Solutions to IAT1
Subject: Microwave Engineering and Antenna Theory

Internal Assessment Test 1 – Sept. 2025

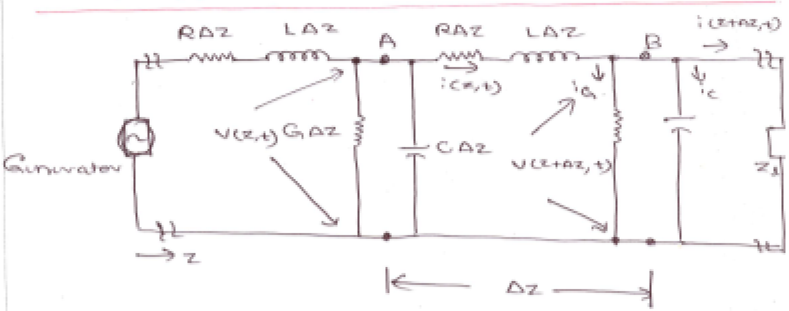
Sl.	Answer any FIVE FULL Questions	Marks	CO	RBT
1	Derive the expression for the voltage and current at any point along a uniform transmission line.	10	1	L2
2	The values of primary constants of an open-wire line per loop kilometers are as follows: R=10Ω, L=3.5mH, C=0.008 μF and G=0.7 μS. For signal frequency of 1000Hz, calculate the characteristic impedance and propagation constant.	10	1	L3
3	A load impedance of Z_l=60-j80 Ω is required to be matched to a 50-ohm coaxial line, by using a short-circuited stub of length 'l' located at a distance 'd' from the load. The wavelength of operation is 1m. Using the Smith chart, find 'd' and 'l'.	10	1	L3
4	Prove symmetric property and unitary property of the S-matrix.	10	2	L2
5	What is a magic tee? Derive the S-matrix of magic tee. Mention its application.	10	2	L2
6	Explain with the neat diagram the precision phase shifter.	10	2	L2
7a	Derive the characteristic impedance of the Microstrip line.	6	2	L2

7b	A certain microstrip line has the following parameters: εr=5.23, h=7 mils, t=2.8 mils, w=10 mils. Calculate the characteristic impedance Z0 of the line.	4	2	L3
8a	Write short notes on parallel strip lines.	5	2	L2
8b	Describe ohmic skin losses and radiation losses in micro strip lines.	5	2	L2

Solutions:

1.

Transmission Lines Equations and Solutions



Elementary section of a transmission line.

By Kirchoff's Voltage law, the summation of the voltage drops around the central loop is given by

$$v(z,t) = R\Delta z\, i(z,t) + L\Delta z \frac{\partial i(z,t)}{\partial t} + v(z+\Delta z, t)$$

$$= R\Delta z\, i(z,t) + L\Delta z \frac{\partial i(z,t)}{\partial t} + v(z,t) + \frac{\partial v(z,t)}{\partial z}\Delta z \quad (1)$$

Rearranging this equation, dividing it by Δz, and then omitting the argument (z,t) which is understood, we get

$$\boxed{-\frac{\partial v}{\partial z} = Ri + L\frac{\partial i}{\partial t}} \qquad (2)$$

Using Kirchoff's current law, the summation of the currents at point B can be expressed as

$$\cancel{i(z,t)} = G\Delta z\, v(z+\Delta z, t) + C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} + i(z+\Delta z, t)$$

$$= G\Delta z\left[v(z,t) + \frac{\partial v(z,t)}{\partial z}\Delta z\right] + C\Delta z\frac{\partial}{\partial t}\left[v(z,t) + \frac{\partial v(z,t)}{\partial z}\Delta z\right] + \cancel{i(z,t)} + \frac{\partial i(z,t)}{\partial z}\Delta z \qquad (3)$$

$$0 = G\,v(z,t) + G\cancel{\frac{\partial v(z,t)}{\partial z}\Delta z} + C\frac{\partial}{\partial t}v(z,t) + C\frac{\partial}{\partial t}\left[\cancelto{0}{\frac{\partial v(z,t)}{\partial z}\Delta z}\right] + \frac{\partial i(z,t)}{\partial z}$$

$$\Rightarrow \quad \boxed{-\frac{\partial i}{\partial z} = Gv + C\frac{\partial v}{\partial t}} \qquad (4)$$

