CBCS SCHEME

USN ☐☐☐☐☐☐☐☐☐☐

BMATS201

Second Semester B.E./B.Tech. Degree Examination, June/July 2025

Mathematics – II for CSE Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$	07	L3	CO1
	b.	Evaluate $\int_{0}^{\infty}\int_{x}^{\infty}\frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration.	07	L3	CO1
	c.	Show that $\beta(m,n)=\frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$	06	L2	CO1
		OR			
Q.2	a.	Evaluate $\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}}(x^2+y^2)\,dy\,dx$ by changing to polar coordinates.	07	L3	CO1
	b.	Using double integration find the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	07	L3	CO1
	c.	Using mathematical tools, write the code to find the volume of the tetrahedron bounded by the planes $x=0$, $y=0$, $z=0$ and $6x+3y+2z=6$.	06	L3	CO5
		Module – 2			
Q.3	a.	Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2, –1, 2).	07	L2	CO2
	b.	Evaluate $\text{div}\,\overline{F}$ and $\text{curl}\,\overline{F}$ at the point (1, 2, 3), given $\overline{F}=\text{grad}(x^3y+y^3z+z^3x-x^2y^2z^2)$	07	L3	CO2
	c.	Express the vector $\overline{F}=2x\hat{i}-3y^2\hat{j}+zx\hat{k}$ in cylindrical form.	06	L3	CO2
		OR			
Q.4	a.	Find the directional derivative of $\phi(x,y,z)=x^2yz+yxz^2$ at (1, –2, 1) in the direction of the vector $2\hat{i}-\hat{j}-2\hat{k}$.	07	L2	CO2
	b.	Show that the vector $\overline{F}=\frac{x\hat{i}+y\hat{j}}{x^2+y^2}$ is both solenoidal and irrotational.	07	L3	CO2
	c.	Using mathematical tool, write the code to find the gradient of xy^3+yz^3.	06	L3	CO5

		Module – 3			
Q.5	a.	Show that the set $W=\{(x,y,z)\,/\,x+y+2z=0\}$ of the vector space $V_3(R)$ is a subspace of $V_3(R)$.	07	L2	CO3
	b.	Find the basis and dimension of the subspace spanned by the vectors $\{(1,-1,0),(0,3,1),(1,2,1),(2,4,2)\}$ in R^3.	07	L3	CO3
	c.	Find the matrix of the linear transformation $T: V_2(R)\to V_3(R)$ such that $T(-1,1)=(-1,0,2)$ and $T(2,1)=(1,2,1)$.	06	L2	CO3
		OR			
Q.6	a.	Determine whether the following set of vectors in 2×2 matrix space is linearly independent or linearly dependent: $S=\{v_1,v_2,v_3,\}$ where $v_1=\begin{bmatrix}2&1\\0&1\end{bmatrix}$, $v_2=\begin{bmatrix}3&0\\2&1\end{bmatrix}$ and $v_3=\begin{bmatrix}1&0\\2&0\end{bmatrix}$	07	L2	CO3
	b.	Prove that $T:R^3\to R^2$ defined by $T(x,y,z)=(x+y,\,y+2z)$ is a linear transformation.	07	L2	CO3
	c.	Verify the Rank – nullity theorem for the linear transformation $T:R^3\to R^2$ defined by $T(x,y,z)=(y-x,\,y-z)$.	06	L2	CO3
		Module – 4			
Q.7	a.	Find a real root of the equation $x\log_{10}x=1.2$ by regula falsi method corrected to 4 decimal places between (2.5, 3). Carry out 4 iterations.	07	L2	CO4
	b.	From the following table of half yearly premium for policies maturing at different ages, estimate the premium for the policies maturing at the age of 46. Age : 45, 50, 55, 60, 65 Premium (in Rs.) 114.84, 96.16, 83.32, 74.48, 68.48	07	L3	CO4
	c.	Use Trapezoidal rule to estimate the integration $\int_{0}^{1}\frac{1}{1+x^2}\,dx$ by taking 10 equal intervals.	06	L3	CO4
		OR			
Q.8	a.	Find by Newton's Raphson method, the root of the equation $\cos x = x e^x$ near to 0.5 , corrected to 4 decimal places.	07	L2	CO4
	b.	Using Newton's divided difference interpolation, find the interpolating polynomial of the given data: x : -4, -1, 0, 2, 5 f(x) : 1245, 33, 5, 9, +1335	07	L3	CO4
	c.	Evaluate using Simpson's $(1/3)^{rd}$ rule, $\int_{4}^{5.2}\log x\,dx$ by dividing the range (4, 5.2) into seven ordinates.	06	L3	CO4

Table for Q.7 b:

Age :	45	50	55	60	65
Premium (in Rs.)	114.84	96.16	83.32	74.48	68.48

Table for Q.8 b:

x :	- 4	- 1	0	2	5
f(x) :	1245	33	5	9	+1335

Module – 5

Q.9	**a.**	Solve $y' = 3x + y^2$; $y(0) = 1$ using Taylor's series method at $x = 0.1$ and $x = 0.2$.	**07**	**L3**	**CO4**
	b.	Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y = 1$ when $x = 0$. Find an approximate y at $x = 0.1$ using Euler's modified method. (Use modified formula twice).	**07**	**L3**	**CO4**
	c.	From the data given below find y at $x = 1.4$ using Milne's method. Given $\frac{dy}{dx} = x^2 + \frac{y}{2}$.	**06**	**L3**	**CO4**

x :	1	1.1	1.2	1.3
y :	2	2.2156	2.4549	2.7514

OR

Q.10	**a.**	Using Runge – Kutta method of order 4, find y at $x = 0.2$, given $\frac{dy}{dx} = \frac{x^2 + y^2}{10}$; $y(0) = 1$ taking $h = 0.1$.	**07**	**L3**	**CO4**
	b.	Using modified Euler's method solve $y' = 3x + \frac{y}{2}$ with $y(0) = 1$ taking $h = 0.2$ at $x = 0.2$. (Use modified Euler's formula twice).	**07**	**L3**	**CO4**
	c.	Using mathematical tools, write the code to solve $\frac{dy}{dx} = x^2 y - 1$; $y(0) = 1$ by Taylor's series method at $x = 0.1\ (0.1)\ 0.3$	**06**	**L3**	**CO5**

