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Internal Assessment Test – I November 2025

Sub:	Mathematics for Computer Science							Code:	BCS301
Date:	04/11/2025	Duration:	90 mins	Max Marks:	50	Sem:	III	Branch:	**CSE/ISE/ CS DS/AIDS**

Question 1 is compulsory and Answer any 6 from the remaining questions.

<table>
<tr><th rowspan="2"></th><th rowspan="2"></th><th rowspan="2">Marks</th><th colspan="2">OBE</th></tr>
<tr><th>CO</th><th>RBT</th></tr>
<tr><td>1</td><td>Find the mean and standard deviation of the Poisson distribution</td><td>[8]</td><td>CO1</td><td>L1</td></tr>
<tr><td>2</td><td>The probability distribution of a finite random variable X is given by the following data. Find the value of K, mean and variance.
<table>
<tr><td>X</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>P(X)</td><td>0.1</td><td>K</td><td>0.2</td><td>2K</td><td>0.3</td><td>K</td></tr>
</table></td><td>[7]</td><td>CO1</td><td>L1</td></tr>
<tr><td>3</td><td>The length of telephone conversation in a booth has been exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made from the booth ends i)less than 5 minutes ii) between 5 and 10 minutes</td><td>[7]</td><td>CO1</td><td>L2</td></tr>
<tr><td>4</td><td>2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains (i) no defective fuses (ii) 3 or more defective fuses.</td><td>[7]</td><td>CO1</td><td>L2</td></tr>
</table>

<table>
<tr><td>5</td><td>In an examination 7% of students score less than 35% marks and 89% of students scored less than 60% marks. Find the mean and standard deviation if the marks are normally distributed. P(1.2263) = 0..39 and P (1.4757) =0.43</td><td>[7]</td><td>CO1</td><td>L3</td></tr>
<tr><td>6</td><td>If x is normally distributed with mean 12 and S.D 4, find the following =.
(i) $P(x \geq 20)$ (ii) $P(x \leq 20)$ [Note: $\Phi(2) = 0.4772$]</td><td>[7]</td><td>CO1</td><td>L3</td></tr>
<tr><td>7</td><td>Every year a man trades his car for a new car in 3 brands of the popular company ‘Tata motors’. If he has a ‘Nexon’ he trades it for a ‘Punch’. If he has a ‘Punch’ he trades if for a ‘Jaguar Land Rover’. If he has a ‘Jaguar Land Rover’ he is just as likely to trade it for a new ‘Jaguar Land Rover’ or for a ‘Punch’ or a ‘Nexon’ one. In 2020 he bought his first car which was ‘Jaguar Land Rover’. Find the probability that he has i) 2022 ‘Jaguar Land Rover’ ii) 2022 ‘Nexon’ iii) 2023 ‘Punch’ iv) 2023 ‘Jaguar Land Rover’</td><td>[7]</td><td>CO1</td><td>L3</td></tr>
<tr><td>8</td><td>The joint distribution of two random variables X and Y is as follows.
<table>
<tr><td>X \ Y</td><td>-4</td><td>2</td><td>7</td></tr>
<tr><td>1</td><td>1/8</td><td>1/4</td><td>1/8</td></tr>
<tr><td>5</td><td>1/4</td><td>1/8</td><td>1/8</td></tr>
</table>
Compute the following:
E(x) and E(Y) (b) E(XY) (c) $\rho(X,Y)$</td><td>[7]</td><td>CO2</td><td>L3</td></tr>
</table>

1). **Mean** μ $= \sum_{x=0}^{n} x(p(x))$

$$\mu = \sum_{x=0}^{n} x \frac{e^{-m} m^{x}}{x!} = \left\{0 + \frac{e^{-m} m^{1}}{1!} + \frac{e^{-m} m^{2}}{2!} + ----\right\}$$

$$\mu = \sum_{x=1}^{n} x \frac{e^{-m} m^{x}}{x(x-1)!}$$

$$\mu = \sum_{x=1}^{n} \cancel{x} \frac{e^{-m} m^{x-1} m}{\cancel{x}(x-1)!}$$

$$\mu = m e^{-m} \sum_{x=1}^{n} \frac{m^{x-1}}{(x-1)!}$$

$$\mu = m e^{-m} \left(0 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + ----\right)$$

$$\mu = m e^{-m} . e^{m} \qquad \left(\because e^{x} = 0 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + --\right)$$

