USN

Internal Assessment Test – II June - 2025

Sub:	Mathematics-II for EC Stream							Code:	BMATE201
Date:	17-06-2025	Duration:	90 mins	Max Marks:	50	Sem:	II		**BRANCH:ECE**

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE CO	OBE RBT
1	Solve the differential equation by using the Laplace transform method $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t},\ y(0) = 0, y'(0) = 0$	[08]	CO3	L3
2	Apply Milnes predictor-corrector formula to compute y at 0.8 , given $\frac{dy}{dx} = x - y^2$, Given y(0) = 0 , y(0.2) = 0.02 , y(0.4) = 0.0795 , y((0.6) = 0.1762	[07]	CO4	L3
3	Given $\frac{dy}{dx} = 1 + \frac{y}{x}$, y=2 at x=1 find y at x=1.4, by using modified Eulers method	[07]	CO4	L3

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3	Given $\frac{dy}{dx} = 1 + \frac{y}{x}$, y=2 at x=1 find y at x=1.4, by using modified Eulers method	[07]	CO4	L3

4	Define a subspace. Show that the intersection of two subspaces of a vector space V is also a subspace of V.	[07]	CO2	L3
5	Define linearly independent set of vectors and linearly dependent set of vectors. Are the vectors $V_1 = (2, 5, 3)$, $V_2 = (1, 1, 1)$, and $V_3 = (4, -2, 0)$ linearly independent? Justify your answer.	[07]	CO2	L3
6	Let $T: V_3(R)\ to\ V_3(R)$ be a linear transformation defined by $T(x, y, z) = (x + y, x - y, 2x + z)$. Find the range, null space, and hence verify the rank-nullity theorem.	[07]	CO2	L3
7	Find the Laplace transform of the triangular wave function $f(t) = \{t,\ if\ 0 \le t \le a, \quad (2a - t),\ if\ a \le t \le 2a\}, f(t + 2a) = f(t)$.	[07]	CO3	L3
8	Express $f(t) = \{cost,\ if\ 0 < t < \pi,\ cos2t,\ if\ \pi < t < 2\pi,\ cos3t,\ if\ t > 2\pi\}$ in terms of the Heaviside unit step function and hence find $L\{f(t)\}$.	[07]	CO3	L3

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Thus, $$\boxed{y(t) = \frac{1}{2}(\cosh kt + \cos kt)}$$

Q1 *Solve the following initial value problem by using Laplace transforms :*

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t} \; ; \; y(0) = 0, \; y'(0) = 0$$ [June 2016]

☞ The given equation is $y''(t) + 4y'(t) + 4y(t) = e^{-t}$

Taking Laplace transform on both sides we have,

$$L[y''(t)] + 4L[y'(t)] + 4L[y(t)] = L(e^{-t})$$

ie., $$\{s^2 L[y(t)] - s\,y(0) - y'(0)\} + 4\{s L[y(t)] - y(0)\} + 4L[y(t)] = \frac{1}{s+1}$$

Using the given initial conditions we obtain,

$$L[y(t)]\{s^2 + 4s + 4\} = \frac{1}{s+1} \quad \text{or} \quad L[y(t)] = \frac{1}{(s+1)(s+2)^2}$$

$\therefore \quad y(t) = L^{-1}\left[\frac{1}{(s+1)(s+2)^2}\right]$ and by partial fractions,

$$\frac{1}{(s+1)(s+2)^2} = \frac{1}{s+1} + \frac{-1}{s+2} + \frac{-1}{(s+2)^2}$$

$$\therefore \quad L^{-1}\left[\frac{1}{(s+1)(s+2)^2}\right] = L^{-1}\left[\frac{1}{s+1}\right] - L^{-1}\left[\frac{1}{s+2}\right] - L^{-1}\left[\frac{1}{(s+2)^2}\right]$$

ie., $$y(t) = e^{-t} - e^{-2t} - e^{-2t} L^{-1}\left(\frac{1}{s^2}\right)$$

Thus, $$\boxed{y(t) = e^{-t} - e^{-2t} - e^{-2t}\,t = e^{-t} - (1+t)e^{-2t}}$$

WORKED PROBLEMS

Q2 *Given that* $\frac{dy}{dx} = x - y^2$ *and the data* $y(0) = 0, y(0.2) = 0.02$, $y(0.4) = 0.0795, y(0.6) = 0.1762$. *Compute* $y(0.8)$ *by applying Milne's method.*

[June 18, Sep 20, Feb 21]

☞ We prepare the following table using the given data which is essentially required for applying the predictor and corrector formulae.