Differentiating Eq. (2) w.r.t. z we get

$$-\frac{\partial^2 v}{\partial z^2} = R\frac{\partial i}{\partial z} + L\frac{\partial}{\partial z}\left(\frac{\partial i}{\partial t}\right) \qquad (5)$$

Differentiating Eq. (4) w.r.t. t we get

$$-\frac{\partial}{\partial t}\left(\frac{\partial i}{\partial z}\right) = G\frac{\partial v}{\partial t} + C\frac{\partial^2 v}{\partial t^2} \qquad (6)$$

Substituting equations (4) & (6) in Eq. (5) we get we get

$$-\frac{\partial^2 v}{\partial z^2} = R\left(-Gv - C\frac{\partial v}{\partial t}\right)$$

$$+L\left(-G\frac{\partial v}{\partial t} - C\frac{\partial^2 v}{\partial t^2}\right)$$

$$= -RGv - RC\frac{\partial v}{\partial t} - LG\frac{\partial v}{\partial t} - LC\frac{\partial^2 v}{\partial t^2}$$

$$\boxed{\frac{\partial^2 v}{\partial z^2} = RGv + (RC+LG)\frac{\partial v}{\partial t} + LC\frac{\partial^2 v}{\partial t^2}} \quad (6)$$

~~Also~~ ~~By~~ Differentiating Eq. (2) w.r.t. t we get

$$-\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial z}\right) = R\frac{\partial i}{\partial t} + L\frac{\partial^2 i}{\partial t^2} \quad (7)$$

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Differentiating Eq. (4) w.r.t. z we get

$$-\frac{\partial^2 i}{\partial z^2} = G\frac{\partial v}{\partial z} + C\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial t}\right) \quad (8)$$

Substituting Eq.s (2), (7) in Eq. (8) we get

$$-\frac{\partial^2 i}{\partial z^2} = G\left(-Ri - L\frac{\partial i}{\partial t}\right)$$

$$+C\left[-R\frac{\partial i}{\partial t} - L\frac{\partial^2 i}{\partial t^2}\right]$$

$$= -RGi - LG\frac{\partial i}{\partial t} - RC\frac{\partial i}{\partial t} - LC\frac{\partial^2 i}{\partial t^2}$$

$$\therefore \boxed{\frac{\partial^2 i}{\partial z^2} = RGi + (LG+RC)\frac{\partial i}{\partial t} + LC\frac{\partial^2 i}{\partial t^2}} \quad (9)$$

- The voltage and current on the line are the functions of both position z and time t.
- The instantaneous line voltage and current can be expressed as

$$v(z,t) = \mathrm{Re}\, V(z)\, e^{j\omega t} \quad (10)$$

$$i(z,t) = \mathrm{Re}\, I(z)\, e^{j\omega t} \quad (11)$$

where Re stands for "real part of".

- The factors V(z) and I(z) are complex quantities of the sinusoidal functions of position z on the line and are known as phasor

If we substitute $j\omega$ for $\frac{\partial}{\partial t}$ in equations ②, ④, ⑥ and ⑨ and divide each equation by $e^{j\omega t}$, the transmission-line equations in phasor form of the frequency domain become

$$\frac{dV}{dz} = -(R + j\omega L)\, I$$
$$= -Z I \qquad (12)$$

$$\frac{dI}{dz} = -(G + j\omega C)\, V$$
$$= -Y V \qquad (13)$$

$$\frac{d^2V}{dz^2} e^{j\omega t} = RGVe^{j\omega t} + (RC + LG)\, j\omega V e^{j\omega t} + LC\,(j\omega)^2 V e^{j\omega t}$$

$$\frac{d^2V}{dz^2} = RGV + (RC + LG)\, j\omega V + LC(j\omega)^2 V$$

$$= \left[RG + RCj\omega + LGj\omega + LC(j\omega)^2\right] V$$

$$= \left[R(G + j\omega C) + j\omega L(G + j\omega C)\right] V$$

$$= (R + j\omega L)(G + j\omega C)\, V$$

$$= ZYV$$

$$\frac{d^2 V}{dz^2} = \gamma^2 V \qquad (14)$$

$$\frac{d^2 I}{dz^2} = \gamma^2 I \qquad (15)$$

in which the following substitutions have been made:

$$Z = R + j\omega L \quad \text{(ohms per unit length)}$$

$$Y = G + j\omega C \quad \text{(mhos per unit length)} \qquad (16)$$

$$\gamma = \sqrt{ZY} = \alpha + j\beta \quad \text{(propagation constant)}$$

α is the attenuation constant in nepers per unit length

β is the phase constant in radians per unit length

Solutions to Transmission-Line Equations

The one possible solution for Eq. (14) is

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$
$$= V_+ e^{-\alpha z} e^{-j\beta z} + V_- e^{\alpha z} e^{j\beta z} \qquad (17)$$

2.

2. Given

$R = 10\,\Omega, \quad L = 3.5\,mH, \quad C = 0.08\,\mu F$

$G = 0.7\,\mu S, \quad f = 1000\,Hz$

$Z_0 = ? \qquad \gamma = ?$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= \sqrt{\frac{10 + j\,2\pi \times 1000 \times 3.5 \times 10^{-3}}{0.7 \times 10^{-6} + j\,2\pi \times 1000 \times 0.08 \times 10^{-6}}}$$

$$= \sqrt{\frac{10 + j\,21.99}{0.7 \times 10^{-6} + j\,5 \times 10^{-4}}}$$

$$= \sqrt{\frac{24.15 \angle 65.54^\circ}{5 \times 10^{-4} \angle 89.91^\circ}}$$

$$= 219.77 \angle -12.18^\circ$$

$$\gamma = \sqrt{ZY} = \sqrt{(10 + j\,22)(0.7 \times 10^{-6} + j\,5 \times 10^{-4})}$$

$$= \sqrt{(24.15 \angle 65.54^\circ)(5 \times 10^{-4} \angle 89.91^\circ)}$$

$$= 0.109 \angle 77.73^\circ$$

3.

$$Z_L = 60 - j80\ \Omega$$

$$Z_0 = 50\ \Omega$$

$$\bar{z}_L = \frac{Z_L}{Z_0} = \frac{60 - j80}{50}$$

$$= 1.2 - j1.6$$

$$\bar{y}_L = \frac{1}{\bar{z}_L} = 0.3 + j0.4$$

$$d = 0.175\lambda - 0.064\lambda$$

$$= 0.111\lambda$$

$$\cancel{\lambda} = \frac{\cancel{c}}{\cancel{f}} = \frac{\cancel{3\times 10^{8}}}{\cancel{1000}}$$