* * * * *

$**$ 3. Evaluate $\int_{-1}^{+1}\int_{0}^{z}\int_{x-z}^{x+z} (x+y+z)\, dy\, dx\, dz.$

$\Rightarrow$ let $I = \int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} (x+y+z)\, dy\, dx\, dz.$

$$= \int_{-1}^{1}\int_{0}^{z} \left[x[y]_{x-z}^{x+z} + \left[\frac{y^2}{2}\right]_{x-z}^{x+z} + z\,[y]_{x-z}^{x+z} \right] dx\, dz$$

$$= \int_{-1}^{1}\int_{0}^{z} \Big[x[\cancel{x}+z-\cancel{x}+z] + \frac{1}{2}[\cancel{x^2}+\cancel{z^2}+2xz-\cancel{x^2}-z^2$$

$$+ 2xz] + z[\cancel{x}+z-\cancel{x}+z] \Big] dx\, dz$$

$$= \int_{-1}^{1}\int_{0}^{z} [2zx + 2xz + 2z^2]\, dx\, dz.$$

$$= \int_{-1}^{1}\int_{0}^{z} [4xz + 2z^2]\, dx\, dz.$$

$$= \int_{-1}^{1} \left[4z\left[\frac{x^2}{2}\right]_0^z + 2z^2[x]_0^z \right] dz.$$

$$= \int_{-1}^{1} [2z[z^2-0] + 2z^2[z-0]]\, dz$$

$$= \int_{-1}^{1} [2z^3 + 2z^3]\, dz$$

$$= 4\int_{-1}^{1} z^3\, dz$$

$$= [z^4]_{-1}^{1}$$

$$I = 0$$

**(i)

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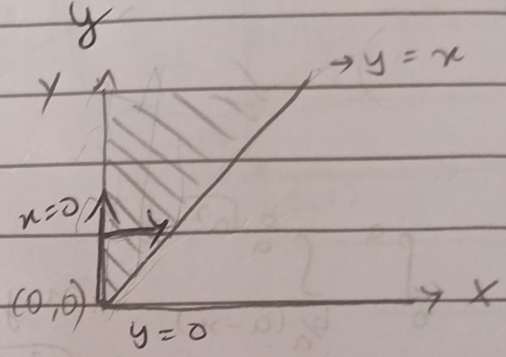
IMP

Q)** Change the order of integration and hence evaluate $I = \int_{x=0}^{\infty}\int_{y=x}^{\infty} \frac{e^{-y}}{y}\, dy\, dx$



→ **Old limits :**

$x = 0$ to $x = \infty$

$y = x$ to $y = \infty$

→ **New limits :**

$y = 0$ to $y = \infty$

$x = 0$ to $x = y$ ✓

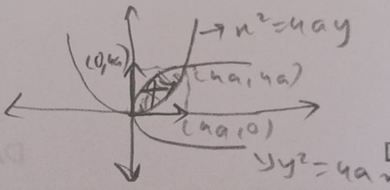
$I = \int_0^{\infty}\int_0^{y} \frac{e^{-y}}{y} (dx)\, dy$ ✓

$I = \int_0^{\infty} \frac{e^{-y}}{y} [x]_0^y\, dy$

$\frac{e^{-y}}{y}[x]_0^y$

$\frac{e^{-y}}{-1}$

$\frac{-e^{-\infty}+e^{-0}}{1}$



$x = y^2/4a$ to $x = 2\sqrt{ay}$

$4a\,y = x^2$ $\quad y^2 = 4ax$

$y = 2\sqrt{ax}$

y

x

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$$I = \int_0^\infty \frac{e^{-y}}{\cancel{y}} [\cancel{y}]\, dy$$

$$I = \int_0^\infty e^{-y}\, dy$$

$$I = -[e^{-y}]_0^\infty = -e^{-\infty} + e^{-0} \quad *[e^{-\infty} = 0]$$

$$I = 0 + 1 = 1$$

** ii) changing order of integration

Relation between Beta and Gamma function

Or Show that $\beta(m,n) = \dfrac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}$

$$\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \, \cos^{2n-1}\theta \, d\theta \rightarrow ①$$

$$\Gamma(n) = 2\int_0^{\infty} e^{-x^2} x^{2n-1} dx \rightarrow ②$$

$$\Gamma(m) = 2\int_0^{\infty} e^{-y^2} y^{2m-1} dy \rightarrow ③$$

$$\Gamma(m+n) = 2\int_0^{\infty} e^{-(x^2+y^2)} r^{2(m+n)-1} dr \rightarrow ④$$

$$\Gamma(m)\cdot\Gamma(n) = 2\int_0^{\infty} e^{-x^2} x^{2n-1} dx \cdot 2\int_0^{\infty} e^{-y^2} y^{2m-1} dy.$$

$$= 4\int_0^{\infty}\int_0^{\infty} e^{-x^2} e^{-y^2} x^{2n-1} y^{2m-1} dx\, dy.$$

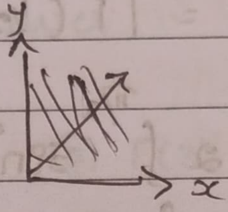
$$= 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\, x^{2n-1}\, y^{2m-1}\, dx\, dy \rightarrow ⑤$$

$x = r\cos\theta$, $y = r\sin\theta$.

$x^2+y^2 = r^2$, $dx\,dy = r\,dr\,d\theta$.

$x = 0$ to $x = \infty$

$y = 0$ to $y = \infty$



$r = 0$ to $r = \infty$

$\theta = 0$ to $\theta = \pi/2$

Eq ⑤ $\Rightarrow$

$$\Gamma(m)\cdot\Gamma(n) = 4\int_0^\infty\int_0^{\pi/2} e^{-r^2} (r\cos\theta)^{2n-1} (r\sin\theta)^{2m-1}\, r\,dr\,d\theta$$

$$= 4\int_0^\infty\int_0^{\pi/2} e^{-r^2}\, r^{2n-1} \cos^{2n-1}\theta\; r^{2m-1} \sin^{2m-1}\theta\; r\,dr\,d\theta$$

$$= 2\int_0^\infty e^{-r^2}\, r^{2n-1}\, r^{2m-1}\, r\,dr \cdot 2\int_0^{\pi/2} \cos^{2n-1}\theta\, \sin^{2m-1}\theta\, d\theta.$$

