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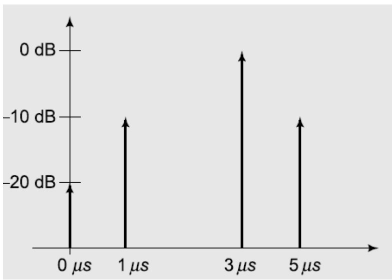
Internal Assessment Test 1 – September 2025

Subject:	Wireless Communication Systems							Code:	BEC703
Date:		Duration:	90 mins	Max Marks:	50	Sem:	7	Branch:	ECE

Answer **Any FIVE FULL** Questions. Each Full Question carries 10 marks.

		Marks	OBE CO	OBE RBT
1.	a. Explain the modeling of a wireless channel, highlighting the key factors like attenuation factor and path delay.	[6]	CO1	L3
	b. Consider a wireless signal with a carrier frequency of f_c = 850 MHz, which is transmitted over a wireless channel that results in L = 4 multipath components at delays of 201, 513, 819, 1223 ns and corresponding to received signal amplitudes of 1, 0.6, 0.3, 0.2, respectively. Derive the expression for the received baseband signal y_b (t) if the transmitted baseband signal is s_b (t).	[4]		
2.	Explain in detail the significance of Coherence Bandwidth in wireless communication. Explore the relation between ISI and Coherence Bandwidth.	[10]	CO1	L2
3.	a. Explain briefly the concept of RMS Delay Spread.	[6]	CO1	L3
	b. Consider the multipath power profile of a wireless channel shown in the Figure below, comprising L = 4 multipath components. Compute the and RMS delay spread σ_T^{RMS} for this wireless channel.	[4]		



		Marks	OBE CO	OBE RBT
4.	a. Explain in brief the Doppler Fading in Wireless Systems.	[6]	CO1	L3
	b. Consider a vehicle moving at 60 miles per hour at an angle of $\theta = 30^0$ with the line joining the base station. Compute the Doppler shift of the received signal at a carrier frequency of f_c = 1850 MHz.	[4]		
5.	Detail the following: a. Generation of Spreading Codes based on Pseudo-Noise Sequences using a Linear Feedback Shift Register. b. Balance property and Run-Length property of PN Sequences.	[10]	CO2	L3
6.	Explain in detail the Advantages of CDMA.	[10]	CO2	L2
7.	With the help of a neat block diagram, explain the implementation of the OFDM transmitter and receiver using IFFT/FFT.	[10]	CO2	L2
8.	Detail the Correlation Properties of Random CDMA Spreading Sequences	[10]	CO2	L2

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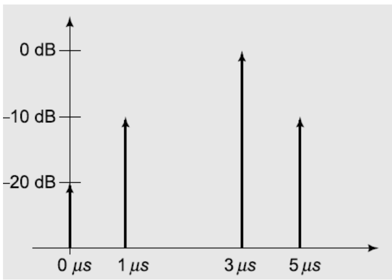
3.	a. Explain briefly the concept of RMS Delay Spread. b. Consider the multipath power profile of a wireless channel shown in the Figure below, comprising L = 4 multipath components. Compute the and RMS delay spread σ_T^{RMS} for this wireless channel.  	[6] [4]	CO1	L3
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SOLUTION:

Q1 :	a. Explain the modeling of a wireless channel, highlighting the key factors like attenuation factor and path delay.
	Ans: a. Explain the modeling of a wireless channel, highlighting the key factors like attenuation factor and path delay.

<u>Modelling of Wireless Systems</u> :→

Let $s_b(t)$ — Base band signal

$s(t)$ — Transmitted signal. (Passband signal)

So the $s(t)$ can be expressed as:

$$s(t) = Re\left\{ s_b(t) \cdot e^{j2\pi f_c t} \right\}$$

Where f_c is the carrier frequency employed for transmission.

<u>Assumptions:</u>

(1) The wireless channel is time invariant.

(2) let's consider a channel with 'L' multipath components

(3) ~~Three~~ Two properties of Channel:

(a) It delays the signal because of the propagation path distance

(b) It attenuates the signal due to the scattering effect.

(4) Let the i^{th} channel (path) be denoted by the quantities α_i & τ_i respectively such that

We know for a Linear Time Invariant System,
the impulse response of an LTI System that attenuates a signal by α_i & delays it by τ_i is given by:

$$h_i(\tau) = \alpha_i\,\delta(\tau-\tau_i) \qquad a_i\,\delta(\tau-\tau_i)$$

where δ is the impulse input.
& $h_i(\tau)$ is the impulse response.

* Wireless channel represents a linear-input-output system hence the signal observed at the receiver is the sum of the different multipath signal impinging on the receive antenna

* Hence a typical Channel Impulse Response (CIR) of a multipath scattering based wireless channel is given by the sum of the above impulse responses corresponding to the individual model.

$$\Longrightarrow \boxed{h(\tau) = \sum_{i=0}^{L-1} \alpha_i\,\delta(\tau-\tau_i)} \longrightarrow \text{I}$$

Eq-I represents the tapped delay-line model because of the nature of the arrival of several progressively delayed components of the signal, where the wireless Channel model consists of 'L' propagation paths arising from the several reflection and scattering multipath Non-Line of Sight (NLOS) components. One of the multipath component can a direct Line of Sight (LOS) component

We also know that in time domain for a LTI system, the o/p of a system is given by the convolution of the input signal and the impulse response of the system

Hence

$$\Longrightarrow y(t) = s(t) * h(t)$$

$$\boxed{y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot s(t-\tau)\, d\tau.} \longrightarrow \text{II}$$

