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Internal Assessment Test – I November 2025

Sub:	AV Mathematics-III for EC Engineering							Code:	BMATEC301
Date:	04/11/2025	Duration:	90 mins	Max Marks:	50	Sem:	III	Branch:	ECE

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE	
			CO	RBT
1	Obtain the Fourier series of y upto 2^{nd} harmonics $f(x)$ is given by	[8]	CO1	L3
2	Find the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $(0, 2\pi)$. Deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\cdots$	[7]	CO1	L2
3	Find the Fourier series of $f(x) = \lvert x \rvert$, in$(-l, l)$.	[7]	CO1	L3
4	Expand $f(x) = 2x - 1$ as a cosine half range Fourier series in $0 < x < 1$.	[7]	CO1	L2

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$	1.98	1.30	1.05	1.03	-0.88	-0.25	1.98

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5	Find the Fourier transform of $f(x) = \begin{cases} 1-\lvert x\rvert, & for\ \lvert x\rvert \le 1 \\ 0, & for\ \lvert x\rvert > 1 \end{cases}$ and hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2}\, dt = \frac{\pi}{2}$.	[7]	CO2	L2
6	Find the Fourier cosine and sine transform of $f(x) = e^{-\alpha x}$, $\alpha > 0$.	[7]	CO2	L3
7	Find the Fourier sine transform of $f(x) = e^{-\lvert x\rvert}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2}\, dx, m > 0$.	[7]	CO2	L2
8	Find the inverse Fourier sine transform of $\frac{e^{-au}}{u}, a > 0$. or Find the Z-transform of i) $\cos n\theta$ and ii) $\sin n\theta$.	[7]	CO3	L2

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IAT-1.

$\frac{\pi}{3} = 60°$

① The Fourier series of y upto 2nd harmonics,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$x°$	$f(x)=y$	$y\cos x$	$y\sin x$	$y\cos 2x$	$y\sin 2x$
0°	1.98	1.98	0	1.98	0.
60°	1.30.	0.65	1.1258	−0.65	1.1258
120°	1.05	−0.525	0.90933	−0.525	−0.90933
180.	1.03	−1.03	0	1.03	0.
240°	−0.88	0.44	0.7621	0.44	−0.7621.
300°	−0.25	−0.125	0.2165	0.125	0.2165
N=6	$\Sigma y = 4.23$.	$\Sigma y\cos x = 1.39$	$\Sigma y\sin x = 3.0137$	$\Sigma y\cos 2x = 2.4$	$\Sigma y\sin 2x = -0.3291$.

cosx	sinx	cos2x	sin2x
1.	0.	1	0.
0.5	0.8660	−0.5	0.8660
−0.5	0.8660.	−0.5	−0.8660
−1.	0.	1	0.
−0.5	−0.8660	−0.5	0.8660
0.5	−0.8660	−0.5	−0.8660.

$N = 6.$

$a_0 = \frac{2}{N}\sum y = \frac{\cancel{2}}{\cancel{6}_3}(4.23) = 1.41.$

$a_1 = \frac{2}{N}\sum y\cos x = \frac{\cancel{2}}{\cancel{6}_3}(1.39) = 0.4633$

$b_1 = \frac{2}{N}\sum y\sin x = \frac{1}{3}(3.0137) = 1.0046.$

$a_2 = \frac{2}{N}\sum y\cos 2x = \frac{1}{3}(2.4) = 0.8$

$b_2 = \frac{2}{N}\sum y\sin 2x = \frac{1}{3}(-0.3291) = -0.1097.$

$f(x) = \frac{a_0}{2} + \left(\sum a_1\cos x + \sum b_1\sin nx\right) + \left(a_2\cos 2x + b_2\sin 2x\right)$ ①

$= \frac{1.41}{2} + (0.4633\cos x + 1.0046\sin x) +$

$(0.8\cos 2x - 0.1097\sin 2x).$

$\therefore f(x) = 0.705 + (0.4633\cos x + 1.0046\sin x) + (0.8\cos 2x - 0.1097\sin 2x).$

is the required fourier series.

2] $f(x) = \frac{\pi - x}{2}$ the given interval is $(0, 2\pi)$.

Given that $f(x) = \frac{\pi - x}{2}$ —— ①.