$$= 2\int_0^\infty e^{-r^2}\, r^{2n-1+2m-\cancel{1}+\cancel{1}}\, dr \cdot 2\int_0^{\pi/2} \cos^{2n-1}\theta\, \sin^{2m-1}\theta\, d\theta.$$

$$= 2\int_0^\infty e^{-r^2}\, r^{2(m+n)-1}\, dr \cdot 2\int_0^{\pi/2} \cos^{2n-1}\theta \cdot \sin^{2m-1}\theta\, d\theta$$

$$\Gamma(m)\cdot\Gamma(n) = \Gamma(m+n)\cdot\beta(m,n)$$

$$\beta(m,n) = \frac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}$$

a let $I = \int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dy\,dx$

$y = 0$ to $y = 1$

[illegible] $x = 0$ to $x = \sqrt{1-y^2}$

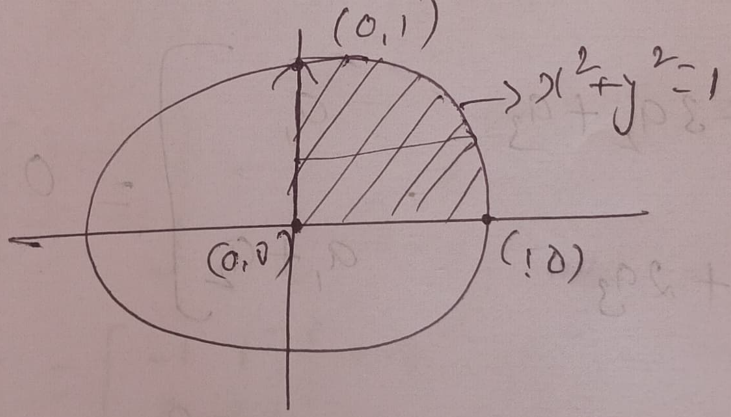
$x^2+y^2 = r^2$, $x = r\cos\theta$ $y = r\sin\theta$ $dx\,dy = r\,dr\,d\theta$

[illegible] [illegible] [illegible], [illegible]

[illegible] , [illegible]

$x^2 = 1-y^2$

$x^2+y^2 = 1$



$\theta = 0$ to $\theta = \pi/2$

$r = 0$ to $r = 1$

$I = \int_0^{\pi/2} \int_0^1 r^2 . r\,dr\,d\theta$

$I = \int_0^{\pi/2} \int_0^1 r^3\,dr\,d\theta$

$I = \int_0^{\pi/2} \left[\frac{r^4}{4}\right]_0^1 d\theta$

$I = \frac{r^4}{4} \int_0^{\pi/2} d\theta$

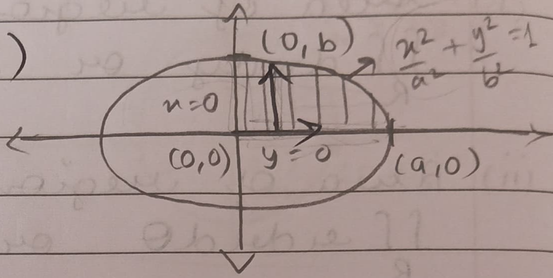
$I = \frac{1}{4}\left[\pi/2\right]$

$\boxed{I = \pi/8}$

Q) Find the area of ellipse $\left[\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right]$

- $x = 0$ to $x = a$ $(\rightarrow)$

$y = 0$ to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(\uparrow)$



$y = 0$ to $y = \frac{b}{a}\sqrt{a^2 - x^2}$

$$A = \int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} dy\, dx$$

$$A = \int_0^a [y]_0^{\frac{b}{a}\sqrt{a^2-x^2}} dx$$

$$A = \int_0^a \frac{b}{a}\sqrt{a^2-x^2}\; dx$$

$$A = \frac{b}{a}\int_0^a \sqrt{a^2-x^2}\; dx$$

$$A = \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left[\frac{x}{a}\right]\right]_0^a$$

$$A = \frac{b}{a}\left[0 + \frac{a^2}{2}\overset{\pi/2}{\sin^{-1}[1]}\right] - [0 + 0]\,)$$

$$A = \frac{b}{\cancel{a}}\left[\frac{\cancel{a^2}}{2}\frac{\pi}{2}\right]$$

$\frac{a^2\pi}{2\;\;2}$ $\frac{b}{\cancel{a}}\frac{a^{\cancel{2}}\pi}{\cancel{4}} \times \cancel{4}$ $\pi ab = A$

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$$A = \frac{\pi ab}{4}$$

$\therefore$ Total Area $= \cancel{4} \times \pi ab/\cancel{4}$

$\boxed{A = \pi ab}$ //

Example 5:
find the area of the cardiod $r=a(1+\cos\theta)$ by double integration

```
from sympy import*
r= Symbol('r')
t= Symbol ('t')
a= Symbol ('a')
a=4
w3= 2*integrate(r,(r,0,a*(1+Cos(t))),(t,0,pi))
pprint (w3)
```

Example: 6
find the Volume of tetrahedron bounded by planes $x=0, y=0, z=0$; $x/a+y/b+z/c=1$

```
from sympy import*
x= Symbol('x')
y= Symbol ('y')
z= Symbol ('z')
a= Symbol ('a')
b= Symbol ('b')
c= Symbol('c')
w2= integrate(1,(z,0,c*(1-x/a-y/b)),(y,0,b*
    (1-x/a)),(x,0,a))
pprint (w2)
```

★ IMP $|\nabla\phi_1|\cdot|\nabla\phi_2|$

★★Q) Find the angle between the surfaces, $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2,-1,2)$

- Let $\phi_1 = x^2 + y^2 + z^2$, $\phi_2 = x^2 + y^2 - z$

W.k.t Angle b/w two surfaces is given by,

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1|.|\nabla\phi_2|} \rightarrow ①$$