Letting eq-I in II we get

$$\cancel{y(t) \left[\sum_{i=0}^{L-1} \alpha_i \delta(\tau-\tau_i)\right] *} h(t)$$

$$y(t) = s(t) * \left[\sum_{i=0}^{L-1} \alpha_i \delta(\tau-\tau_i)\right]$$

$$\Rightarrow y(t) = \int_{-\infty}^{\infty} \left[\sum_{i=0}^{(L-1)} \alpha_i \delta(\tau-\tau_i)\right] \cdot s(t-\tau)\, d\tau.$$

$$= \sum_{i=0}^{L-1} \left[\int_{-\infty}^{\infty} \alpha_i \delta(\tau-\tau_i) \cdot s(t-\tau)\, d\tau\right]$$

$$y(t) = \sum_{i=0}^{L-1} \alpha_i \left[\int_{-\infty}^{\infty} \delta(\tau-\tau_i) \cdot s(t-\tau)\, d\tau\right]$$

$$\Rightarrow y(t) = \sum_{i=0}^{L-1} \alpha_i \; s(t-\tau_i) \longrightarrow \text{III}$$

Now we can express $s(t-\tau_i)$ in terms of $s_b(t)$ & $f_c(t)$

$$\Rightarrow y(t) = \sum_{i=0}^{L-1} \alpha_i \; s_b(t-\tau_i) \cdot e^{j2\pi f_c(t-\tau)}$$

$$y(t) = Re\left\{\sum_{i=0}^{L-1} \alpha_i \; s_b(t-\tau_i) \cdot e^{j2\pi f_c(t-\tau_i)}\right\}$$

$$= Re\left\{\sum_{i=0}^{L-1} \alpha_i \; s_b(t-\tau_i) \cdot e^{-j2\pi f_c \tau_i} \cdot e^{j2\pi f_c t}\right\}$$

$$= Re\left\{\left(\sum_{i=0}^{L-1} \alpha_i \; s_b(t-\tau_i) \cdot e^{-j2\pi f_c \tau_i}\right) e^{j2\pi f_c t}\right\}$$

$$= Re\left\{\underbrace{\left(\sum_{i=0}^{L-1} \alpha_i \, e^{-j2\pi f_c \tau_i} \cdot s_b(t-\tau_i)\right)}_{y_b(t)} e^{j2\pi f_c t}\right\}$$

Hence the complex base band signal equivalent of $y(t)$ can be expressed as:

$$y_b(t) = \sum_{i=0}^{L-1} \alpha_i \, e^{-j2\pi f_c \tau_i} \cdot s_b(t-\tau_i) \Longrightarrow \text{IV}$$

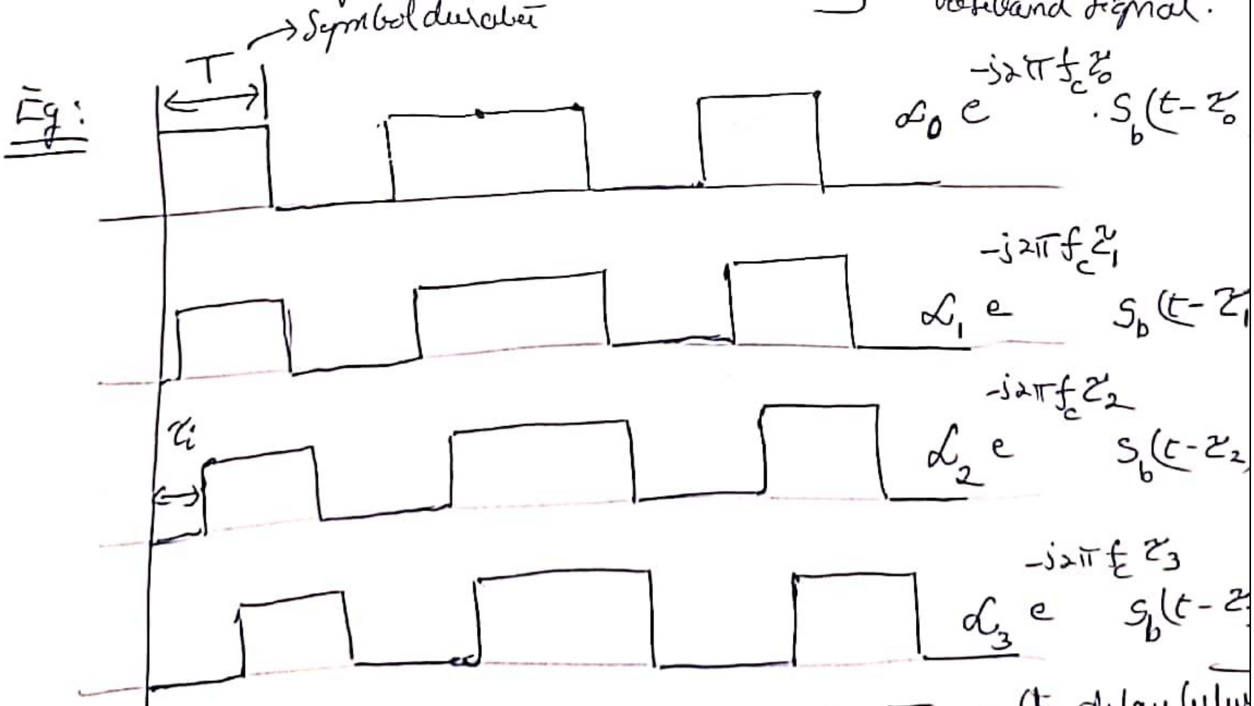
Hence it is observed from (IV) that the received baseband signal consists of multiple delayed copies of $s_b(t-\tau_i)$ of the transmitted signal $s_b(t)$ &

each i^{th} signal copy arising from the i^{th} multipath component is associated with the following three parameters:

a) Attenuation factor α_i
b) The delay path τ_i
c) The phase factor $e^{-j2\pi f_c \tau_i}$

} are the important parameters in the received complex baseband signal.