$$\mu = m.$$

Variance $v = \sum_{x=0}^{n} x^2 p(x) - \mu^2$

$$= \sum_{x=0}^{n} (x(x-1) + x) p(x) - \mu^2$$

$$v = \sum_{x=0}^{n} x(x-1) p(x) + x p(x) - \mu^2$$

$$v = \sum_{x=0}^{n} x(x-1) \frac{e^{-m} m^{x}}{x!} + \sum_{x=0} x p(x) - \mu^2$$

$$v = \sum_{x=2}^{n} \cancel{x(x-1)} \frac{e^{-m} m^{x-2} m^{2}}{\cancel{x}(\cancel{x-1})(x-2)!} + m - (m)^2$$

$$v = m^2 e^{-m} . \sum_{x=2}^{n} \frac{m^{x-2}}{(x-2)!} + m - m^2$$

$$v = m^2 e^{-m}\left(0 + \frac{m}{1!} + \frac{m^2}{2!} + \cdots\right) + m - m^2$$

$$v = m^2 e^{-m} . e^{m} + m - m^2$$

$$v = \cancel{m^2} + m - \cancel{m^2}$$

$$v = m_{//}$$

2)

x	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	k

The probability distribution should follow two conditions

(i) $\sum_{i=0}^{n} p(x) = 1$ (ii) $\sum_{i}^{n} p(x) \geq 0.$

$$\sum_{x=0}^{n} P(x) = 1$$

$$= p(x_{-2}) + P(x_{-1}) + P(x_0) + P(x_1) + P(x_2) + P(x_3) = 1$$

$$= 0.1 + k + 0.2 + 2k + 0.3 + k = 1$$

$$= 4k + 0.6 = 1$$

$$4k = 1 - 0.6$$

$$4k = 0.4$$

$$k = \frac{0.4}{4}$$

$$k = 0.1_{//}$$

The probability distribution table

x	-2	-1	0	1	2	3
P(x)	0.1	0.1	0.2	0.2	0.3	0.1

Mean $\mu = \sum_{x=0}^{n} x p(x)$

$= -2(0.1) + (-1)(0.1) + (0.2)(0) + (1)(0.2) + 2(0.3) + 3(0.1)$

$= -0.2 - 0.1 + 0 + 0.2 + 0.6 + 0.3$

$\mu = 0.8$

Variance $v = \sum_{x}^{n} x^2 p(x) - \mu^2$

$= (-2)^2(0.1) + (-1)^2(0.1) + (0)^2(0.2) + (1)^2(0.2) + (2)^2(0.3) + (3)^2(0.1) - (0.8)^2$

$= 0.4 + 0.1 + 0 + 0.2 + 1.2 + 0.9 - 0.64$

$= 2.8 - 0.64$

$v = 2.16$ //

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3). Let mean $\mu = 5 = \frac{1}{\alpha}$

$\alpha = \frac{1}{5}$

(i) $p(x<5) = \int_0^\infty \alpha e^{-\alpha x} dx$

$= \int_0^5 \frac{1}{5} e^{-1/5 x} dx$

$= \frac{1}{5} \left[\frac{e^{-1/5 x}}{-1/5} \right]_0^5$

$= \frac{1}{\cancel{5}} \times \frac{-\cancel{5}}{1} \left[e^{-1} - e^{0} \right]$

$= -\left[e^{-1} - 1 \right]$

$= -e^{-1} + 1 = -0.367 + 1 = 0.633$ //

(ii) $$P(5 \le x \le 10) = \int_{5}^{10} \frac{1}{5} e^{-1/5 x} \, dx$$

$$= \frac{1}{5}\left[\frac{e^{-1/5 x}}{-1/5}\right]_{5}^{10}$$

$$= \frac{1}{\cancel{5}} \times \frac{-\cancel{5}}{1}\left[e^{-2} - e^{-1}\right]$$

$$= -\left[0.1353 - 0.3678\right]$$

$$= +0.2325 \,//$$

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4) Let the fuses manufactured are defective $= p$

$$= \frac{2}{100} = 0.02$$

Let n be the number of fuses in a box $= 200$

The mean of the probability $m = np$

$$m = 200 \times 0.02$$

$$m = 4$$

(i) $$P(x=0) = \frac{e^{-m} m^{x}}{x!}$$

$$= \frac{e^{-4} 4^{0}}{0!} = e^{-4} = 0.0183 \,//$$

(ii) $$P(x \ge 3) = 1 - p(x_0) + p(x_1) + p(x_2)$$

$$= 1 - \left(\frac{e^{-4} 4^{0}}{0!} + \frac{e^{-4} 4^{1}}{1!} + \frac{e^{-4} 4^{2}}{2!}\right)$$