Now,

$$\nabla\phi_1 = \frac{\partial\phi_1}{\partial x}\hat{i} + \frac{\partial\phi_1}{\partial y}\hat{j} + \frac{\partial\phi_1}{\partial z}\hat{k}$$

$$\nabla\phi_1 = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

$$\nabla\phi_1(2,-1,2) = 2(2)\hat{i} + 2(-1)\hat{j} + 2(2)\hat{k}$$

$$\nabla\phi_1(2,-1,2) = 4\hat{i} - 2\hat{j} + 4\hat{k} \rightarrow ②$$

$$\nabla\phi_2 = \frac{\partial\phi_2}{\partial x}\hat{i} + \frac{\partial\phi_2}{\partial y}\hat{j} + \frac{\partial\phi_2}{\partial z}\hat{k}$$

$$\nabla\phi_2 = 2x\hat{i} + 2y\hat{j} - \hat{k}$$

$$\nabla\phi_2(2,-1,2) = 2(2)\hat{i} + 2(-1)\hat{j} - \hat{k}$$

$$= 4\hat{i} - 2\hat{j} - \hat{k} \rightarrow ③$$

32 4 36 $\sqrt{16+4+16}$

$\nabla\phi_1 = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$

~~$\nabla\phi_1(2,-1,2) = 2(2)\hat{i} + 2(-1)\hat{j} + 2(2)\hat{k}$~~

$= 4\hat{i} - 2\hat{j} + 4\hat{k}$

$|\nabla\phi_1| = 6.$ $16+4+1$

~~$\phi_2 = x^2+y^2-3-3$~~

$\nabla\phi_2 = 2x\hat{i} + 2y\hat{j} - \hat{k}$

$\nabla\phi_2(2,-1,2) = 4\hat{i} - 2\hat{j} - \hat{k}$

$|\nabla\phi_2| = \sqrt{21}$

$\cos\theta = \frac{4\hat{i}-2\hat{j}+4\hat{k} \cdot 4\hat{i}-2\hat{j}-\hat{k}}{6\sqrt{21}}$

$\cos\theta = \frac{16+4-4}{6\sqrt{21}}$

$\cos\theta = \frac{\cancel{16}\ 8}{\cancel{6}_{3}\sqrt{21}}$

$\cos\theta = \frac{8}{3\sqrt{21}}$

$\theta = \cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$

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By ①,

$$\cos\theta = \frac{(4\hat{i} - 2\hat{j} + 4\hat{k})(4\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{16+4+16} \cdot \sqrt{16+4+1}}$$

$$= \frac{8+4+4}{\sqrt{36} \cdot \sqrt{21}} = \frac{\cancel{16}\ 8}{3\cancel{6}\sqrt{21}}$$

$$\cos\theta = \frac{8}{3\sqrt{21}}$$

$$\theta = \cos^{-1}\left(\frac{8}{3\sqrt{21}}\right) \quad //$$

[DD = ∇

$xy - z^2 = 0$

$\nabla\phi_1 [y\hat{i} + x\hat{j} - 2z\hat{k}]$

$\nabla\phi_1(4,1,2) = \hat{i} + 4\hat{j} - 4\hat{k}$

$\nabla\phi_2(3,3,-3) = 3\hat{i} + 3\hat{j} + 6\hat{k}$

$$\cos\theta = \frac{\hat{i} + 4\hat{j} - 4\hat{k} \cdot 3\hat{i} +}{\sqrt{33} \cdot \sqrt{5}}$$

$$= \frac{3 + 12 - 24}{\sqrt{33} \cdot \sqrt{54}} = -\frac{9}{\sqrt{33}}$$

Find the angle bet

3 b

$$\vec{F} = grad\,(x^3y + y^3z + z^3x - x^2y^2z^2)$$
$$(1,2,3)$$

w.r.t

$$\vec{F} = \nabla\phi$$

$$\vec{F} = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$\vec{F} = (3x^2y + z^3 - 2y^2xz^2)\hat{i} + (x^3 + 3y^2z - 2x^2yz^2)\hat{j}$$
$$+ (y^3 + 3z^2x - 2x^2y^2z)$$

$$div\,\vec{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$div\,\vec{F} = (6xy - 2y^2z^2) + (6yz - 2x^2z^2) +$$
$$(6xz - 2x^2y^2)$$

$$div\,\vec{F}_{(1,2,3)} = (6(1)(2) - 2(4)9) +$$
$$(6(2)(3) - 2(1)(9)) +$$
$$(6(1)(3) - 2(1)4)$$

$$div\,\vec{F} = (12 - 72) + (36 - 18) + (18 - 8)$$

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$$\mathrm{div}\,\vec{E} = (-60) + (18) + (10)$$

$$\underline{\underline{\mathrm{div}\,\vec{E} = -32}}$$

$| \vec{E} =$

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$$\text{curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (3x^2y + z^3 - 2y^2xz^2) & (x^3 + 3y^2z - 2x^2yz^2) & (y^3 + 3z^2x - 2x^2y^2z) \end{vmatrix}$$

~~$6xy - 2y^2z^2$~~

$$\left[(3y^2 - 4x^2yz) - (3y^2 - 4x^2yz)\right] - \hat{j}\left[(3z^2 - 4xy^2z) - (3z^2 - 4y^2xz)\right] + \hat{k}\left[(3x^2 - 4xyz^2) - (3x^2 - 4yxz^2)\right]$$

$(1, 2, 3)$

~~$\hat{i}\left[(3(4) - 4(1)(2)(3))\right]$~~

$\text{curl } \vec{F} = 0$

Express the vector $\vec{A} = 2x\hat{i} - 3y^2\hat{j} + xz\hat{k}$ in cylindrical cordinates.

$A = A_1\hat{e}_1 + A_2\hat{e}_2 + A_3\hat{e}_3$

$x = \rho\cos\phi \qquad y = \rho\sin\phi \qquad z = z$

$\vec{A} = 2x\hat{i} - 3y^2\hat{j} + xz\hat{k}$

$= 2\rho\cos\phi\,\hat{i} - 3\rho^2\sin^2\phi\,\hat{j} + \rho\cos\phi\, z\,\hat{k}$

$$A_1 = \vec{A} \cdot \hat{e}_1$$

$$= (2\rho \cos\phi \, \hat{i} - 3\rho^2 \sin^2\phi \, \hat{j} + \rho \cos\phi \, z \, \hat{k}) \cdot$$

$$(\cos\phi \, \hat{i} + \sin\phi \, \hat{j})$$

$$= 2\rho \cos^2\phi - 3\rho^2 \sin^3\phi$$

$$A_2 = \vec{A} \cdot \hat{e_2}$$

$$= (2\rho \cos\phi \, \hat{i} - 3\rho^2 \sin^2\phi \, \hat{j} + \rho \cos\phi z \, \hat{k}) \cdot$$

$$(-\sin\phi \, \hat{i} + \cos\phi \, \hat{j})$$

$$= -2\rho \cos\phi \sin\phi - 3\rho^2 \sin^2\phi \cos\phi$$

$$A_3 = \vec{A} \cdot \hat{e_3}$$

$$= (2\rho \cos\phi \, \hat{i} - 3\rho^2 \sin^2\phi \, \hat{j} + \rho \cos\phi z \, \hat{k}) \cdot \hat{k}$$

$$= \rho \cos\phi \, z$$

$$\vec{A} = (2\rho \cos^2\phi - 3\rho^2 \sin^3\phi) \, \hat{e}_1 - (2\rho \cos\phi \sin\phi + 3\rho^2 \sin^2\phi \cos\phi) \, \hat{e}_2 + (\rho \cos\phi \, z) \, \hat{e}_3$$