$$l = 0.344\lambda - 0.25\lambda$$

$$= 0.094\lambda$$

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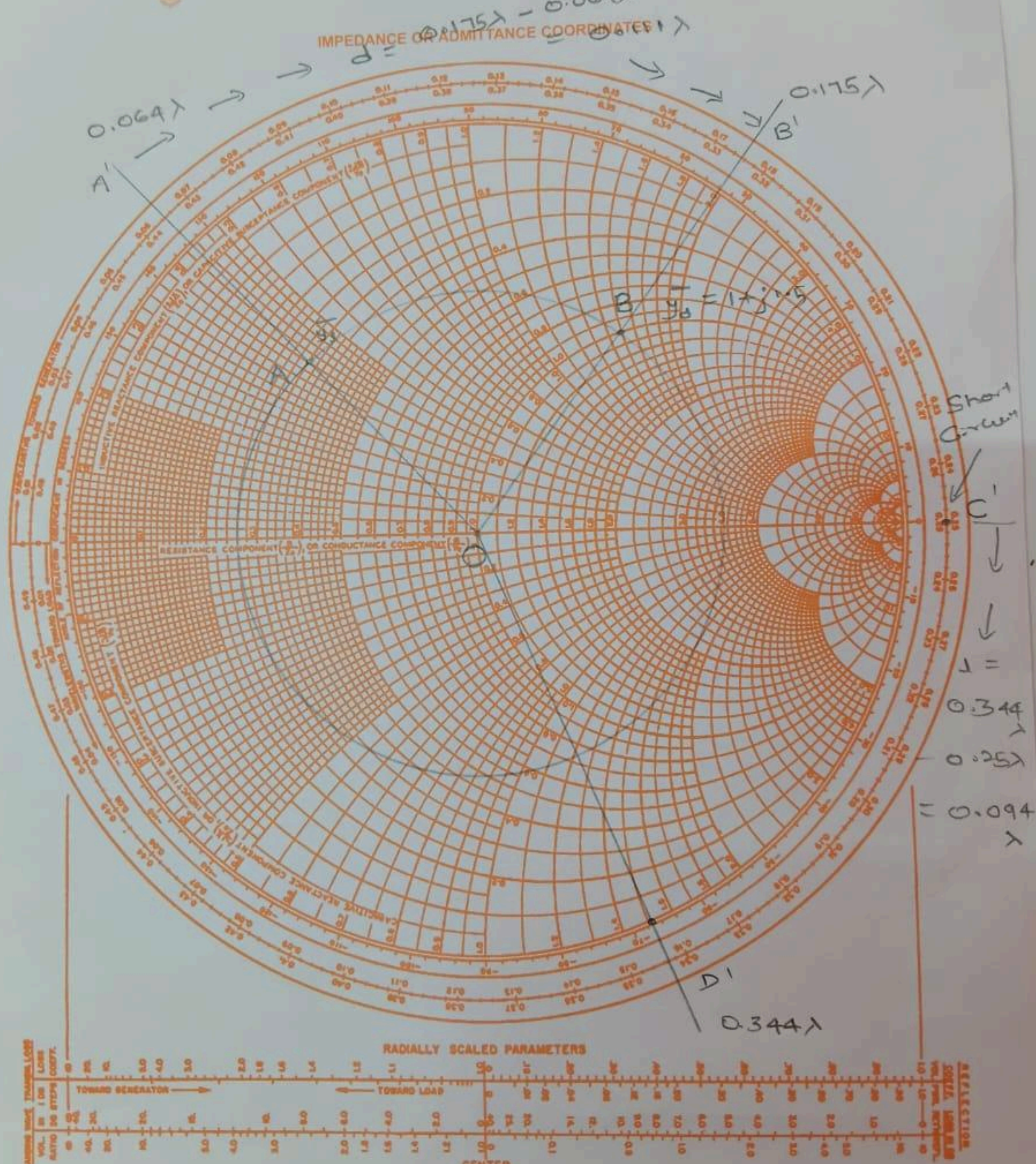

4.

To show that the [S] matrix for a reciprocal network is symmetric.

Assuming the characteristic impedances (Z_{0n}) of all the ports are identical. ($\leftarrow \cancel{Z_{0n} = 1}$)

Also, setting $Z_{0n} = 1$ (for convenience)

