4^c = 2.4649 + \frac{0.1}{3}(2.67243 + 4 \times 3.0657 + 3.4995)$$

$$= 2.4649 + 0.0333(18.4347)$$

$$= 2.4649 + 0.61358$$

$$\boxed{= 3.0787}$$

$$\boxed{y(1.4) = 3.0787}$$

Q)

2) $\begin{pmatrix}-1 & 7\\ 8 & -1\end{pmatrix} = a\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix} + b\begin{pmatrix}2 & -3\\ 0 & 2\end{pmatrix} + c\begin{pmatrix}0 & 1\\ 2 & 0\end{pmatrix}$

$$\begin{pmatrix}-1 & 7\\ 8 & -1\end{pmatrix} = \begin{pmatrix}a & 0\\ 2a & a\end{pmatrix} + \begin{pmatrix}2b & -3b\\ 0 & 2b\end{pmatrix} + \begin{pmatrix}0 & c\\ 2c & 0\end{pmatrix}$$

$$\begin{pmatrix}-1 & 7\\ 8 & -1\end{pmatrix} = \begin{pmatrix}a+2b & -3b+c\\ 2a+2c & a+2b\end{pmatrix}$$

$\therefore \quad a+2b = -1 \quad$ —①

$-3b+c = 7 \rightarrow \boxed{b = \frac{7-c}{-3}}$

$2a+2c = 8 \rightarrow a+c = 4$

$\boxed{a = 4-c}$

putting values of a & b in ①

$$4-c+2\left(\frac{7-c}{-3}\right) = -1$$

$$4-c+2\left(\frac{c-7}{3}\right) = -1$$

$$4-c+\frac{2c}{3} - \cancel{7}\frac{14}{3} = -1$$

$$-\frac{3c+2c}{3} = -1+\frac{14}{3}-4$$

$$\frac{-c}{3} = \frac{-3+14-12}{3}$$

$$-c = -1$$

$$\boxed{c = 1}$$

$b = \frac{7-1}{-3} = \frac{6}{-3} = -2$

$a = 4-1 = 3$

It is a linear Combinations & $a=3$, $b=-2$, $c=1$

Q.no. 3)

solution: Given $W = \{(x,y,z) / x-3y+4z=0)\}$

let $\alpha, \beta \in R$

We need to show, $c_1\alpha + c_2\beta \in W$, where. $c_1 c_2 \in R$

Eg let,
$$\left.\begin{aligned}\alpha &= x_1, y_1, z_1\\ \beta &= x_2, y_2, z_2\\ \alpha &= x_1 - 3y_1 + 4z_1\\ \beta &= x_2 - 3y_2 + 4z_2\end{aligned}\right\} - ①$$

Now,
$$c_1\alpha + c_2\beta = c_1(x_1, y_1, z_1) + c_2(x_2, y_2, z_2)$$
$$= (c_1x_1 + c_2x_2, c_1y_1 + c_2y_2 + c_1z_1 + c_2z_2)$$
$$= (c_1x_1 + c_2x_2) - 3(c_1y_1 + c_2y_2) + 4(c_1z_1 + c_2$$
$$= c_1(x_1 - 3y_1 + 4z_1) + c_2(x_2 - 3y_2 + 4z_2)$$
$$= c_1 \cdot 0 + c_2 \cdot 0$$
$$= 0$$
$$= 0 \in W$$

Therefore, the subset W of the vector space R^3 is a of R^3.

4. $f(x) = \tan x - x$, $x = 4.5$

let, $x_0 = 1$, $f(1) = 0.5574 > 0$

$x_0 = 2$, $f(2) = -4.185 < 0$

$x_0 = 3$, $f(3) = -3.142 < 0$

$x_0 = 4$, $f(4) = -2.842 < 0$

$f'(x) = \sec^2 x - 1 = \tan^2 x$

Using Newton Raphson method

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{0.5574}{2.455}$$

$$= 0.77$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.77 - \left(\frac{0.1996}{0.94}\right)$$

$$= 0.557$$

let $x_0 = 4.6$

$$x_1 = 4.6 - \frac{4.26}{18.5}$$

$$x_1 = 4.5457$$

$x_1 = \cdot \cdot$

$$x_2 = 4.5457 - \frac{5.9}{34.8976}$$

x

5.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	
0	7					
		4				
5	11		-1			
		3		2		
10	14		1		-1	
		4		1		0
15	18		2		-1	
		6		0		
20	24		2			
		8				
25	32					

To find $f(8)$ using Newton's forward interpolation:

$h = x_1 - x_0 = 5$, $x = 8$

$$r = \frac{x - x_0}{h} = \frac{8-0}{5} = 1.6$$

$$f(8) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0$$

$$= 7 + 1.6(4) + \frac{1.6(0.6)}{2}(-1) + \frac{(1.6)(0.6)(-0.4)}{6}(2) + \frac{(1.6)(0.6)(-0.4)(-1.4)}{24}$$

$$\boxed{f(8) = 11.6176}$$

To find f(22) using Newton's backward interpolation method.