Represent

4. a. $\phi(x, y, z) = x^2yz + yxz^2$ $(1, -2, 1)$

$\vec{d} = 2\hat{i} - \hat{j} - 2\hat{k}$

D.D $= \nabla\phi \cdot \hat{n}$. $\hat{n} = \frac{\vec{d}}{|\vec{d}|}$

~~$\nabla\phi = (2xyz)\hat{i} +$~~

$\nabla\phi = (2xyz + yz^2)\hat{i} + (x^2z + xz^2)\hat{j} + (x^2y + 2xyz)$

$(1, -2, 1)$

$\nabla\phi = (2(1)(-2)1 + (-2)1)\hat{i} + (1(1) + 1(1))\hat{j}$

$+ (-2(1) + 2(1)(-2)(1))\hat{k}$

$$\nabla\phi = (-4-2)\hat{i} + (2)\hat{j} + (-2-4)\hat{k}$$

$$\nabla\phi = \underline{\underline{-6\hat{i} + 2\hat{j} - 6\hat{k}}}$$

$$\hat{n} = \frac{2\hat{i} - \hat{j} - 2\hat{k}}{\sqrt{4+1+4}} = \frac{2\hat{i} - \hat{j} - 2\hat{k}}{\sqrt{9}}$$

$$D.D = \nabla\phi \cdot \hat{n}$$

$$D.D = (-6\hat{i} + 2\hat{j} - 6\hat{k}) \cdot \frac{2\hat{i} - \hat{j} - 2\hat{k}}{\sqrt{9}}$$

$$D.D = \frac{1}{3}\left[-12 - 2 + 12\right]$$

$$\underline{\underline{D.D = -2/3}}$$

LAB 3: FINDING GRADIENT, DIVERGENT AND CURL.

Find gradient of $\phi = x^2y+2xz-4$

```
from sympy.vector import *
from sympy import symbols
N= CoordSys3D('N')
x, y, z = symbols('x y z')
A=N.x**2*N.y+2*N.x*N.z-4
delop = Del()
display(delop(A))
gradA= gradient(A)
print(f"\n gradient of {A} is \n")
display(gradA)
```

5

a, let $\alpha = (x_1, y_1, z_1) \Rightarrow x_1 + y_1 + 2z_1 = 0$

$\beta = (x_2, y_2, z_2) \Rightarrow x_2 + y_2 + 2z_2 = 0$

① $c_1\alpha + c_2\beta \in W$

Consider $c_1\alpha + c_2\beta$

$c_1(x_1, y_1, z_1) + c_2(x_2, y_2, z_2)$

$(c_1x_1, c_1y_1, c_1z_1) + (c_2x_2, c_2y_2, c_2z_2)$

$(c_1x_1 + c_2x_2, c_1y_1 + c_2y_2, c_1z_1 + c_2z_2)$

$x + y + 2z = 0$

$\underline{c_1x_1 + c_2x_2} + c_1y_1 + c_2 2y_2 + 2c_1z_1 + 2c_2z_2 = 0$

$c_1(x_1 + y_1 + 2z_1) + c_2(x_2 + y_2 + 2z_2) = 0$

$c_1(0) + c_2(0) = 0$

$c_1\alpha \in W$

sider $c_1\alpha = c_1(x_1, y_1 z_1)$

$= (c_1x_1, c_1y_1, c_1z_1)$

$x+y+2z=0$

$c_1x_1+c_1y_1+2c_1z_1=0$

$c_1(x_1+y_1+2z_1)=0$

$c_1(0)=0$

$\therefore$ w is a subspace of $V_3(R)$

Given $V = \{ (1,-1,0), (0,3,1), (1,2,1), (2,4,2) \}$

let $$V = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 1 \\ 1 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_1$, $R_4 \rightarrow R_4 - 2R_1$

$$V = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 1 \\ 0 & 3 & 1 \\ 0 & 6 & 2 \end{bmatrix}$$

$4+2$

2

$R_4 \rightarrow R_4 - 2R_3$

$$V = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 - R_2} V = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis $= \{ (1,-1,0), (0,3,1) \}$

Dimension is 2

C.

$T(-1,1) = (-1,0,2)$

$T(2,1) = (1,2,1)$

let $X = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$ $\quad Y = \begin{bmatrix} -1 & 1 \\ 0 & 2 \\ 2 & 1 \end{bmatrix}$

$Y = A.X$

let $A = Y.X^{-1}$

let $X = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$

$|X| = (-1)(1) - (2)(1) = -3$

$\therefore X^{-1} = \frac{1}{-3}\begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -1/3 & 2/3 \\ 1/3 & 1/3 \end{bmatrix}$

$A = YX^{-1} = \begin{bmatrix} -1 & 1 \\ 0 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -1/3 & 2/3 \\ 1/3 & 1/3 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 2/3 & -1/3 \\ 2/3 & 2/3 \\ -1/3 & 5/3 \end{bmatrix}$

a.

$$S = \{ v_1, v_2, v_3 \}, \; v_1 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \; v_2 = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}, \; v_3 = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$$

W.K.T $a_1 v_1 + a_2 v_2 + a_3 v_3 = 0$

$$a_1 \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} + a_2 \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} + a_3 \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2a_1, & a_1 \\ 0 & a_1 \end{bmatrix} + \begin{bmatrix} 3a_2 & 0 \\ 2a_2 & a_2 \end{bmatrix} + \begin{bmatrix} a_3 & 0 \\ 2a_3 & 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2a_1 + 3a_2 + a_3 & a_1 \\ 2a_2 + 2a_3 & a_1 + a_2 \end{bmatrix} = 0$$