The total voltage and current at the nth port can be written as

$$V_n = V_n^+ + V_n^- \qquad (1)$$

$$I_n = V_n^+ - V_n^- \qquad (2)$$

Adding equations (1) and (2) we obtain

$$2V_n^+ = V_n + I_n$$

$$V_n^+ = \frac{1}{2}(V_n + I_n)$$

Adding equations (1) and (2) we obtain

$$2V_n^+ = V_n + I_n$$

$$V_n^+ = \frac{1}{2}(V_n + I_n)$$

Or $\quad [V^+] = \frac{1}{2}([V] + [I]) = \frac{1}{2}([Z][I] + [I])$

$$[V^+] = \frac{1}{2}\{[Z] + [U]\}[I] \qquad (3)$$

$\cancel{[V^-]}$

Subtracting equation (2) from equation (1) we obtain

$$2V_n^- = V_n - I_n$$

$$V_n^- = \frac{1}{2}(V_n - I_n)$$

$$[V^-] = \frac{1}{2}([V] - [I])$$

$$= \frac{1}{2}\{[Z][I] - [I]\}$$

$$= \frac{1}{2}\{[Z] - [U]\}[I] \qquad (4)$$

Dividing equation (4) by equation (3),

$$\frac{[V^-]}{[V^+]} = \{[Z] - [U]\}\{[Z] + [U]\}^{-1}$$

$$[V^-] = \{[Z] - [U]\}\{[Z] + [U]\}^{-1}[V^+] \qquad (5)$$

So that

$$[S] = \{[Z] - [U]\}\{[Z] + [U]\}^{-1} \qquad (6)$$

Taking transpose of (6) gives

$$[S]^t = \{([Z] - [U])^t\}\{([Z] + [U])^t\}^{-1}$$

Now $[U]$ is diagonal, so $[U]^t = [U]$

and if the network is reciprocal, $[Z]$ is symmetric so that

$$[Z]^t = [Z].$$

The above then reduces to

$$[S]^t = ([Z] - [U])([Z] + [U])^{-1} \qquad (7)$$

Comparing equations ⑥ and ⑦, we obtain

$$[S]^t = [S]$$

for reciprocal networks.

To show that the [S] matrix for a lossless network is unitary.

→ If the network is lossless, then no real power can be delivered to the network.

→ Thus, if the characteristic impedances of all the ports are identical and assumed to be unity, the average power delivered to the network is

$$P_{av} = \frac{1}{2} Re\{[V]^t [I]^*\}$$

$$= \frac{1}{2} Re\{([V^+]^t + [V^-]^t)([V^+]^* - [V^-]^*)\}$$

$$P_{av} = \frac{1}{2} Re\{[V^+]^t[V^+]^* - [V^+]^t[V^-]^* + [V^-]^t[V^+]^* - [V^-]^t[V^-]^*\}$$

→ Since the terms $-[V^+]^t[V^-]^* + [V^-]^t[V^+]^*$ are of the form $A - A^*$, and so are purely imaginary.

$$P_{av} = \frac{1}{2} [V^+]^t[V^+]^* - \frac{1}{2} [V^-]^t[V^-]^* = 0 \quad ⑧$$

→ In equation ⑧ the term $\frac{1}{2}[V^+]^t[V^+]^*$ represents the total incident power and the term $\frac{1}{2}[V^-]^t[V^-]^*$ represents the total reflected power.

So for a lossless ~~network~~ network or junction, we have the intuitive result that the incident and reflected powers are equal

$$[V^+]^t [V^+]^* = [V^-]^t [V^-]^* \quad (9)$$

Using $[V^-] = [S][V^+]$ in (9) gives

$$[V^+]^t [V^+]^* = [S]^t [V^+]^t [S]^* [V^+]^*$$

If $[V^+]$ is nonzero,

$$[S]^t [S]^* = [U] \quad (10)$$

or $$[S]^* = \{[S]^t\}^{-1} \quad (11)$$

5.

Magic Tee (E-H Tee)

→ Magic Tee is a combination of E-pla[ne] Tee and H-plane Tee.

→ In this waveguide is cut at both widt[h] and breadth and side arms in the direction of magnetic field and elect[ric] field are inserted, respectively.

→ Ports (1) and (2) form collinear ports, port (3) is called H-arm and port (4) is called E-arm.

S-matrix of Magic Tee

$$[S]_{MT} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad (1)$$

$S_{13} = S_{23}$ (2) (From H-plane tee action)

$S_{14} = -S_{24}$ (3) (From E-plane tee action)

Due to the geometry, power fed at port 3 cannot come out of port 4 and vice versa.

$$S_{34} = S_{43} = 0 \quad (4)$$

Assume ports 3 and 4 are matched

$$S_{33} = S_{44} = 0 \quad (5)$$

Magic Tee is reciprocal. Hence, Symmetric property also holds good.

$$S_{ij} = S_{ji} \quad (6)$$

Using equations (2) to (6) in equation (1), the S-matrix simplifies to the following matrix

$$[S]_{MT} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \quad (7)$$

$$[S][S]^{*} = [U] \quad (8)$$

(from Unitary property)

$$
\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix}
\begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix}
$$

$$
= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \qquad (10)$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \qquad (11)$$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \qquad (12)$$