Fourier series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx. \quad —— ②$$

$$f(x) = \frac{\pi - x}{2}$$

$$f(2\pi - x) = \frac{\pi - (2\pi - x)}{2} = \frac{\pi - 2\pi + x}{2} = \frac{-\pi + x}{2} = -\frac{(\pi - x)}{2} = -f(x).$$

f(x) is a odd function from above analysis.

If a function is odd. then $a_0 = 0$; $a_n = 0$.

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \sin nx \, dx.$$

$$= \frac{2}{\pi}\int_0^{\pi} \left(\frac{\pi - x}{2}\right) \sin nx \, dx$$

$$= \frac{2}{\pi}\left[\left(\frac{\pi - x}{2}\right)\left(\frac{-\cos nx}{n}\right) - \left(-\frac{1}{2}\right)\left(\frac{-\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\left(\frac{\pi - x}{2}\right)\frac{\cos nx}{n} - \overset{0}{\cancel{\frac{\sin nx}{n^2}}}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\left(\frac{\pi - x}{2}\right)\frac{\cos nx}{n}\right]_0^{\pi}$$

$$\boxed{\begin{array}{l}\sin n\pi = 0 \\ \text{and } \sin 0° = 0.\end{array}}$$

$$= \frac{2}{\pi}\left[-\frac{(\overset{0}{\cancel{\pi - \pi}})}{2}\frac{\cos n\pi}{n} - \left(-\left(\frac{\pi - 0}{2}\right)\frac{\cos 0°}{n}\right)\right]$$

$$= \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\cancel{\pi}}{\cancel{2}}\left(\frac{1}{n}\right)\right] = \frac{1}{n}.$$

$b_n = \frac{1}{n}$.

∴ the fourier series is given by —— from ②.

$$f(x) = \frac{0}{2} + 0 + \sum_{n=1}^{\infty} b_n \sin nx$$

② —— $$= \sum_{n=1}^{\infty} \frac{1}{n} (\sin nx).$$

let take $x = \frac{\pi}{2}$

$$f(\pi/2) = \frac{\pi - \pi/2}{2} = \frac{\pi/2}{2} = \frac{\pi}{4}.$$

$$f(\pi/2) = \sum_{n=1}^{\infty} \frac{1}{n} (\sin n\pi/2) = \frac{\pi}{4}.$$

$$\frac{\pi}{4} = \frac{\sin(1)\frac{\pi}{2}}{(1)} + \frac{\sin(2)\frac{\pi}{2}}{(2)} + \frac{\sin(3)\frac{\pi}{2}}{(3)} + \frac{\sin(4)\frac{\pi}{2}}{(4)} + \frac{\sin(5)\frac{\pi}{2}}{(5)} + \cdots$$

$$\frac{\pi}{4} = \frac{1}{1} + 0 - \frac{1}{3} + 0 + \frac{1}{5} - \frac{1}{7} + \cdots$$

∴ the deduced form will be

$$\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}.$$

3] Given that $f(x) = |x|$

The interval is $(-l, l)$.

$l \Rightarrow \frac{l-(-l)}{2} = \frac{2l}{\cancel{2}}$

$$f(x) = \begin{cases} -x, & x<0 \quad (-l, 0) \\ x, & x>0 \quad (0, l). \end{cases} \text{——} ①$$

Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) \quad \text{—} ②$$

$f(x) = |x| = x.$

$f(-x) = x$

It is an even function, so $b_n = 0$. here.

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx = \frac{1}{l}\left[\int_{-l}^{0} -x\,dx + \int_{0}^{l} x\,dx\right]$$

$$= \frac{1}{l}\left[-\left[\frac{x^2}{2}\right]_{-l}^{0} + \left[\frac{x^2}{2}\right]_{0}^{l}\right] = \frac{1}{l}\left[-\left[0 - \frac{l^2}{2}\right] + \left[\frac{l^2}{2} - 0\right]\right]$$

$$a_0 = \frac{1}{l}\left[\frac{l^2}{2} + \frac{l^2}{2}\right] = \frac{1}{\cancel{l}}\left(\frac{\cancel{2}\, l^{\cancel{2}}}{\cancel{2}}\right) = \boxed{l = a_0}$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx. = \frac{1}{l}\int_{-l}^{l}$$