$2a_1 + 3a_2 + a_3 = 0, \quad a_1 = 0$

$2a_2 + 2a_3 = 0, \quad a_1 + a_2 = 0,$

$\therefore a_1 = 0, \; a_2 = 0, \; a_3 = 0, \; a_4 = 0$

hence , it is linearly dependent.

c. $T: V_3(R) \to V_2(R)$

$T(x, y, z) = (x+y,\ y+2z)$

let $\alpha = (x_1, y_1, z_1)$

$T(\alpha) = (x_1+y_1,\ y_1+2z_1)$

$\beta = (x_2, y_2, z_2)$

$T(\beta) = (x_2+y_2,\ y_2+2z_2)$

$T(\alpha+\beta) = T(\alpha) + T(\beta)$

Consider

$\alpha+\beta = (x_1, y_1, z_1) + (x_2, y_2, z_2)$

$= (\underline{x_1+x_2},\ \underline{y_1+y_2},\ \underline{z_1+z_2})$

$T(\alpha+\beta) = T(x_1+x_2,\ y_1+y_2,\ z_1+z_2)$

$T(\alpha+\beta) = (x_1+x_2+y_1+y_2,\ y_1+y_2+2z_1+2z_2) \to ①$

$T(\alpha)+T(\beta) = (x_1+y_1,\ y_1+2z_1) + (x_2+y_2,\ y_2+2z_2)$

$T(\alpha)+T(\beta) = (x_1+y_1+x_2+y_2,\ y_1+2z_1+y_2+2z_2) \to ②$

from ① & ②

$T(\alpha+\beta) = T(\alpha)+T(\beta)$

$T(c\alpha) = c\,T(\alpha)$

Consider

$c\alpha = c(x_1, y_1, z_1)$

$c\alpha = (cx_1, cy_1, cz_1)$

$T(c\alpha) = T(cx_1, cy_1, cz_1)$

$T(c\alpha) = (cx_1 + cy_1, cy_1 + 2cz_1)$

$T(c\alpha) = c(x_1 + y_1, y_1 + 2z_1)$

$T(c\alpha) = c\,T(\alpha)$

hence T is L-T

verify the Rank - Nullity theorem for linear transformation $T : V_3(R) \rightarrow V_2(R)$, $T(x, y, z) = (y-x, y-z)$

$V_3(R) = \{(1,0,0), (0,1,0), (0,0,1)\}$

$T(1,0,0) = [-1 \quad 0]$

$T(0,1,0) = [1 \quad 1]$

$T(0,0,1) = [0 \quad -1]$

$$V = \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 1$

$$V = \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2$.

$$V = \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$r(v) = 2$.

$n(v) = 1$

$r(v) + n(v) = d(v)$

$2 + 1 = 3$.

Hence, rank nullity theorem is verified.

3. Find the real root of the eq^n $x \log_{10} x - 1.2$ by Regula-false method between 2 and 3 carry out 3 iterations.

$f(x) = x \log_{10} x - 1.2$.

$[a, b] = [2, 3]$

$f(2) = -0.5979 < 0$.

$f(3) = 0.2314 > 0$

$f(2.1) = -0.5233$

$f(2.2) = -0.4467$

$f(2.3) = -0.3680$

$f(2.4) = -0.2875$.

$f(2.5) = -0.2051$

$f(2.6) = -0.1211$

$f(2.7) = -0.0353$

$f(2.8) = 0.0520 > 0$.

$\therefore [2.7, 2.8]$

$a = 2.7$ $\quad b = 2.8$

I: $x_1 = \dfrac{a f(b) - b(f(a))}{f(b) - f(a)}$

$= \dfrac{2.7 \times 0.0520 - 2.8 \times (-0.0353)}{0.0520 + 0.0353}$

$x_1 = 2.7404$

II $f(2.7404) = -0.0002 < 0$

$\therefore [2.7, 2.8] \longrightarrow [2.7404, 2.8]$

$a = 2.7404$ $\quad b = 2.8$

$f(a) = -0.0002$ $\quad f(b) = 0.0520$

$x_2 = 2.7404 \times 0.0520 - 2.8 (-0.0002)$

3. Find the real root of the eq^n $x \log_{10} x - 1.2$ by Regula-false method between 2 and 3 carry out 3 iterations.

$f(x) = x \log_{10} x - 1.2$

$[a, b] : [2, 3]$

$f(2) = -0.5979 < 0.$

$f(3) = 0.2314 > 0$

$f(2.1) = -0.5233$

$f(2.2) = -0.4467$

$f(2.3) = -0.3680$

$f(2.4) = -0.2875.$

$f(2.5) = -0.2051$

$f(2.6) = -0.1211$

$f(2.7) = -0.0353$

$f(2.8) = 0.0520 > 0.$

$\therefore [2.7, 2.8]$

$a = 2.7 \qquad b = 2.8$

I: $x_1 = \dfrac{a f(b) - b(f(a))}{f(b) - f(a)}$

$= \dfrac{2.7 \times 0.0520 - 2.8 \times (-0.0353)}{0.0520 + 0.0353}$

$x_1 = 2.7404$

II $f(2.7404) = -0.0002 < 0$

$\therefore [2.7, 2.8] \longrightarrow [2.7404, 2.8]$

$a = 2.7404 \qquad b = 2.8$

$f(a) = -0.0002 \qquad f(b) = 0.0520$

$x_2 = \dfrac{2.7404 \times 0.0520 - 2.8 (-0.0002)}{0.0520 + 0.0002}$

$x_2 = 2.7426.$

$f(2.7426) = 0.000$[illegible]

$\therefore [2.7404, 2.8] \longrightarrow [2.7404, 2.7426]$

$$x_3 = \frac{2.7404 \times 0 - 2.7426(-0.0002)}{0 + 0.0002}.$$

$x_3 = 2.7406.$

$\therefore \underline{x = 2.7406}$

Find the real root of the eqn. $e^x - 3x - \sin x = 0$ b/w 0 and 1

7. b

Age(x)	premium(y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
45	114.84	-18.68	5.84	-1.84	0.68
50	96.16	-12.84	4	-1.16	
55	83.32	-8.84	2.84		
60	74.48	-6			
65	68.48				