$$|S_{14}|^2 + |S_{14}|^2 = 1 \qquad (13)$$

$$S_{13} S_{11}^* + S_{13} S_{12}^* = 0 \qquad (14)$$

From (5), $2|S_{13}|^2 = 1$

$$|S_{13}|^2 = \frac{1}{2}$$

$$S_{13} = \frac{1}{\sqrt{2}} \qquad (15)$$

from (13), $2|S_{14}|^2 = 1$

$$|S_{14}|^2 = \frac{1}{2}$$

$$S_{14} = \frac{1}{\sqrt{2}} \quad (16)$$

Using (15) and (16) in (10) and (11),

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0 \quad (17)$$

$$|S_{12}|^2 + |S_{22}|^2 = 0 \quad (18)$$

$$\Rightarrow \quad S_{11} = S_{22} \quad (19)$$

Equation (17) is possible only when $S_{11} = S_{12} = 0$

$$[S]_{MT} = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

Applications of Magic Tee

(i) As an impedance bridge for measurement of impedance at microwave frequencies

(ii) As balanced mixer in superheterodyne receiver

(iii) As E-H tuner for impedance matching

(iv) As power combiner

(v) As a duplexer in radar system

6.

Rotary phase shifter (Precision type phase shifter)

→ Because of its accuracy, the rotary phase shifter is used as a calibration standard in most microwave laboratories.