Eg:



$$\boxed{T_m < \tfrac{1}{8} T}.$$

where T_m is the delay betw the 1st & last arriving copies of the signal which is also called the <u>delay spread</u>

Typical BPSK information signal $s_b(t)$ received from i^{th} multipath.

	b. Consider a wireless signal with a carrier frequency of f_c = 850 MHz, which is transmitted over a wireless channel that results in L = 4 multipath components at delays of 201, 513, 819, 1223 ns and corresponding to received signal amplitudes of 1, 0.6, 0.3, 0.2, respectively. Derive the expression for the received baseband signal y_b (t) if the transmitted baseband signal is s_b (t).
	Ans: (b):

Sol:

We know that the expression for the received baseband signal for a wireless medium with 'L' multipath component is

$$y_b(t) = \sum_{i=0}^{L-1} \alpha_i e^{-j2\pi f_c \tau_i} \cdot s_b(t-\tau_i) \longrightarrow (A)$$

where α_i & τ_i are the attenuation & delay introduced due to the i^{th} path.

We have:

$f_c = 850$ MHz.

$L = 4$. $[\tau_0 = 201$ ms, $\tau_1 = 513$ ms, $\tau_2 = 819$ ms, $\tau_3 = 1223$ msec$]$

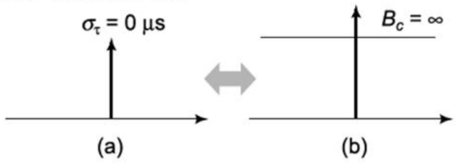
$[\alpha_0 = 1, \alpha_1 = 0.6, \alpha_2 = 0.3, \alpha_3 = 0.2]$

Hence for path 0,

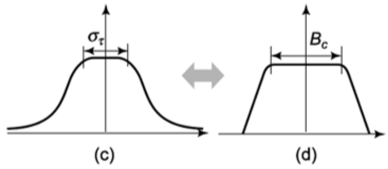
$$\alpha_0 e^{-j2\pi f_c \tau_0} = 1 \times e^{-j2\pi \times 850 \times 10^6 \times 201 \times 10^{-9}}$$

$$= e^{-j2\pi \times 850 \times 10^{-3}}$$

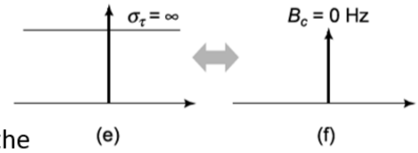
$$= \underline{0.587 + 0.809j}.$$

	Sol: We know that the expression for the received baseband signal for a wireless medium with 'L' multipath components is: $$y_b(t) = \sum_{i=0}^{L-1} \alpha_i e^{-j2\pi f_c \tau_i} \cdot s_b(t-\tau_i) \longrightarrow (A)$$ where α_i & τ_i are the attenuation & delay introduced due to the i^{th} path. We have: $f_c = 850$ MHz. $L = 4$. $[\tau_0 = 201\,ms,\ \tau_1 = 513\,ms,\ \tau_2 = 819\,ms,\ \tau_3 = 1223\,msec]$ $[\alpha_0 = 1,\ \alpha_1 = 0.6,\ \alpha_2 = 0.3,\ \alpha_3 = 0.2]$ Hence for path 0, $$\alpha_0 e^{-j2\pi f_c \tau_0} = 1 \times e^{-j2\pi \times 850\times10^{6} \times 201\times10^{-9}}$$ $$= e^{-j2\pi\times 850\times 10^{-3}}$$ $$= 0.584 + 0.809j.$$
Q-2:	Explain in detail the significance of Coherence Bandwidth in wireless communication. Explore the relation between ISI and Coherence Bandwidth.
Sol:	**Coherence Bandwidth in Wireless Communications** 1. let us define the frequency response H (f) of the wireless channel as: $$H(f) = \int_0^\infty h(\tau)\, e^{-j2\pi f\tau}\, d\tau$$  2. Let us begin by considering a simple case corresponding to $\sigma_\tau = 0$, 3. In this scenario, since the delay spread is zero, the wireless channel comprises a single propagation path. 4. Hence, the delay profile h (τ) is given as: $h(\tau) = \delta(\tau)$. 5. The corresponding frequency response H (f) is given as: $$H(f) = \int_0^\infty \delta(\tau)\, e^{-j2\pi f\tau}\, d\tau = 1$$

- As the delay spread σ_τ increases in Figure(c), the time spread of this response increases, leading to a decrease in the bandwidth of the response H (f) as shown in Figure (d).



- Finally, as the time spread of the response becomes ∞ as shown in Figure (e), the channel filter becomes an impulse δ (f) as shown in Figure (f) and the bandwidth of the channel filter reduces to 0.