$$= \frac{1}{l}\left[\int_{-l}^{0} -x\cos\left(\frac{n\pi x}{l}\right)dx + \int_{0}^{l} x\cos\left(\frac{n\pi x}{l}\right)dx\right]$$

$$= \frac{1}{l}\left(\left[\left(\frac{-x\left(\sin\left(\frac{n\pi x}{l}\right)\right)}{n\pi/l} - (-1)\frac{\left(-\cos\frac{n\pi x}{l}\right)}{(n\pi/l)^2}\right)\right]_{-l}^{0} + \right.$$

$$\left.\left[\frac{x\sin\frac{n\pi x}{l}}{n\pi/l} - (1)\frac{\left(-\cos\frac{n\pi x}{l}\right)}{(n\pi/l)^2}\right]_{0}^{l}\right)$$

$$= \frac{1}{l}\left[\left[\frac{-x\sin\frac{n\pi x}{l}}{n\pi/l} - \frac{\cos\frac{n\pi x}{l}}{(n\pi/l)^2}\right]_{-l}^{0} + \left[\frac{x\sin\frac{n\pi x}{l}}{n\pi/l} + \frac{\cos\frac{n\pi x}{l}}{(n\pi/l)^2}\right]_{0}^{l}\right]$$

$$= \frac{1}{l}\left[\left[0 - \frac{1}{(n\pi/l)^2} - \left(0 - \frac{\cos n\pi}{(n\pi/l)^2}\right)\right] + \left[0 + \frac{\cos n\pi}{(n\pi/l)^2} - \left(0 + \frac{1}{(n\pi/l)^2}\right)\right]\right]$$

$$= \frac{1}{l}\left[\frac{-1}{(n\pi/l)^2} + \frac{(-1)^n}{(n\pi/l)^2} + \frac{(-1)^n}{(n\pi/l)^2} - \frac{1}{(n\pi/l)^2}\right] = \frac{1(l)^2}{l(n\pi)^2}\left[-2 + 2(-1)^n\right]$$

$$= \frac{l^2}{(n\pi)^2} \times \frac{1}{l} \left[-2+2(-1)^n\right]$$

$$= \frac{l}{(n\pi)^2} (-2) \left[1-(-1)^n\right]$$

$$a_n = \frac{-2l}{(n\pi)^2} \left[1-(-1)^n\right]$$

$a_0 = l$

$b_n = 0$

Substituting a_0, a_n in eq^n ②

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + 0$$

$$= \frac{l}{2} + \sum_{n=1}^{\infty} \frac{-2l}{n^2\pi^2} \left[1-(-1)^n\right] \cos\frac{n\pi x}{l}.$$

$$= \frac{l}{2} - \frac{2l}{\pi^2} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^2} \cos\left(\frac{n\pi x}{l}\right).$$

here $1-(-1)^n$ function give 2 when n is odd. and 0 when n is even.

$$\therefore f(x) = \frac{l}{2} - \frac{2l}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{(2)}{n^2} \cos\left(\frac{n\pi x}{l}\right).$$

$$\boxed{f(x) = \frac{l}{2} - \frac{4l}{\pi^2} \sum_{n=1,3-}^{\infty} \frac{\cos\left(\frac{n\pi x}{l}\right)}{n^2}.} = \frac{l}{2} - \frac{4l}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos(2k-1)\pi x}{(2k-1)^2} //.$$

where k represents odd numbers

4] $f(x) = 2x-1. \rightarrow ①$ is the given function

interval $0<x<1$ or $(0,1) \leftrightarrow (0,l)$.