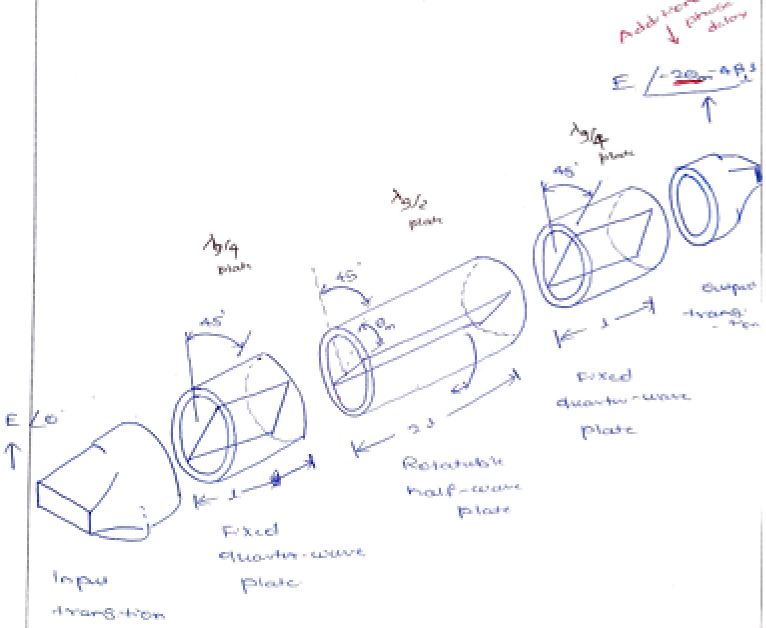

Fig 1 Rotary Phase Shifter

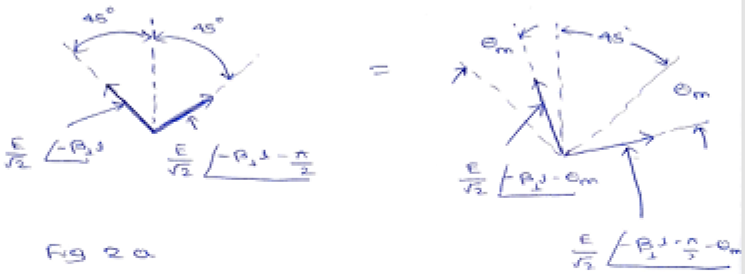

Fig 2a

Vector - phasor components at the input to the rotatable Section

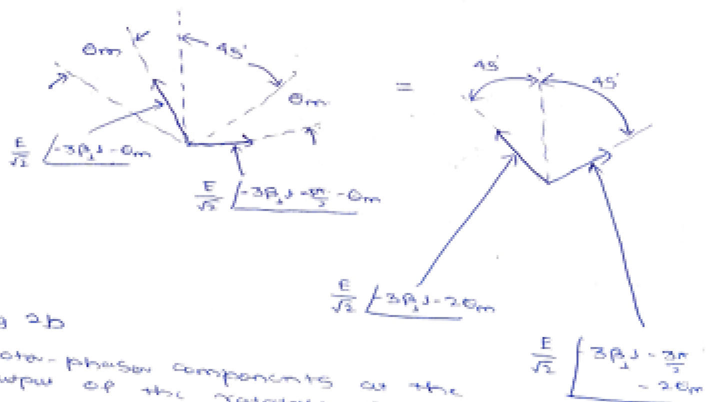

Fig 2b

Vector - phasor components at the output of the rotatable Section

Vector - phasor representation

Working

- → The vector-phasor $E\angle 0$ represents the vertically polarized input wave.
- → It may be decomposed into two components, one perpendicular and one parallel to the dielectric slab of the input quarter-wave plate.
- → The value of each component is $\frac{E}{\sqrt{2}}\angle 0$.
- → When $\theta_m = 0$, all three dielectric slabs are in ~~phase~~ line. In this case, the perpendicular and parallel components arrive at the output phase delayed $4\beta_\perp l$ and $4\beta_\perp l + 2\pi$, respectively, where $\beta_\perp$ represents the phase constant of the circular guide for the perpendicular component and $4l$ is the overall length of the three circular sections.
- → With the two components in phase (2π rad being equivalent to zero phase).

the output is a vertically polarized wave of value $E \angle -4\beta_\perp l$.

→ The orientation of the output transition is such that the wave is delivered to the output rectangular guide without reflection or loss.

→ The following analysis shows that the output wave experiences an additional phase delay of $2\theta_m$ when the half wave plate is rotated by an angle θ_m.

→ Also, the output remains vertically polarized which means that the phase shifter is lossless and reflectionless for any position of the rotatable section.

→ As indicated earlier, the input wave may be decomposed into two components. The effect of the input quarter-wave plate is to delay the perpendicular component $\beta_\perp l$ and the parallel component $\beta_\perp l + \pi/2$, which results in a clockwise circularly polarized wave at the input to the rotatable section.

→ When $\theta_m \neq 0$, it is convenient to replace these components ~~~~ by two ~~~~ other components, one perpendicular to the dielectric slab in the rotatable section and one parallel to it.

→ Both sets of components are shown in ~~Fig~~ ~~part~~ Fig 2 (a).

→ With the length of the half-wave plate equal to $2l$, the perpendicular and parallel components are further delayed $2\beta_1 l$ and $2\beta_1 l + \pi$, respectively.

→ The resultant components are shown in Fig 2 (b).

→ This now represents a counterclockwise circularly polarized wave.

→ As before, these components may be replaced by an equivalent set that are perpendicular and parallel to the output dielectric slab.

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→ Their vector representation is shown on the right side of ~~part c in~~ Fig 2 b.

→ Propagation through the output quarter-wave plate delays these components an additional $\beta_{\parallel} l$ and $\beta_{\perp} l + \pi/2$, respectively.

→ As a result, the value of the output components are

$$\frac{E}{\sqrt{2}} \angle -4\beta_{\perp} l - 2\theta_m \quad \text{and} \quad \frac{E}{\sqrt{2}} \angle -4\beta_{\perp} l - 2\pi - 2\theta_m$$

→ With the waves in phase, vector addition results in a vertically polarized output wave of value $E \angle -4\beta_{\perp} l - 2\theta_m$, which is shown in Figure 1.

→ Thus, the analysis verifies that rotating the center section an angle θ_m causes the output to be phase delayed an additional $2\theta_m$.

→ Also, with the orientation of the output transition as shown, the vertically

7.

Characteristic Impedance

Obtained from wire-over ground model

$\epsilon_{re} = 0.475\ \epsilon_r + 0.67$

- ϵ_{re} is called effective dielectric constant
- ϵ_r is called relative dielectric constant

$Z_0 = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln\left[\frac{5.98h}{0.8w+t}\right]$ for h < 0.8w

$Z_0 = \frac{377}{\sqrt{\epsilon_r}} \frac{h}{w}$ for w >> h

Example 11-1-1: Characteristic Impedance of Microstrip Line

A certain microstrip line has the following parameters:

$$\epsilon_r = 5.23$$
$$h = 7 \text{ mils}$$
$$t = 2.8 \text{ mils}$$
$$w = 10 \text{ mils}$$

Calculate the characteristic impedance Z_0 of the line.