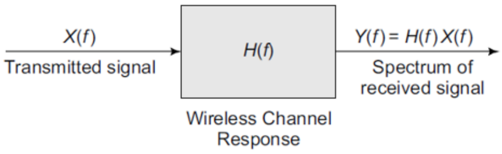


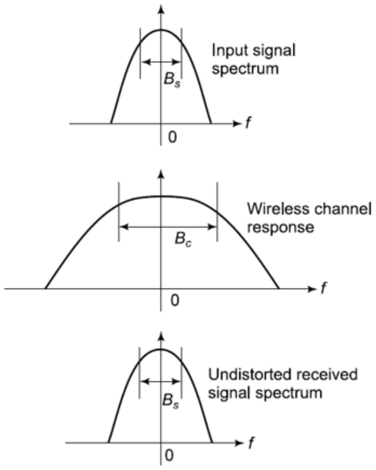
- The coherence bandwidth Bc is then defined as the bandwidth of the response H (f), i.e., the frequency band over which the response H (f) is flat as shown in Figure (d)

Significance of this quantity Bc

Consider any signal x (t) transmitted over the wireless channel, with corresponding Fourier transform X (f).

$$Y(f) = H(f)X(f)$$





- If the bandwidth Bs of the signal x (t) is less than Bc, then X (f) spans the flat part of the channel response H (f).
- Hence, the output : Y (f) = H (f) X (f)

  is simply a scaled version of X (f) corresponding to the magnitude of the flat part.
- Thus, the input signal spectrum X (f) is undistorted at the output.
- Such a wireless channel is termed a **flat-fading channel.**

Fig: Signal bandwidth Bs less than coherence bandwidth Bc implying no distortion

Case-2:

- The signal bandwidth Bs is greater than the coherence bandwidth Bc. [Bs > Bc]
- In this scenario, different parts of the signal spectrum X (f) experience different attenuations,

  i.e., the attenuation is frequency-selective.
- Thus, the output spectrum Y (f) is a distorted version of the input spectrum X (f).
- Such a wireless channel is termed a frequency-selective channel due to the frequency-dependent nature of the attenuation of the signal.
- Hence we can summarize the above as:

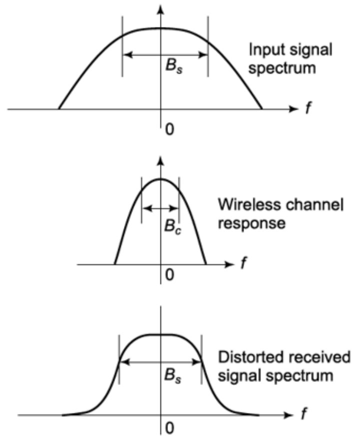


Fig: Signal bandwidth B_s greater than coherence bandwidth B_c leading to distortion in spectrum of received signal

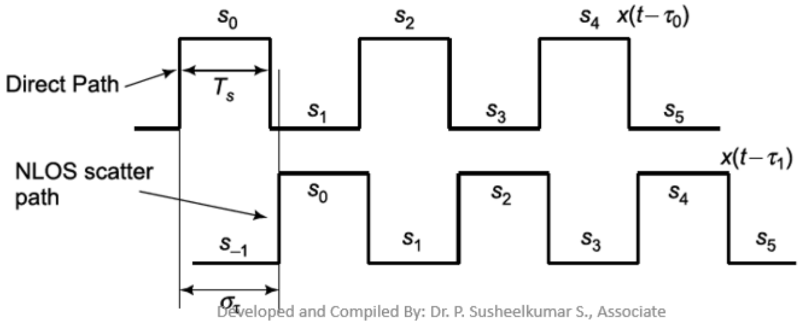
$$B_s \leq B_c \quad \Rightarrow \quad \text{No } \textit{distortion} \text{ in received signal, i.e., } \textit{flat} \text{ fading}$$

$$B_s \geq B_c \quad \Rightarrow \quad \textit{Distortion} \text{ in received signal, i.e., } \textit{frequency-selective} \text{ fading}$$

Relation Between ISI and Coherence Bandwidth

- Consider a Pulse Amplitude Modulated (PAM) signal **x (t)** of symbol time **Ts** transmitted by the base station.

- Let us also consider the presence of a **scatter component** at a delay of **τ1 = Td** in addition to the **direct line-of-sight component** with a delay **τ0 = 0**.



- The net signal sensed by the receiver is the sum of the direct and scatter components, i.e., x (t) and x (t − τ0).

- From Figure we can say that if the delay spread $\sigma_\tau = \tau_1 - \tau_0$ is comparable to the symbol time Ts, and
  - When these two signals are superposed at the receiver, the symbol s0 from x (t) adds to a different symbol from x (t − τ0).

- For instance, in the figure, s0 adds to s−1, i.e., the previous symbol.

- As the delay spread increases, and the number of interfering paths correspondingly increases, the severity of ISI increases,
  - with several symbols superposing at the receiver. This can be clearly seen in Figure below:

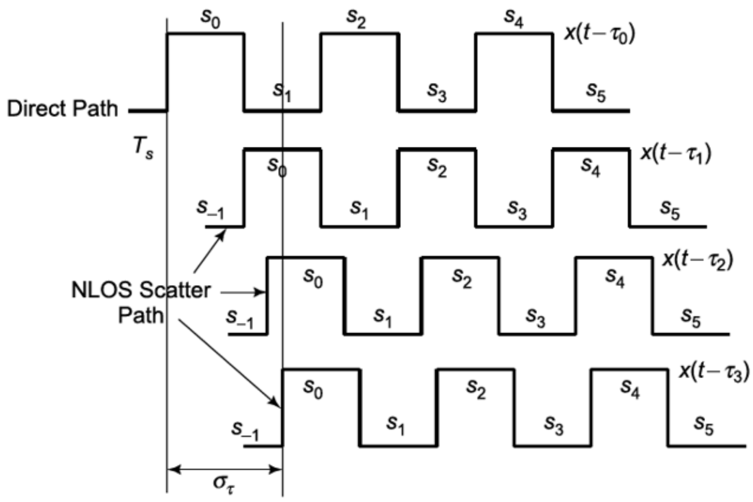
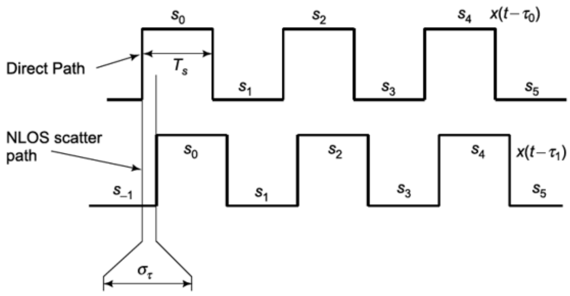