Fourier half range cosine series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + 0. \text{ ——②} \quad \text{here } b_n = 0.$$

$l = 1-0 = 1.$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx = 2\int_0^1 (2x-1)\,dx$$

$$= 2\left[\frac{\cancel{2}x^2}{\cancel{2}} - x\right]_0^1 = 2[(1-1)-(0-0)] = 0.$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx. = \frac{2}{1}\int_0^1 (2x-1)\cos n\pi x\,dx$$

$$= \frac{2}{1}\left[(2x-1)\frac{\sin n\pi x}{n\pi} - (2)\frac{(-\cos n\pi x)}{(n\pi)^2}\right]_0^1$$

$$= 2\left[(2x-1)\frac{\overset{0}{\cancel{\sin n\pi x}}}{n\pi} + 2\frac{\cos n\pi x}{(n\pi)^2}\right]_0^1$$

$$= 2\cdot 2\left[\frac{\cos n\pi x}{(n\pi)^2}\right]_0^1 = \frac{4}{(n\pi)^2}[\cos n\pi - \cos 0]$$

$$\boxed{a_n = \frac{4}{(n\pi)^2}[(-1)^n - 1]}$$

$b_n = 0$

Substitute a_0, a_n in eq — ②

$$f(x) = 0 + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2}[(-1)^n - 1]\cos\frac{n\pi x}{1} + 0.$$

$$= \frac{4}{\pi^2}\left[\sum_{n=1}^{\infty} \frac{((-1)^n-1)}{n^2}\cos n\pi x\right]$$

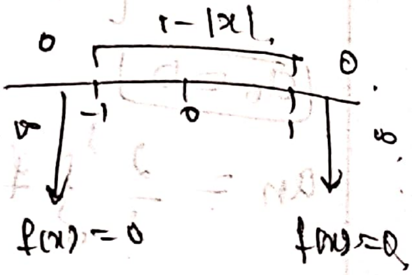
$[(-1)^n - 1]$
↳ odd → gives -2
↳ even → gives 0.

$$= \frac{4}{\pi^2}\sum_{n=1,3,\ldots}^{\infty} \frac{(-2)}{n^2}\cos n\pi x$$

$$f(x) = -\frac{8}{\pi^2}\sum_{n=1,3,--}^{\infty}\frac{\cos n\pi x}{n^2}$$ here k represents an odd number

$$f(x) = -\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(2k-1)\pi x}{(2k-1)^2}.$$

5] Given $f(x) = \begin{cases} 1-|x|, & |x| \le 1 \\ 0 & |x| > 1. \end{cases}$



Fourier transform of f(x) is.

$$F_c(u) = \int_{-\infty}^{\infty} f(x)\, e^{iux}\, dx$$

$$= \int_{-1}^{1} f(x) e^{iux} dx.$$

$$= \left[\int_{-1}^{0}(1+x)e^{iux}dx + \int_{0}^{1}(1-x)e^{iux}dx\right]$$

$$= \left[(1+x)\frac{e^{iux}}{iu} - (1)\frac{e^{iux}}{i^2u^2}\right]_{-1}^{0} + \left[(1-x)\frac{e^{iux}}{iu} - (-1)\frac{e^{iux}}{i^2u^2}\right]_{0}^{1}$$

$$= \left[(1+x)\frac{e^{iux}}{iu} + \frac{e^{iux}}{u^2}\right]_{-1}^{0} + \left[(1-x)\frac{e^{iux}}{iu} - \frac{e^{iux}}{u^2}\right]_{0}^{1}$$

$$= \left[1\cdot\frac{e^0}{iu} + \frac{e^0}{u^2} - \left(0 + \frac{e^{-iu}}{u^2}\right)\right] + \left[\left(0 - \frac{e^{iu}}{u^2}\right) - \left(1\cdot\frac{e^0}{iu} - \frac{e^0}{u^2}\right)\right]$$

$$= \left[\frac{1}{iu} + \frac{1}{u^2} - \frac{e^{-iu}}{u^2}\right] + \left(-\frac{e^{iu}}{u^2} - \frac{1}{iu} + \frac{1}{u^2}\right)$$

$$= \cancel{\frac{1}{iu}} + \frac{1}{u^2} - \frac{e^{-iu}}{u^2} - \frac{e^{iu}}{u^2} - \cancel{\frac{1}{iu}} + \frac{1}{u^2}$$

$$= \frac{2}{u^2} - \frac{1}{u^2}\left(e^{-iu} + e^{+iu}\right).$$

$e^{ix} + e^{-ix} = 2\cos x$

$$= \frac{2}{u^2} - \frac{1}{u^2}(2\cos u).$$

$$F_c(u) = \frac{2}{u^2}(1 - \cos u) = \frac{2}{u^2}\left(\sin^2 u/2\right)$$

Inverse Fourier Transform

$$f(x) = \frac{2}{\pi}\int_0^\infty F_c(u)\,\overset{e^{-iux}}{\cancel{\cos ux}}\,du$$

$$f(0) = \frac{2}{\pi}\int_0^\infty \frac{2\sin^2 u/2}{u^2}\,\overset{e^{-iux}}{\cancel{\cos ux}}\,du.$$

let $x = 0$.