$x = 46$

$$p = \frac{x - x_0}{h} = \frac{46-45}{5} = \frac{1}{5} = 0.2$$

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$y = 114.84 + (0.2 \times -18.68) + \frac{(0.2)(0.2-1)}{2} 5.84$$

$$+ \frac{0.2(0.2-1)(0.2-2)}{6}(-1.84) + \frac{0.2(0.2-1)(0.2-2)(0.2-3}{24}$$

$y = 110.52$

DATE ☐☐ ☐☐ ☐☐☐☐

$$I = \int_0^1 \frac{dx}{1+x^2}$$

$n = 10$ $\qquad h = \frac{b-a}{n} = \frac{1-0}{10} = \frac{1}{10}$

x	$y = 1/1+x^2$
0	1
0.1	0.9900
0.2	0.9615
0.3	0.9174
0.4	0.8620
0.5	0.8
0.6	0.7352
0.7	0.6711
0.8	0.6097
0.9	0.5524
1	0.5

$$I = \frac{h}{2}\left[(y_0 + y_{10}) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9)\right]$$

$I = 0.78493$

5. Find the real root of the eqⁿ $\cos x = x e^x$ whe is near to $x = 0.5$ by N-R method correct to 3 decimal places.

$f(x) = \cos x - x e^x$

& $x_0 = 0.5$

$f'(x) = -\sin x - [x e^x + e^x]$

$= -\sin x - x e^x - e^x$

$= -\sin x - e^x [x + 1]$ //

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 0.5 - \frac{0.053}{(-2.953)}$

$0.5 + 0.018$

$=$ ~~0.4982~~ 0.518

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.518 - \frac{(-0.001)}{-3.043}$

$= 0.518 - 0.000$

$x_2 = 0.518$

$\therefore x = 0.518$ //

indepen

[25] *Determine $f(x)$ as a polynomial in x for the following data using Newton's divided difference formula.*

x	-4	-1	0	2	5
y	1245	33	5	9	1335

[Dec 18]

☞ The divided difference table is formed first.

x	$y = f(x)$	1st D.D	2nd D.D	3rd D.D.	4th D.D.
$x_0 = -4$	$f(x_0) = 1245$	$f(x_0, x_1)$ $\frac{33 - 1245}{-1 + 4} = -404$	$f(x_0, x_1, x_2)$ $\frac{-28 + 404}{0 + 4} = 94$		
$x_1 = -1$	$f(x_1) = 33$	$f(x_1, x_2)$ $\frac{5 - 33}{0 + 1} = -28$	$f(x_1, x_2, x_3)$ $\frac{2 + 28}{2 + 1} = 10$	$f(x_0, x_1, x_2, x_3)$ $\frac{10 - 94}{2 + 4} = -14$	$f(x_0, x_1, x_2, x_3, x_4)$ $\frac{13 + 14}{5 + 4} = 3$
$x_2 = 0$	$f(x_2) = 5$	$f(x_2, x_3)$ $\frac{9 - 5}{2 - 0} = 2$	$f(x_2, x_3, x_4)$ $\frac{442 - 2}{5 - 0} = 88$	$f(x_1, x_2, x_3, x_4)$ $\frac{88 - 10}{5 + 1} = 13$	
$x_3 = 2$	$f(x_3) = 9$	$f(x_3, x_4)$ $\frac{1335 - 9}{5 - 2} = 442$			
$x_4 = 5$	$f(x_4) = 1335$				

On substituting in the Newton's divided difference formula, we have,

$$\boxed{f(x) = 3x^4 - 5x^3 + 6x^2 - 14x + 5}$$

[39] Evaluate $\int_{4}^{5.2} \log_e x \, dx$ *taking 6 equal strips by applying Simpson's 1/3rd rule.*

Here $h = (5.2 - 4) / 6 = 0.2$; $n = 6$.

The values of x and $y = \log_e x$ are tabulated.

x	4	4.2	4.4	4.6	4.8	5.0	5.2
y	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Substituting the values from the table in the rule as in [34], we obtain the value of the integral as $\boxed{1.82785}$

[40] *Use Trapezoidal rule to compute the area bounded by the curve $y = f(x)$,*

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a. Sol^n

Given $y' = 3x + y^2$; $y(0)=1$, $x=0.1$ & $x=0.2$

$x_0 = 0$, $y_0 = 1$

W.K.T

$$y(x) = y(x_0) + (x-x_0)\,y_1(x_0) + \frac{(x-x_0)^2}{2!}\,y_2(x_0) + \frac{(x-x_0)^3}{3!}\,y_3(x_0)$$

$$y(x) = y(0) + x\,y_1(0) + \frac{x^2}{2}\,y_2(0) + \frac{x^3}{6}\,y_3(0) \rightarrow ①$$

$y(0)=1$, $y_1(x) = 3x+y^2$: $y_1(0) = 1$

$y_2(x) = 3 + 2yy_1 \Rightarrow y_2(0) = 3+2(1)(1) = 5$

$y_3(x) = 2[yy_2 + y_1y_1] \Rightarrow 2[yy_2 + y_1^2]$

$y_3(0) = 2[1.5 + 1^2] = 2[6] = 12$

$$y(x) = 1 + x(1) + \frac{x^2}{2}(5) + \frac{x^3}{6}\,12$$

$$y(x) = 1 + x + \frac{5x^2}{2} + 2x^3$$

$y(0.1) = 1+0.1+0.025+0.001 = 1.126$

$y(0.2) = 1+0.2+0.100+0.008 = 1.308$

b. Sol^{n}

Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ $\quad y=1, \quad x=0, \quad y=?$ at $x=0.1$

$f(x,y) = \frac{y-x}{y+x}$ $\quad y_0 = 1, \quad x_0 = 0$

$x_1 = x_0 + h$

$0.1 = 0 + h \quad \therefore \boxed{h=0.1}$ //

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$y_1^{(0)} = 1 + (0.1) f(0,1)$

$y_1^{(0)} = 1 + (0.1)[1 \quad] = \underline{\underline{1.100}}$

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$y_1^{(1)} = 1 + \frac{0.1}{2}[f(0,1) + f(0.1, 1.1)]$

$y_1^{(1)} = 1 + 0.05[1 + 0.833]$

$y_1^{(1)} = \underline{\underline{1.092}}$

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$y_1^{(2)} = 1 + \frac{0.1}{2}[f(0,1) + f(0.1, 1.092)]$