$$= 1 - \left(e^{-4} + 4e^{-4} + \frac{\overset{8}{\cancel{16}} e^{-4}}{\cancel{2}}\right)$$

$$= 1 - \left(13e^{-4}\right)$$

$= 1 - 0.2381$

$= 0.7619$ //

No defective $= 200 \times 0.0183 = 3.66 \approx 4$

3 or more defective pieces $= 200 \times 0.7619 = 152.38$

5). $P(x < 35) = 7\% = 0.07$

$P(x < 60) = 89\% = 0.89$

$P(x < 35) = z_1 = \frac{x-\mu}{\sigma} = \frac{35-\mu}{\sigma}$ — ①

$P(x < 60) = z_2 = \frac{x-\mu}{\sigma} = \frac{60-\mu}{\sigma}$ — ②

$P(x < 35) = P(z < z_1) = 0.07$

$= 0.5 + P(z_1) = 0.07$

$= P(z_1) = 0.07 - 0.5$

$P(z_1) = -0.43$

$z_1 = -1.4757$

$P(x < 60) = P(z < z_2) = 0.89$

$= 0.5 + P(z_2) = 0.89$

$= P(z_2) = 0.39$

$z_2 = 1.2263$

Subtitute values of z_1 and z_2 in eqn ① & ②

$\frac{35-\mu}{\sigma} = -1.4757$

$35 - \mu = -1.4757\sigma$ — ③

$\frac{60-\mu}{\sigma} = 1.2263$

$60 - \mu = 1.2263\sigma$ — ④

$60 - \cancel{\mu} = 1.22636$

$\underline{\underset{(-)}{35} \underset{(+)}{-} \cancel{\mu} = \underset{(+)}{-}1.47576}$ $\qquad \mu = 48.653$

$25 = 2.70206$

$\sigma = \frac{25}{2.7020}$

$\sigma = 9.2524$ //

$60 - \mu = (1.2263)(9.2524)$

$\mu = 60 - 11.3462$

$\mu = 48.6538$ //

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6. given that mean $\mu = 12$

Standard deviation $\sigma = 4$

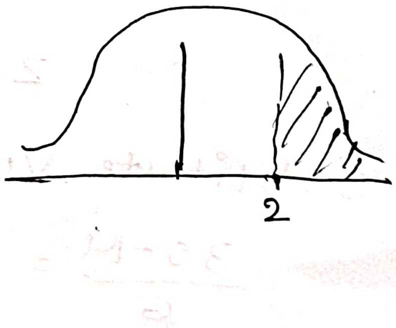
(i) $P(x \geq 20) = \quad z = \frac{x - \mu}{\sigma}$

$z = \frac{20 - 12}{4} = \frac{8}{4} = 2$

$P(x \geq 20) = P(x \geq 2)$

$= 0.5 - A(0, 2)$

$= 0.5 - \int_0^2 \phi(z)\, dz$



$= 0.5 - 0.4772$

$= 0.0228$

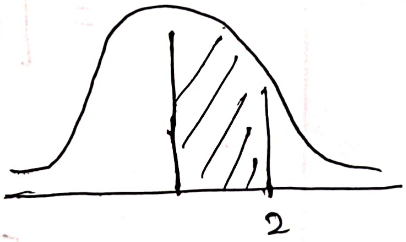
(ii) $p(x \leq 20) = 2 = \frac{x-\mu}{\sigma} = \frac{20-12}{4} = \frac{8}{4} = 2$

$p(x \leq 20) = p(x \leq 2)$

$= A(0, 2)$

$= \int_0^2 \phi(z)\, dz$

$= 0.4772$ //



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7). The state space of the matrix $= [a_1, a_2, a_3]$

a_1 :- Nexon car

a_2 : Punch car

a_3 = Jaguar Land Rover

The Initial probability matrix $= \overset{N}{[0}, \overset{P}{0}, \overset{J}{1}] = p^{(0)}$

The transition probability

$$P = \begin{array}{c|ccc} & N & P & J \\ \hline N & 0 & 1 & 0 \\ P & 0 & 0 & 1 \\ J & 1/3 & 1/3 & 1/3 \end{array}$$

(i) Probability that he has Jaguar Land rover in 2022

$p^{(2)} = p^{(0)} . p^2$

$$p^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$1/3 + 1/9$

$= \frac{3+1}{9} = 4$

$$= \begin{bmatrix} 0+0+0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{bmatrix}$$

$$p(2) = p^0 . p^2$$

$$= [0, 0, 1] \begin{bmatrix} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{bmatrix}$$

$$= \left[1/9 , 4/9 , 4/9\right] //$$

(ii) probability that he has Nexon in 2022.