Fig: Severe ISI caused by multiple scatter components

Criterion for occurrence of ISI



- When the symbol time Ts is much larger than the delay spread Td, there is almost no ISI.
- However, as the delay spread Td becomes comparable to Ts, it leads to ISI.
- Thus one can empirically state the criterion for ISI as:

$$T_d \geq \frac{1}{2} T_s$$

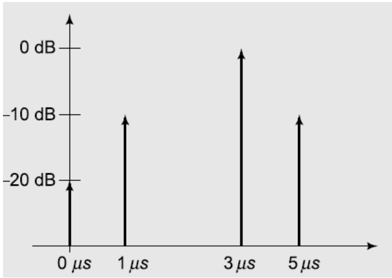
- Thus one can empirically state the criterion for ISI as: $T_d \geq \frac{1}{2} T_s$
- The Symbol Time Ts is related to the signal Bandwidth and is expressed as: $T_s = \frac{1}{B_s}$
- Also the max delay spread is related as: $B_c = \frac{1}{2\tau_d}$
- Hence we have $\tau_d = \frac{1}{2B_c}$
- Therefore, $\frac{1}{2B_c} \geq \frac{1}{2B_s}$ $\rightarrow$ $\boxed{B_s \geq B_c}$; which is the same

  condition for frequency selective signal distortion
- Hence frequency-selective distortion and inter-symbol interference are essentially both sides of the same coin.

- In the time domain, if the delay spread is much larger compared to the symbol time, it results in inter-symbol interference.
- Correspondingly, in the frequency domain, this implies that the bandwidth of the signal is much larger than the coherence bandwidth of the channel.
- Thus, when one tries to push a signal of much higher bandwidth through a channel filter, with a much smaller bandwidth, we get frequency-selective distortion.
- Thus, to correct for the inter-symbol interference at the receiver, one needs to intuitively multiply by the inverse of the channel response filter, i.e., $\frac{1}{H(f)}$, to convert the frequency selective channel into a system with a net flat-fading response.
- This process, termed **equalization** is the ***different frequency components are being equalized to a common flat-level***.

3. a. Explain briefly the concept of RMS Delay Spread.

b. Consider the multipath power profile of a wireless channel shown in the Figure below, comprising L = 4 multipath components. Compute the and RMS delay spread σ_T^{RMS} for this wireless channel.



RMS Delay Spread: → σ_{τ}^{RMS}

1) The paths which arrive later are significantly lower in power due to larger propagation distances & weaker reflections.

2) This leads to a larger value of the maximum delay spread σ_T^{max}, even though several of the later paths comprise weak scatter ~~fo~~ components with negligible power.

3) Hence the maximum delay spread metric is not a reliable indicator of the true power spread of the arriving multipath signal components.

4) Hence RMS delay spread is a more realistic indicator of the spread of the signal power in the arriving components, since it weighs the delays in proportion to the signal power in the multipath components. Therefore it is not susceptible to distortion in scenarios with a large number of trailing weak components in comparison to maximum delay spread.

We know that

$$h(t) = \sum_{i=0}^{L-1} a_i \cdot \delta(t - \tau_i)$$

where each $\delta(t-\tau_i)$ corresponds to delaying the signal by τ_i

& a_i is the attenuation associated with each i^{th} path

Illy the power profile of the multipath channel is expressed as:

$$\phi(t) = \sum_{i=0}^{L-1} |a_i|^2 \delta(t-\tau_i)$$

$$\phi(t) = \sum_{i=0}^{L-1} g_i \, \delta(t-\tau_i) \quad , \text{ where}$$

where $g_i = |a_i|^2$ is the power gain of the i^{th} path.

Here we define a new quantity b_i as:

$$b_i = \frac{g_i}{g_0 + g_1 + \cdots + g_{L-1}}.$$

where $(g_0 + g_1 + \cdots + g_{L-1})$ is the total power in the multipath power profile.

1) Hence here 'b_i' represents the fraction of power in the i^{th} multipath component. ($b_i > 0$)

& $b_0 + b_1 + \cdots + b_{L-1} = 1$.

2) Hence we define average delay $\bar{\tau}$ as the mean of the above power distribution

$$\Rightarrow \quad \bar{\tau} = b_0 \tau_0 + b_1 \tau_1 + b_2 \tau_2 + \cdots + b_{L-1} \tau_{L-1}$$

$$\bar{\tau} = \sum_{i=0}^{L-1} b_i \tau_i$$

$$= \sum_{i=0}^{L-1} \frac{g_i}{\sum_{j=0}^{L-1} g_j} \tau_i$$

$$\bar{\tau} = \frac{\sum_{i=0}^{L-1} g_i \tau_i}{\sum_{j=0}^{L-1} g_j}$$

3) Hence we can now calculate RMS delay spread

σ_τ^{RMS} from the standard deviation of the power distribution,

given as

$$\left(\sigma_\tau^{RMS}\right)^2 = b_0(\tau_0 - \bar{\tau})^2 + b_1(\tau_1 - \bar{\tau})^2 + \cdots + b_{L-1}(\tau_{L-1} - \bar{\tau})^2$$

$$= \sum_{i=0}^{L-1} b_i (\tau_i - \bar{\tau})^2$$

$$= \sum_{i=0}^{L-1} \frac{g_i}{\left(\sum_{j=0}^{L-1} g_j\right)} (\tau_i - \bar{\tau})^2$$