x	y	$y' = x - y^2$
$x_0 = 0$	$y_0 = 0$	$y_0' = 0 - 0^2 = 0$
$x_1 = 0.2$	$y_1 = 0.02$	$y_1' = 0.2 - (0.02)^2 = 0.1996$
$x_2 = 0.4$	$y_2 = 0.0795$	$y_2' = 0.4 - (0.0795)^2 = 0.3937$
$x_3 = 0.6$	$y_3 = 0.1762$	$y_3' = 0.6 - (0.1762)^2 = 0.5689$
$x_4 = 0.8$	$y_4 = ?$	

We have the predictor formula : $y_4^{(P)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$

$$\therefore \quad y_4^{(P)} = 0 + \frac{4(0.2)}{3}[2(0.1996) - 0.3937 + 2(0.5689)] = 0.3049$$

Now, $y_4' = x_4 - y_4^2 = 0.8 - (0.3049)^2 = 0.707$

Next we have the corrector formula : $y_4^{(C)} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4')$

$$y(0.8) = y_4^{(c)} = 0.0795 + \frac{0.2}{3}[0.3937 + 4(0.5689) + 0.707] = \boxed{0.3046}$$

Q3 *Apply Milne's method to compute* $y(1.4)$ *corrector to four decimal places given*

dy ...

$x = 1.4$ by ... [Dec 18, Sep 2...

☞ The problem has to be worked in two stages.

I Stage : $x_0 = 1,\ y_0 = 2,\ f(x,y) = 1 + (y/x),\ h = 0.2$

$x_1 = x_0 + h = 1.2,\ y(x_1) = y_1 = y(1.2) = ?\ ;\ f(x_0, y_0) = 1 + (2/1) = 3$

We have Euler's formula : $y_1^{(0)} = y_0 + h\, f(x_0, y_0)$ ...

$$\therefore \quad y_1^{(0)} = 2 + (0.2)(3) = 2.6$$

Further we have modified Euler's formula :

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$ ...

$$y_1^{(1)} = 2 + (0.1)[3 + (1 + y_1^{(0)}/x_1)] = 2 + (0.1)[4 + 2.6/1.2] = 2.6\ldots$$

Next approximation $y_1^{(2)}$ is got just by replacing the value of $y_1^{(1)}$ in place of ...

Now, $y_1^{(2)} = 2 + (0.1)[4 + 2.6167/1.2] = 2.6181$

$y_1^{(3)} = 2 + (0.1)[4 + 2.6181/1.2] = 2.6182\ ;\ y_1^{(4)} = 2.6182 = y(1.2)$

II Stage : We repeat the process by taking $y(1.2) = 2.6182$ as the i... condition.

$$x_0 = 1.2,\ y_0 = 2.6182\ ;\ f(x_0, y_0) = 1 + (y_0/x_0) = 3.1818$$

$$x_1 = x_0 + h = 1.4,\ y(x_1) = y_1 = y(1.4) = ?$$

From (1), $y_1^{(0)} = 2.6182 + (0.2)(3.1818) = 3.2546$

From (2), $y_1^{(1)} = 2.6182 + (0.1)[3.1818 + (1 + y_1^{(0)}/x_1)]$

i.e., $y_1^{(1)} = 2.6182 + (0.1)[4.1818 + 3.2546/1.4] = 3.2689$

$y_1^{(2)} = 2.6182 + (0.1)[4.1818 + 3.2689/1.4] = 3.2699$

$y_1^{(3)} = 2.6182 + (0.1)[4.1818 + 3.2699/1.4] = 3.2699$

Thus,

$$\boxed{y(1.4) = 3.2699 \approx 3.27}$$

Q4

4. Define a Subspace. Show that the intersection of two subspaces of a vector space V is also a subspace of V.

Sol^n: A subset V' of a vector space $V(F)$ is called a subspace of $V(F)$ if V' is also a vector space over F.

Let U and W be subspaces of the vector space $V(F)$

We need to show that $U \cap W$ is also a subspace of V.

ie, We need to show that

i) $U \cap W$ is closed under ~~scalar~~ vector addition &

ii) $U \cap W$ is closed under scalar multiplication

~~when~~

~~Let $u \in U \cap W$ $w \in W$.~~

Let u & $w \in U \cap W$

i) To Prove $U \cap W$ is closed under vector addition

Since $u, w \in U \cap W$

$u, w \in U$ & $u, w \in W$

$\therefore$ $u+w \in U$ & $u+w \in W$

Since U & W are subspaces.

$\Rightarrow$ $u+w \in U \cap W$

hence (i) is true.

ii) To show that $U \cap W$ is closed under Scalar multiplication

Let $c \in F$ be a random scalar in the Field F.

Then $c \cdot u \in U$ & $c \cdot u \in W$

Since U & W are subspaces of V over F.

$\Rightarrow$ $cu \in U \cap W$.

hence (ii) is True.

Since (i) & (ii) are true. $U \cap W$ is a subspace of V(F).