Solution

$$Z_0 = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln\left[\frac{5.98h}{0.8w + t}\right]$$
$$= \frac{87}{\sqrt{5.23 + 1.41}} \ln\left[\frac{5.98 \times 7}{0.8 \times 10 + 2.8}\right]$$
$$= 45.78\ \Omega$$

Ohmic Losses

- Comes into picture due to finite conductivity of microstrip conductor

- Attenuation constant due to ohmic loss of a wide microstrip line (w>h) is given by

$$\alpha_c = \frac{8.686}{Z_0 w}\sqrt{\frac{\pi f \mu}{\sigma}} \quad \text{dB/cm}$$

Radiation Losses

- At microstrip frequencies, the microstrip line acts as an resulting in radiation losses.
- The ratio of radiated power to the total dissipated pow microstrip is given by

$$\frac{P_{rad}}{P_t} = \frac{R_r}{Z_0}$$

- R_r = radiation resistance of the microstrip

$$R_r = 240\pi^2 \left(\frac{h}{\lambda_0}\right)^2 F(\epsilon_{re})$$

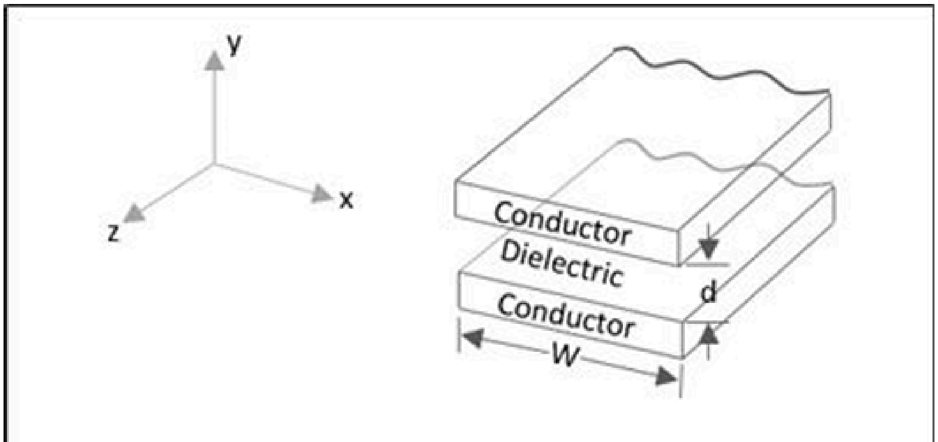
Where

$$\lambda_0 = \text{Free} - \text{space wavelength}$$

$$F(\epsilon_{re}) = \text{radiation factor given by}$$

$$F(\epsilon_{re}) = \frac{\epsilon_{re} + 1}{\epsilon_{re}} - \frac{\epsilon_{re} - 1}{2(\epsilon_{re})^{3/2}} \, ln \frac{\sqrt{\epsilon_r} + 1}{\sqrt{\epsilon_r - 1}}$$

Parallel Strip Lines



Parallel Strip Line

If w >> d, then the fringing capacitance becomes negligibly small.
The inductance along the two conducting strips is given by

$L = \frac{\mu_c d}{w}$ henry/m

Where μ_c = permeability of the conductor

The capacitance between the two parallel conducting strips is given by

$C = \frac{\epsilon_d w}{d}$ farad/m

The series resistance of both strips is given by

$$R = \frac{2R_s}{w} = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} \quad \frac{\Omega}{\mathrm{m}}$$

Where R_s = surface resistance

$\sigma_c = \text{conductivity of the strips in mhos/m}$

The shunt conductance of the parallel strip line is

$$G = \frac{\sigma_c w}{d} \quad \text{mhos/m}$$

Where σ_c is conductivity of the dielectric material between the two strips

Characteristic Impedance Z_0

$$Z_0 = \frac{377}{\sqrt{\epsilon_{rd}}}\left(\frac{d}{w}\right) \quad \Omega$$

The phase velocity of the TEM wave propagating through the parallel strip line is given by

$$v_p = \frac{c}{\sqrt{\epsilon_{rd}}} \quad \text{m/s}$$