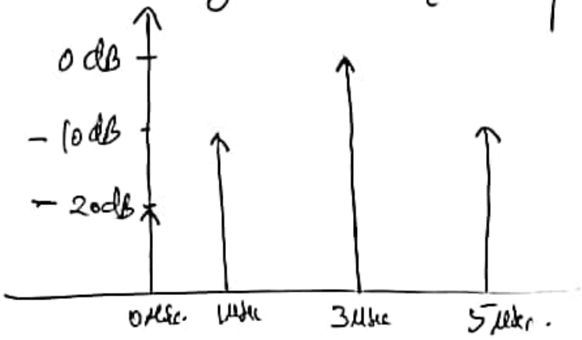
$$\Rightarrow \left(\sigma_\tau^{RMS}\right)^2 = \frac{\sum_{i=0}^{L-1} g_i (\tau_i - \bar{\tau})^2}{\sum_{j=0}^{L-1} g_j}$$

$$\Rightarrow \sigma_\tau^{RMS} = \sqrt{\frac{\sum_{i=0}^{L-1} g_i (\tau_i - \bar{\tau})^2}{\sum_{j=0}^{L-1} g_j}}$$

$$\sigma_\tau^{RMS} = \sqrt{\frac{\sum_{i=0}^{L-1} |a_i|^2 (\tau_i - \bar{\tau})^2}{\sum_{j=0}^{L-1} |a_j|^2}}$$

4) Thus we observe that the RMS metric to characterise the delay spread is not sensitive to spurious multipath components of weak power since it weighs each delay in proportion to its power, thereby automatically suppressing the contribution of weaker paths

<u>Prob:</u> Consider the multipath power profile of a wireless channel shown below, comprising $L = 4$ multipath components. Compute the RMS delay spread σ_τ^{RMS} for this wireless channel.



<u>Sol:</u>

1) Path $L_0 \rightarrow \tau_0 = \mu\text{sec}$, $g_0(dB) = -20$ dB

$\therefore \quad 10 \cdot \log_{10} g_0 = -20 \Rightarrow \log_{10} g_0 = -2$

$\Rightarrow g_0 = 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.010 = \underline{0.01}$

$\therefore a_0 = \sqrt{g_0} = \underline{0.1}$

2) Path $L_1 \rightarrow 1\,\mu\text{sec}\,(\tau_1)$, $g_1(dB) = -10$ dB

$\Rightarrow g_1 = 0.1$ & $a_1 = 0.3162$

3) Path $L_2 \rightarrow 3\,\mu\text{sec}\,(\tau_2)$, $g_2(dB) = 0$ dB.

$\Rightarrow g_2 = 1$ & $a_2 = 1$

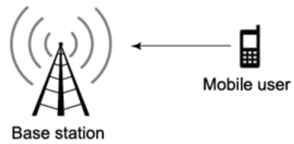
4) Path $L_3 \rightarrow 5\,\mu\text{sec}\,(\tau_3)$; $g_3(dB) = -10$

$\Rightarrow g_3 = 0.1$, $a_3 = 0.3162$

	Hence $\bar{\tau}$ (mean delay) for the channel: $$\bar{\tau} = \frac{\sum_{i=1}^{L-1} g_i \tilde{\tau}_i}{\sum_{i=0}^{L-1} g_i} = \frac{0.01 \times 0 + 0.1 \times 1 + 1 \times 3 + 0.1 \times 5}{0.01 + 0.1 + 1 + 0.1}$$ $$= 2.9752 \ \mu sec.$$ Also. $$\sigma_{\tau}^{RMS} = \sqrt{\frac{\sum_{i=0}^{L-1} g_i (\tau_i - \bar{\tau})^2}{\sum_{i=0}^{L-1} g_i}}$$ $$= \sqrt{\frac{0.01 \times (0 - 2.9752)^2 + 0.1 \times (1 - 2.9752)^2 + 1 \times (3 - 2.9752)^2 + 0.1 \times (5 - 2.9752)^2}{(0.01 + 0.1 + 1 + 0.1)}}$$ $$= \sqrt{\frac{0.088518 + .3901415 + 0.00061504 + 0.4099815}{1.21}}$$ $$= \sqrt{0.734922} = 0.8573 \ \mu sec$$
4.	a. Explain in brief the Doppler Fading in Wireless Systems. b. Consider a vehicle moving at 60 miles per hour at an angle of $\theta = 30^0$ with the line joining the base station. Compute the Doppler shift of the received signal at a carrier frequency of $f_c = 1850$ MHz.

Doppler Fading in Wireless Systems

- The Doppler shift associated with an electromagnetic wave is defined as the perceived change in the frequency of the wave due to relative motion between the transmitter and receiver.



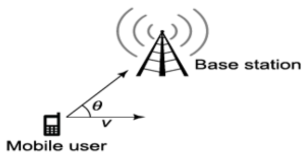
- The perceived frequency is higher than the true frequency if the transmitter is moving towards the receiver and lower otherwise.

- Doppler fading is inherent in wireless communications due to the inherent nature of mobile transceivers, which enables mobility in wireless systems, leading to relative
- motion between the transmitter and the receiver.