Hence proved

5) Define linearly independent & linearly dependent set of vectors. Are the vectors $V_1 = (2, 5, 3)$ $V_2 = (1, 1, 1)$ & $V_3 = (4, -2, 0)$ linearly independent? Justify your answer.

Sol^n: A set of vectors $\{v_1, v_2 \dots v_n\}$ of $V(F)$ is said to be linearly independent if there exist scalars $c_1, c_2 \dots c_n \in F$ such that $c_1 v_1 + c_2 v_2 + \dots + c_n v_n = 0$ implies $c_1 = c_2 = \dots c_n = 0$.

Otherwise if at least one c_i is non zero the ~~set of~~ vectors v_i ~~are~~ linearly dependent.

Setting the vectors as a matrix we get,

$$\begin{bmatrix} 2 & 5 & 3 \\ 1 & 1 & 1 \\ 4 & -2 & 0 \end{bmatrix}$$

Ideally we would find the RREF of the above matrix and check for zero rows. If zero rows exist then the vectors are linearly dependent & linearly independent otherwise.

But since the above matrix is square we would just have to find the determinant.

if the determinant is non zero, implying that the matrix has full rank, then the vectors are linearly independent
if determinant is zero then ∃ a zero row and ∴ linearly dependent.

$$\therefore \begin{vmatrix} 2 & 5 & 3 \\ 1 & 1 & 1 \\ 4 & -2 & 0 \end{vmatrix} = 2(0-(-2)) - 5(0-4) + 3((-2)-4)$$

$$= 4 + 20 - 18$$

$$= \underline{\underline{6}}$$

~~hen~~ Since the determinant of the matrix is non zero, the vectors (2, 5, 3) (1, 1, 1) & (4, -2, 0) are linearly independent.

Q6 $T(x,y,z) = (x+y, x-y, 2x+z)$

$T(1,0,0) = (1,1,2) : T(0,1,0) = (1,-1,0) ; T(0,0,1) = (0,0,1)$

Consider, $[A] = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underset{\sim}{R_2 \to -R_1 + R_2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \underset{\sim}{R_2 \to -1/2 \cdot R_1}$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \therefore r(T) = 3 \text{ and } n(T) = 0$$

$r(T)+n(T) = 3+0 = 3 = d[V_3(R)]$. **Theorem is verified.**

$R(T) = L(S) = \{x_1(1,1,2)+x_2(0,1,1)+x_3(0,0,1)\}$

$\therefore \quad R(T) = \{(x_1, x_1+x_2, 2x_1+x_2+x_3) / x_1, x_2, x_3 \in R\}$

Next, consider $T(x,y,z) = 0$.

That is, $x+y = 0,\ x-y = 0,\ 2x+z = 0$

On solving $x = 0, y = 0, z = 0 \quad \therefore \quad N(T) = \{(0,0,0)\}$

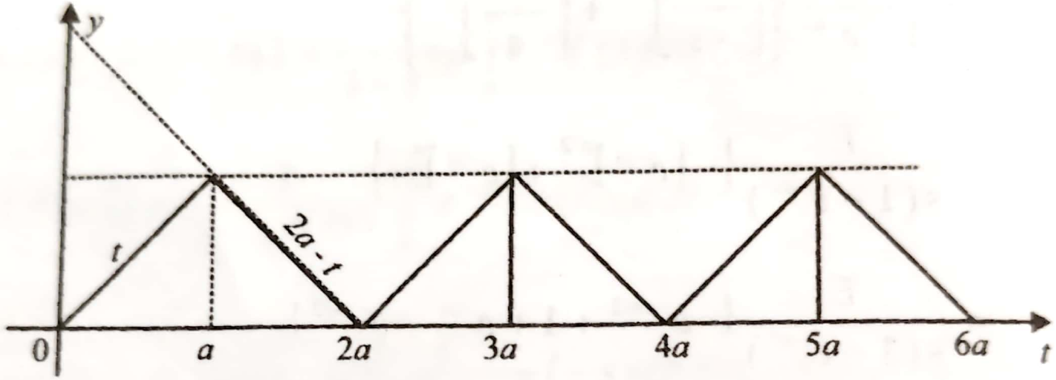
$\{-E,\ a<t<2a$ ... 2017]

Q7 If $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a, \end{cases} \quad f(t+2a) = f(t)$

(i) *Sketch the graph of f (t) as a periodic function*

(ii) *Show that* $L[f(t)] = \dfrac{1}{s^2} \tanh(as/2)$ **[June & Dec 2016]**

☞ (i) Let $f(t) = y$ and $y = t$ is a straight line passing through the origin making an angle 45^0 with the t-axis. $y = 2a - t$ or $y + t = 2a$ or $t/2a + y/2a = 1$ is a straight line passing through the points $(2a, 0)$ and $(0, 2a)$. The graph of $y = f(t)$ is as follows.