$f_c(0) = 1 - |0| = 1.$

$$1 = \frac{2}{\pi}\int_0^\infty \frac{2\sin^2 u/2}{u^2}\,du$$

let $u/2 = t$

$u = 2t$

$1 = 2\frac{dt}{du}$

$2dt = du.$

$$1 = \frac{4}{\pi}\int_0^\infty \frac{\sin^2 u/2}{u^2}\,du$$

$$\frac{\pi}{4} = \int_0^\infty \frac{4\sin^2 u/2}{u^2}\,du = \int_0^\infty \frac{\sin^2 u/2}{(u/2)^2}\,du$$

$$= \int_0^\infty \frac{\sin^2 t}{t^2}\,2dt$$

$$1 = \frac{2}{\pi}\int_0^\infty \frac{\sin^2 t}{t^2}\,dt.$$

$$\boxed{\frac{\pi}{2} = \int_0^\infty \frac{\sin^2 t}{t^2}\,dt}$$

6] Given $f(x) = e^{-\alpha x}$, $\alpha > 0$ — ①.

The Fourier cosine transform is given by

$$F_c(u) = \int_{0}^{\infty} f(x) \cos ux \, dx.$$

$$= \int_0^\infty e^{-\alpha x} \cos ux \, dx.$$

we have $\int_0^\infty e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx$

$$= \left[\frac{e^{-\alpha x}}{\alpha^2+u^2}(-\alpha\cos ux + u\sin ux)\right]_0^\infty$$

$$= \left[\frac{\cancel{e^{-\infty}}\to 0}{\alpha^2+u^2}[-\alpha\cos ux + u\sin ux) - \frac{e^0 \to 1}{\alpha^2+u^2}(-\alpha)\right] = \frac{\alpha}{\alpha^2+u^2}$$

$$\boxed{F_c(u) = \frac{\alpha}{\alpha^2+u^2}.}$$

The fourier sine transform is

$$F_s(u) = \int_0^\infty f(x) \sin ux \, dx.$$

$$= \int_0^\infty e^{-\alpha x} \sin ux \, dx$$

we have $\int_0^\infty e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx$

$$F_s(u) = \left[\frac{e^{-\alpha x}}{\alpha^2+u^2}(-\alpha\sin ux - u\cos ux)\right]_0^\infty$$

$$F_s(u) = \left[\cancelto{0}{\frac{e^{-\infty}}{\alpha^2+u^2}}(-\alpha\sin ux - u\cos ux) - \frac{e^{0}}{\alpha^2+u^2}(0 - u)\right]$$

$$\boxed{F_s(u) = \frac{u}{\alpha^2+u^2}}$$

8] z Transform

i) $\cos n\theta$. ii) $\sin n\theta$.

let $k = e^{i\theta} = \cos\theta + i\sin\theta$

$k^n = e^{in\theta} = \cos n\theta + i\sin n\theta$.

$$Z_T(k^n) = \frac{z}{z-k}$$

$$Z_T(e^{i\theta n}) = \frac{z}{z-e^{i\theta}}$$

$$Z_T[\cos n\theta + i\sin n\theta] = \frac{z}{z-e^{i\theta}} \quad \text{——} \ ①$$

$$= \frac{z}{z-e^{i\theta}} \times \frac{z-e^{-i\theta}}{z-e^{-i\theta}}$$

$$= \frac{z^2 - ze^{-i\theta}}{z^2 - ze^{-i\theta} - ze^{i\theta} + e^{0}}$$

$$= \frac{z^2 - z\cancel{e}(\cos\theta - i\sin\theta)}{z^2 - z(e^{-i\theta} + e^{i\theta}) + 1}$$

$$= \frac{z^2 - z\cos\theta + iz\sin\theta}{z^2 - z\,2\cos\theta + 1} \quad \text{——} \ ②$$