$$p(2) = p^0 . p^2$$

$$= [0, 0, 1] \begin{bmatrix} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{bmatrix}$$

$$= \left[1/9 , 4/9 , 4/9\right] //$$

(iii) Probability that he has punch car in 2023

$$p(3) = p^{(0)} . p^3 = p(0) . p^2 . p$$

$$p^3 = \begin{bmatrix} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$\frac{1}{27} + \frac{4}{27}$$

$$= \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \\ 4/27 & 7/27 & 16/27 \end{bmatrix}$$

$p(3) = p^0 \cdot p^3$

$$= [0, 0, 1] \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \\ 4/27 & 7/27 & 16/27 \end{bmatrix}$$

$$= [4/27, 7/27, 16/27] //$$

(iv) Probability that he has Jaguar Land Rover

$p3 = p^0 \cdot p^3$

$$= [0, 0, 1] \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \\ 4/27 & 7/27 & 16/27 \end{bmatrix}$$

$$= [4/27, 7/27, 16/27] //$$

8

x \ y	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

$P(x=1) = 1/8 + 1/4 + 1/8 = 4/8 = 1/2$

$P(x=5) = 1/4 + 1/8 + 1/8 = 4/8 = 1/2$

$P(y=-4) = 1/8 + 1/4 = 3/8$

$P(y=2) = 1/4 + 1/8 = 3/8$

$P(y=7) = 1/8 + 1/8 = 2/8 = \frac{1}{4}$

The Marginal distribution of x

x	1	5
P(x)	1/2	1/2

Marginal distribution of y

y	-4	2	7
$P(y)$	$3/8$	$3/8$	$1/4$

(a) $E(x) = \sum_{i=x}^{n} x\, p(x)$

$= (1)\, 1/2 + (5)(1/2)$

$= 1/2 + 5/2$

$= \frac{6}{2} = 3_{//}$

$E(y) = \sum_{i=y}^{n} y\, P(y)$

$= -4(3/8) + 2(3/8) + 7(1/4)$

$= -\frac{12}{8} + \frac{6}{8} + \frac{7}{4}$

$= 1$

(b) $E(xy) = \sum_{x}\sum x y\, P(x = x, y = y)$

$= x_1 y_1\, P(x_1, y_1) + x_1 y_2 P(x_1, y_2) + x_1 y_3\, P(x_1, y_3)$
$+ x_2 y_1\, P(x_2, y_1) + x_2 y_2 P(x_2, y_2) + x_2 y_3\, P(x_2\, y_3)$

$= (1)(-\overset{1}{\cancel{4}})(1/\underset{2}{\cancel{8}}) + (1)(2)(1/4) + (1)(7)(1/8) +$
$(5)(-\cancel{4})(1/\cancel{4}) + (5)(\overset{1}{\cancel{2}})(1/\underset{4}{\cancel{8}}) + (5)(7)(1/8)$

$= -1/2 + 1/2 + 7/8 + (-5) + \frac{5}{4} + \frac{35}{8}$

$E(xy) = 3/2_{//}$

c) $\text{cov} = E(XY) - E(X)E(Y)$

$$= 3/2 - 3(1)$$

$$= 3/2 - 3$$

$$= \cancel{9/2} - 3/2 \;//$$

$$p(x,y) = \frac{\text{cov}\, xy}{\sigma_x \sigma_y}$$

$$\sigma_x^2 = E(x^2) - [E(x)]^2$$

$$= \sum_x x^2 p(x) - \mu^2$$

$$= (1^2)(1/2) + (5)^2(1/2) - 9$$

$$= 1/2 + \frac{25}{2} - 9$$

$$= \frac{26}{2} - 9$$

$$\sigma_x^2 = 4$$

$$\sigma_x = 2 \;//$$

$$\sigma_y^2 = E(y)^2 - [E(y)]^2$$

$$= \sum_y y^2 p(y) - \mu^2$$

$$= (-4)^2(3/8) + 2^2(3/8) + 7^2\left(\frac{1}{4}\right) - 1^2$$

$$= \frac{48}{8} + 3/2 + \frac{49}{4} - 1$$

$$= \frac{79}{4} - 1$$

$$\sigma_y^2 = \frac{75}{4} = 18.75$$

$$\sigma_y = 4.33 \;//$$

$$p(x,y) = \frac{-3/2}{2 \times 4.33} = \frac{-1.5}{8.66}$$

$$p(x,y) = -0.1732 \;//$$