The periodic function $f(t)$ is called the *triangular wave function.*

(ii) We have, $T = 2a$ and $L[f(t)] = \dfrac{1}{1-e^{-sT}} \displaystyle\int_0^T e^{-st} f(t)\,dt$

$$L[f(t)] = \frac{1}{1-e^{-2as}} \int_0^{2a} e^{-st} f(t)\,dt$$

$$L[f(t)] = \frac{1}{1-e^{-2as}}\left\{\int_0^a t\,e^{-st}\,dt + \int_a^{2a}(2a-t)e^{-st}\,dt\right\}$$

Applying Bernoulli's rule to each of the integrals we have,

$$L[f(t)] = \frac{1}{1-e^{-2as}}\left\{\left[t\cdot\frac{e^{-st}}{-s}-(1)\frac{e^{-st}}{s^2}\right]_0^a + \left[(2a-t)\frac{e^{-st}}{-s}-(-1)\frac{e^{-st}}{s^2}\right]_a^{2a}\right\}$$

$$L[f(t)] = \frac{1}{1-e^{-2as}}\left\{\frac{-1}{s}(ae^{-as}-0)-\frac{1}{s^2}(e^{-as}-1)-\frac{1}{s}(0-ae^{-as})+\frac{1}{s^2}(e^{-2as}-e^{-as})\right\}$$

$$L[f(t)] = \frac{1}{s^2(1-e^{-2as})}(-e^{-as}+1+e^{-2as}-e^{-as})$$

$$= \frac{1}{s^2(1-e^{-2as})}(1-2e^{-as}+e^{-2as}) = \frac{(1-e^{-as})^2}{s^2(1-e^{-as})(1+e^{-as})}$$

$$L[f(t)] = \frac{(1-e^{-as})}{s^2(1+e^{-as})} = \frac{e^{as/2}-e^{-as/2}}{s^2(e^{as/2}+e^{-as/2})}$$

where we have multiplied both the numerator and denominator by $e^{as/2}$.

$$\therefore \quad L[f(t)] = \frac{2\sinh(as/2)}{s^2\cdot 2\cosh(as/2)} = \frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$$

Thus, $\boxed{L[f(t)] = 1/s^2\cdot\tanh(as/2)}$

Note: Similar Problems

(i) Find $L[f(t)]$ if $f(t) = \begin{cases} t, & 0 \le t \le 1 \end{cases}$ [June 201

Q8 $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$

$$f(t) = \cos t + (\cos 2t - \cos t)\,u(t-\pi) + (\cos 3t - \cos 2t)\,u(t-2\pi)$$

$$L[f(t)] = L(\cos t) + L[(\cos 2t - \cos t)\,u(t-\pi)] + L[(\cos 3t - \cos 2t)\,u(t-2\pi)] \quad \ldots (1)$$

Let $F(t-\pi) = \cos 2t - \cos t$; $G(t-2\pi) = \cos 3t - \cos 2t$

$\Rightarrow$ $F(t) = \cos 2(t+\pi) - \cos(t+\pi)$ and

$$G(t) = \cos 3(t+2\pi) - \cos 2(t+2\pi)$$

ie, $F(t) = \cos 2t + \cos t$; $G(t) = \cos 3t - \cos 2t$

$\therefore$ $\bar{F}(s) = \dfrac{s}{s^2+4} + \dfrac{s}{s^2+1}$; $\bar{G}(s) = \dfrac{s}{s^2+9} - \dfrac{s}{s^2+4}$

But $L[F(t-\pi)\,u(t-\pi)] = e^{-\pi s}\,\bar{F}(s)$ and

$$L[G(t-2\pi)\,u(t-2\pi)] = e^{-2\pi s}\,\bar{G}(s)$$

ie, $L[(\cos 2t - \cos t)\,u(t-\pi)] = e^{-\pi s}\left(\dfrac{s}{s^2+4} + \dfrac{s}{s^2+1}\right)$

and $\quad L[(\cos 3t - \cos 2t)\, u(t-2\pi)] = e^{-2\pi s}\left(\dfrac{s}{s^2+9} - \dfrac{s}{s^2+4}\right)$

Hence (1) becomes

$$L[f(t)] = \frac{s}{s^2+1} + e^{-\pi s}\left(\frac{s}{s^2+4} + \frac{s}{s^2+1}\right) + e^{-2\pi s}\left(\frac{s}{s^2+9} - \frac{s}{s^2+4}\right)$$

Thus $\boxed{L[f(t)] = \dfrac{s}{s^2+1} + s\,e^{-\pi s}\left(\dfrac{1}{s^2+4} + \dfrac{1}{s^2+1}\right) - \dfrac{5s\,e^{-2\pi s}}{(s^2+4)(s^2+9)}}$