$y_1^{(2)} = 1 + 0.05[1 + 0.832] = \underline{\underline{1.092}}$

$$\therefore \underline{\underline{y(0.1) = 1.092}}$$

[21] *Apply Milne's method to compute* $y\,(1.4)$ *corrector to four decimal places given* $\frac{dy}{dx} = x^2 + \frac{y}{2}$ *and following data :*

$$y\,(1) = 2,\ y\,(1.1) = 2.2156,\ y\,(1.2) = 2.4649,\ y\,(1.3) = 2.7514$$

☞ First we shall prepare the following table.

x	y	$y' = x^2 + \frac{y}{2}$
$x_0 = 1$	$y_0 = 2$	$y'_0 = 1^2 + \frac{2}{2} = 2$
$x_1 = 1.1$	$y_1 = 2.2156$	$y'_1 = (1.1)^2 + \frac{2.2156}{2} = 2.3178$
$x_2 = 1.2$	$y_2 = 2.4649$	$y'_2 = (1.2)^2 + \frac{2.4649}{2} = 2.67245$
$x_3 = 1.3$	$y_3 = 2.7514$	$y'_3 = (1.3)^2 + \frac{2.7514}{2} = 3.0657$
$x_4 = 1.4$	$y_4 = ?$	

We have $y_4^{(P)} = y_0 + \frac{4h}{3}(2y'_1 - y'_2 + 2y'_3)$

$$\therefore \quad y_4^{(P)} = 2 + \frac{4(0.1)}{3}[2(2.3178) - 2.67245 + 2(3.0657)] = 3.0793$$

Hence, $y'_4 = x_4^2 + \frac{y_4}{2} = (1.4)^2 + \frac{3.0793}{2} = 3.49965$

Now consider, $y_4^{(C)} = y_2 + \frac{h}{3}(y'_2 + 4y'_3 + y'_4)$

$$y(1.4) = y_4^{(C)} = 2.4649 + \frac{0.1}{3}[2.67245 + 4(3.0657) + 3.49965] = \boxed{3.0794}$$

[22] *Use Taylor's series method (upto third derivative term) to find y at* $x = 0.1, 0.2, 0.3$ *given that* $\frac{dy}{dx} = x^2 + y^2$ *with* $y(0) = 1$. *Apply Milne's predictor-corrector formulae to find* $y(0.4)$ *using the generated set of initial values.*

☞ Referring to Problem - **[1]**, we have obtained $y(0.1) = 1.1113$, $y(0.2) = 1.2507$, $y(0.3) = 1.426$. Using these values along with $y(0) = 1$ initiall

ollowing table.

10

a $\frac{dy}{dx} = \frac{x^2+y^2}{10}$ ∴ $y(0)=1$, $h=0.2$

$y(0.2) = ?$ $x_1 = x_0 + h$, $x_0 = 0$

$x_1 = 0+0.2 = \underline{\underline{0.2}}$ $y_0 = 1$

$f(x,y) = \frac{x^2+y^2}{10}$

$k_1 = h f(x_0, y_0)$

$k_1 = 0.2 f(0, 1)$

$k_1 = 0.2[0.1] = \underline{\underline{0.020}}$

$k_2 = h f\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right]$

$k_2 = 0.2 f\left[0 + \frac{0.2}{2}, 1 + \frac{0.020}{2}\right]$

$k_2 = 0.2 f[0.1, 1.010]$

$\underline{\underline{k_2 = 0.021}}$

$k_3 = h f\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right]$

$k_3 = 0.2 f\left[0.1, 1 + \frac{0.021}{2}\right]$

$k_3 = 0.2 f[0.1, 1.011]$

$\underline{\underline{k_3 = 0.021}}$

$k_4 = h f(x_0 + h, y_0 + k_3)$

$k_4 = 0.2 f[0 + 0.2, 1 + 0.021]$

$k_4 = 0.2 f[0.2, 1.021]$

$\underline{\underline{k_4 = 0.022}}$

$y(x_0 + h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$$y(0.2) = 1 + \frac{1}{6}\left[0.020 + (2 \times 0.021) + (2 \times 0.021) + 0.022\right]$$

$\boxed{y(0.2) = 1.021}$ //

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b.

Solution

$f(x,y) = 3x + \frac{y}{2}$, $y(0) = 1$, $h = 0.2$

$y(0.2) = ?$

$x_0 = 0$, $y_0 = 1$, $h = 0.2$

$y_1^{(0)} = y_0 + h\left[f(x_0, y_0)\right]$

$y_1^{(0)} = 1 + 0.2\left[f(0,1)\right]$

$y_1^{(0)} =$ 1.100 [illegible] = [illegible]

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$

$y_1^{(1)} = 1 + \frac{0.2}{2}\left[f(0,1) + f(0.2\ 1.[illegible])\right]$

$y_1^{(1)} = 1 + 0.1\left[0.5 + [illegible]\right]$

$y_1^{(1)} =$ [illegible] 1.165

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)}) \right]$$

$$y_1^{(2)} = 1 + 0.1\left[0.5 + f(0.2,\ 1.165) \right]$$

$$y_1^{(2)} = 1 + 0.1\left[1.198 \right]$$

$$y_1^{(2)} = 1.168$$

$$\therefore\ y(0.2) = 1.168$$

$$\boxed{y(0.2) = 1.168}$$ //

```
from numpy import*
def taylor(deriv, x, y, xstop, h):
→X=[]
→Y=[]
→X.append(x)
 Y.append(y)
 while x < xstop:
     D=deriv(x,y)
     H=1.0.
     for j in range(3):
         H=H*h/(j+1)
         y=y+D[j]*H
     x=x+h
     X.append(x)
     Y.append(y)
 return array(X), array(Y)
def deriv(x,y):
     D=zeros((4,1))
     D[0]=[2*y[0]+3*exp(x)]
     D[1]=[4*y[0]+9*exp(x)]
     D[2]=[8*y[0]+21*exp(x)]
     D[3]=[16*y[0]+45*exp(x)]
     return D
```

```
x=0.0
xstop =0. 3
y= array([0.0])
h=0.1
x,y=taylor(deriv,x,y,xstop,h)
print('the required values are : at x=%0.2f,
 y=%0.5f, x=%0.2f, y=%0.5f, x=%0.2f, y=%0.5f,
x=%0.2f, y=%0.5f'%(x[0],y[0],x[1],y[1],
x[2],y[2],x[3],y[3]))
```