$r = 1.6 \quad h = 5$

$$f(22) = y_0 + r\nabla y_0 + \frac{r(r+1)}{2!}\nabla^2 y_0 + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_0 + \frac{r(r+1)(r+2)(r+3)}{4!}$$

$$= 32 + 1.6(8) + \frac{(1.6)(2.6)}{2}(2) + \frac{(1.6)(2.6)(3.6)}{6}(0) + \frac{(1.6)(2.6)(3.6)(4.6)}{24}(-1)$$

$$= 26.8$$

6 $\int_{0}^{0.6} e^{-x^2}\,dx$

$n = 6$ $\qquad x_0 + nh = 0.6$, $x_0 = 0$

$f(x) = e^{-x^2}$ $\qquad 0 + 6h = 0.6$

$h = 0.1$

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$: y	1	0.99	0.96	0.91	0.852	0.77	0.69

Using Simpson's $\frac{1}{3}^{rd}$ rule :

$$I = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + y_5 \cdots y_{n-1}) + 2(y_2 + y_4 + \cdots y_{n-2}\right.$$

$$I = \frac{0.1}{3}\left[(1 + 0.69) + 4(0.99 + 0.91 + 0.77) + 2(0.96 + 0.852)\right]$$

~~= 0.583~~

$I = 0.53313$

$\int_{0}^{0.6} e^{-x^2}\,dx$ by 6 sub-intervals is 0.53313

⑦ $y' = x - y^2$

$y(0) = 1$

$x_0 = 0$

$y_0 = 1$

$h = 0.1$

$y' = x - y^2$

$y'' = 0 - 2yy'$

$y''' = -2[yy'' + y'y']$

$y'''' = -2[yy''' + (\overset{y''}{y''})^2 + y'y'' + y'y'']$

$y'''' = -2[yy''' + (y'')^2 + 3(y'y'')]$.

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \frac{(x-x_0)^4}{4!}$$

$y''''(x_0)$.

$y'(0) = -1$

$y''(0,1) = -2(1)(-1) = 2$.

$y'''(0,1) = -2(2+1) = -6$

$y''''(0,1) = -2(-6+4+2(-1\times 2) = -2(-6+4+2(-2)) = -2(-4) = -8$.

$$y(\overset{0.1}{x}) = 1 + (0.1-0)(-1) + \frac{(0.1)^2}{2}(2) + \frac{(0.1)^3}{6}(-6) + \frac{(0.1)^4}{24}(-8$$

$y(0.1) = 0.9 + 0.01 - 0.001 - 0.000033$

$y(0.1) =$ ~~0.8896~~ 0.90896

$y(0.1) = 0.9 + 0.01 - 0.001 - 0.0001$

$y(0.1) = 0.8889$

8) $x = 0.1$

$x_0 = 0$

$y_0 = 1$

$h = 0.1$

$\frac{dy}{dx} = 3x + \frac{y}{2}$

$k_1 = h\ f(x_0, y_0)$

$= 0.1\ f(0, 1)$

$= 0.1 \times 0.5$

$\boxed{= 0.05}$

$k_2 = h\ f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$k_2 = 0.1\ f(0.05, 1.025)$

$= 0.1 \times \left(3(0.05) + \frac{(1.025)}{2}\right)$

$0.1 \times (0.15 + 0.5125)$

0.1×0.6625

$\boxed{= 0.06625}$

$k_3 = 0.1\ f\left(0.05, y_0 + \frac{k_2}{2}\right)$

$= 0.1 \times f(0.05, 1.033)$

$= 0.1 \times \left(3(0.05) + \frac{(1.033)}{2}\right)$

$0.1 \times (0.15 + 0.5165)$

$= \boxed{0.06665}$

$K_4 = hf(x_0 h, y_0 + k_3)$

$hf(0.1, 1.06665)$

$0.1 \times \left(3(0.1) + \frac{1.06665}{2}\right)$

$0.1 \times (30.3 + 0.533325$

$\boxed{= 0.0833325}$

$y = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$1 + \frac{1}{6}(0.05 + 2(0.06625) + 2(0.06665) + 0.0833325)$

$1 + 0.066522$

$\boxed{= 1.06652}$