- This is different compared to the conventional wired communications, where the inherent nature of the fixed radio-access medium does not allow for mobility.

Doppler Shift Computation

- Suppose the mobile station is moving with a velocity v at an angle θ with the line joining the mobile and base station.



- Let the carrier frequency be fc.

- The Doppler shift for this scenario is given as:

$$f_d = \left(\frac{v}{c}\cos\theta\right) f_c$$

where $c = 3 \times 10^8$m/s is the velocity of light, i.e., velocity of an electromagnetic wave in free space.

- It can be clearly seen that the Doppler shift increases with the velocity v.

- It depends critically on the angle θ between the direction of motion and the line joining the transmitter and receiver.

- For instance, the Doppler shift is maximum when θ = 0, π, i.e., when the relative motion is along the line joining the transmitter and receiver.

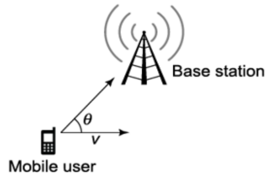
- However, when θ = π/2 , i.e., the motion is perpendicular to the receive direction, the Doppler shift is zero.

- Also, the Doppler shift is positive in the sense that the perceived frequency is higher if:
    - $0 \le \theta \le \pi/2$, in which case $\cos\theta > 0$.

- On the other hand, it is negative, leading to a lower perceived frequency than the transmit frequency if: $\pi/2 \le \theta \le \pi$.

Problem:

Consider a vehicle moving at 60 miles per hour at an angle of θ = 30◦ with the line joining the base station. Compute the Doppler shift of the received signal at a carrier frequency of fc = 1850 MHz.



Solution:

$$60\,\text{mph} = 60 \times 1.61\,\text{kmph}$$
$$= 60 \times 1.61 \times \frac{5}{18}\,\text{m/s}$$
$$= 26.8\,\text{m/s}$$

Doppler shift fd is expressed as: $$f_d = \left(\frac{v}{c}\cos\theta\right) f_c$$

$$f_d = \frac{26.8}{3 \times 10^8} \times \cos(30°) \times 1850 \times 10^6$$
$$= 143\,\text{Hz}$$

- Thus, the Doppler shift is **fd = 143 Hz**.
- Since the mobile user is moving towards the base station, the Doppler shift is positive, i.e., the perceived frequency fr is higher compared to the carrier frequency fc and
  - Hence **fr = fc + fd = 1850 MHz + 143 kHz = 1993 MHz**

5. Detail the following:

a. Generation of Spreading Codes based on Pseudo-Noise Sequences using a Linear Feedback Shift Register.

b. Balance property and Run-Length property of PN Sequences.

<u>Spreading Codes based on Pseudo-Noise (PN) Sequences:</u>

- Consider the code c_2 = [1, −1, 1, −1].
- Observe that the code looks like a random sequence of +1, −1, or a pseudo-noise (PN) sequence.
- This is so termed since it only resembles a noise sequence, but is not actually a noise sequence.
- One method to generate such long spreading codes based on PN sequences for a significantly large N is through the employment of a *<u>Linear Feedback Shift Register</u>* (LFSR).

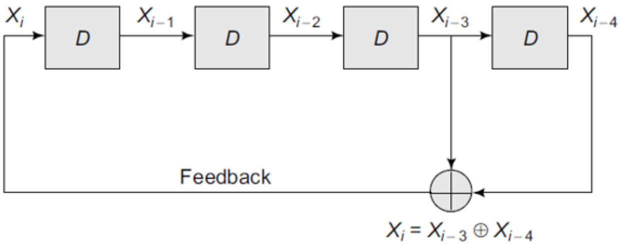


Fig: Linear feedback shift register

- Shift register architecture where the element D represents delays.
- Thus, the digital circuit therein contains D = 4 delay elements or shift registers.
- The input on the left is denoted by X_i
- And the outputs of the different delays are : X_{i-1}, X_{i-2}, X_{i-3}, X_{i-4}
- Let X_{i-4} also denote the final output of the system.
- Thus, the governing equation of the circuit: $X_i = X_{i-3} \oplus X_{i-4}$

- Thus, the governing equation of the circuit: $X_i = X_{i-3} \oplus X_{i-4}$

- The above equation is a Linear Equation
- Since it implements a linear relation, with feedback and uses delay elements or shift registers,
  - The Circuit is also termed a ***Linear Feedback Shift Register (LFSR)*** architecture

- The next input (Current State of the system), i.e., X_i depends on X_{i-1}, X_{i-2}, X_{i-3}, X_{i-4}, this can also be thought of as the current state of the system.
- Consider initializing the system in the state $X_{i-1}=1$, $X_{i-2}=1$, $X_{i-3}=1$, $X_{i-4}=1$.
- Thus, we have the corresponding Xi given

$$X_i = X_{i-3} \oplus X_{i-4} = 1 \oplus 1 = 0$$

- Consider initializing the system in the state $X_{i-1}=1$, $X_{i-2}=1$, $X_{i-3}=1$, $X_{i-4}=1$.
- Xi = 0
- At the next instant, X_i shifts to X_{i-1} and subsequent shifts to yield:
  $X_{i-1}=0$, $X_{i-2}=1$, $X_{i-3}=1$, $X_{i-4}=1$.
- Hence the states change from:

1111 ➡ 0111 ➡ 0011 ➡ 0001 ➡ 1000 ➡ 0100 ➡ 0010 ➡ 1001 ➡ 1100 ➡

➡ 0110 ➡ 1011 ➡ 0101 ➡ 1010 ➡ 1101 ➡ 1110 ➡ 1111

- Here the LFSR goes through the sequence of 15 states before reentering the state 1111.