WKT $e^{-ix} = \cos x - i\sin x$

$e^{ix} + e^{-ix} = 2\cos x$

Seperate imaginary term from eq^n ② we get,

$$Z_T[\cos n\theta + i\sin n\theta] = \frac{z^2 - z\cos\theta}{z^2 - z2\cos\theta + 1} + i\frac{(z\sin\theta)}{z^2 - z2\cos\theta + 1}$$

Equating the terms.

$$Z_T[\cos n\theta] = \frac{z^2 - z\cos\theta}{z^2 - z\,2\cos\theta + 1}$$

$$i Z_T[\sin n\theta] = i\left[\frac{z\sin\theta}{z^2 - z2\cos\theta + 1}\right]$$

$\therefore$

i) $$Z_T[\cos n\theta] = \frac{z^2 - z\cos\theta}{z^2 - z2\cos\theta + 1}$$

ii) $$Z_T[\sin n\theta] = \frac{z\sin\theta}{z^2 - z2\cos\theta + 1}$$

7] $f(x) = e^{-|x|}$.

Fourier sine transform

$$F_s(u) = \int_0^\infty f(x)\sin ux\,dx.$$

$$= \int_0^\infty e^{-x}\sin ux\,dx.$$

using $\int_0^\infty e^{a}\sin bx$ formula.

$$F_s(u) = \left[\frac{e^{-x}}{1+u^2}\left[-\sin ux - u\cos ux\right]\right]_0^\infty$$

$$F_s(u) = \left[0 - \frac{e^0}{1+u^2}(0-u)\right] = \frac{u}{1+u^2}$$

Inverse Fourier transform

$$f_s(x) = \frac{2}{\pi}\int_0^\infty F_s(u)\sin ux\, du.$$

$$= \frac{2}{\pi}\int_0^\infty \frac{u}{1+u^2}\sin ux\, du.$$

Subt e^{-m} in the above eqn

$$f_s(m) = \frac{2}{\pi}\int_0^\infty \frac{u}{1+u^2}\sin ux\, du$$

$$e^{-m} = \frac{2}{\pi}\int_0^\infty \frac{x}{1+x^2}\sin mx\, dm.$$

. replacing u by x.

$$\frac{\pi}{2}e^{-m} = \int_0^\infty \frac{x\sin mx}{1+x^2}\, dx..$$

$$\int_0^\infty \frac{x\sin mx}{1+x^2}\, dx = \frac{\pi}{2}e^{-m}, \quad m > 0.$$

⑧ Inverse fourier sine transform of $F_s[u]$ is

$$f_s(x) = \frac{2}{\pi}\int_0^\infty F_s[u].\sin ux.du$$

$$F_s[u] = \frac{e^{-au}}{u},\ a>0$$

$$f_s(x) = \frac{2}{\pi}\int_0^\infty \frac{e^{-au}}{u}.\sin ux.du$$

we can't integrate this so differentiate w.r.to x.

$$\frac{d[f_s(x)]}{dx} = \frac{2}{\pi}\int_0^\infty \frac{e^{-au}}{\cancel{x}}.\underline{\cos ux}.\cancel{x}.du$$

$$\frac{d}{dx}[f_s(x)] = \frac{2}{\pi}\int_0^\infty e^{-au}.\cos ux.dx$$

$$= \frac{2}{\pi}\left[\frac{e^{-au}}{a^2+x^2}\left[-a\left[\cos xu + x\sin xu\right]\right]\right.$$

$$= \frac{2}{\pi}\left[e^{-\infty} - \frac{e^{0}}{a^{2}+x^{2}}\left[-a\left[\cos x(0) + x(0)\right]\right]\right]$$

$$= \frac{2}{\pi}\left[+\frac{a}{a^{2}+x^{2}}\right]$$

$$\frac{d}{dx}\left(F_s(x)\right) = \frac{2}{\pi}\left[\frac{a}{a^{2}+x^{2}}\right]$$

$$F_s(x) = \frac{2}{\pi}\int\frac{a}{a^{2}+x^{2}}$$

$$\boxed{F_s(x) = \frac{2}{\pi}\tan^{-1}\left(x/a\right).}$$

ou

$$\Big]_{0}^{\infty}$$