- Hence the system goes through
- $2^D - 1 = 2^4 - 1 = 15$ states.
- The maximum number of possible states for D = 4 is $2^D = 16$.
- However, the LFSR can be seen to go through all the possible states except one,
  - which is the 0000 or the all-zero state
- If the LFSR is initialized in the 0000 state, it continues in the 0000 state
- LFSR never gets out of the all zero states
- Hence, it is desired that the LFSR never enter the all-zero state.
- Such an LFSR circuit which goes through the maximum possible 2D − 1 states, without entering the all-zero state is termed :
  - Maximum-Length Shift Register circuit or maximum length LFSR
- The generated PN sequence is termed a *maximum-length PN sequence*.

1111 ➡ 0111 ➡ 0011 ➡ 0001 ➡ 1000 ➡ 0100 ➡ 0010 ➡ 1001 ➡ 1100 ➡

➡ 0110 ➡ 1011 ➡ 0101 ➡ 1010 ➡ 1101 ➡ 1110 ➡ 1111

- For the above LFSR, the maximum-length PN sequence is the sequence of outputs at X_{i-4} given as:

$$\text{PN Sequence} = 1\,1\,1\,1\,0\,0\,0\,1\,0\,0\,1\,1\,0\,1\,0$$

- We map the bits 1, 0 to the ***BPSK symbols***
  - -1, +1 to get the modulated PN sequence,

$$\text{PN sequence} = -1\ -1\ -1\ -1\ +1\ +1\ +1\ -1\ +1\ +1\ -1\ -1\ +1\ -1\ +1$$

Properties of PN Sequences

1. Balance Property

- Counting the number of −1 and +1 chips in the sequence, it is seen that the number of −1s is one more than the number of +1s. This is termed the balance property of the PN sequence.
- This fundamentally arises from the noiselike properties of PN sequences.
  - If we are generating random noise of +1, −1 chips, with P (Xi = +1) = P (Xi = −1) = ½
    - we expect to find on an average that half the chips are +1 and the rest are −1.
    - In the above case, however, as the total number of chips is an odd number, i.e., 15, it is not possible to have an exactly even number of +1, −1s. Hence, we observe that the number of +1, −1s is close to half the total number, i.e., eight −1s and seven +1s.
- Thus, the balance property basically supports the notion of a noiselike PN chip sequence.

2. Run-Length Property:

- A run is defined as a string of continuous values.

- There are a total of 8 runs in this PN sequences.
  - For instance, the first run −1, −1, −1, −1 is a run of length 4.
  - Thus, there is one run of length 4.

- Similarly, there is one run +1, +1, +1 of length 3, and
  - two runs of length 2, viz., −1, −1, +1, +1.

- Finally, it can also be seen that there are 4 runs of length 1,
  - viz., two runs of +1 and two runs of −1.

- Thus, there are a total of $2^{(D-1)} = 8$ runs.
  - Out of the 8 runs, it can be seen that:
    - 1, i.e., 1/8 of the runs are of length 3,
    - 1/4 of the runs are of length 2 and
    - 1/2 of the runs are of length 1.

- This is termed the run-length property of PN sequences and can be generalized as follows.

- Consider a maximal length PN sequence of length 2^D -1
- Out of the total number of runs in the sequence,
  - 1/2 of the runs are of length 1,
  - 1/4 of the runs are of length 2,
  - 1/8 of the runs are of length 3,
  - and so on.
- This is again in tune with the noiselike properties of PN sequences.

- For instance, consider a random IID sequence of +1, −1.
- In such a sequence, one would expect the average number of +1 or −1 to be half the total chips.
- Further, the number of strings +1, +1 or −1, −1, i.e., runs of length two would be expected to comprises 1/4 of the total runs.

- This arises since the probability of seeing two consecutive +1, +1 symbols is

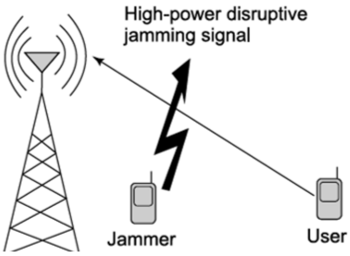
$$P(X_i = +1,\, X_{i+1} = +1) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

6. Explain in detail the Advantages of CDMA.

Advantages of CDMA

Advantage 1: Jammer Margin

- CDMA is inherently characterized by Jammer Suppression over other conventional cellular systems.
- A jammer is basically a malicious user in a communication network who transmits with a very high power to cause interference, thus leading to disruption of communication links.
- Jammers are of significant concern, especially in the context of highly secure communication systems such as those used for military and defense purposes.
- The effect of jammer suppression in a CDMA system can be understood as follows.
- Consider a communication system in which the signal x (n) of the power P is received in the presence of additive white Gaussian noise w (n) of power σ^2_w



- <u>The effect of jammer suppression in a CDMA system can be understood as follows:</u>
- Consider a communication system in which the signal x (n) of the power P is received in the presence of additive white Gaussian noise w (n) of power σ^2_w
- The baseband system model for this communication system can be expressed as:

$$y(n) = x(n) + w(n)$$

- Hence, the SNR at the receiver is SNR = $\frac{P}{\sigma_w^2}$.
- However, in the presence of a jamming signal x_j (n) of power P_j , the received signal y (n) is:

$$y(n) = x(n) + x_j(n) + w(n)$$

- Thus, the jammer interferes with the signal reception and the signal-to-interference-noise power ratio (SINR) can be calculated as :
    - SINR = $\frac{P}{P_j + \sigma_w^2}$

<u>From a CDMA Systems Perspective:</u> Consider now a CDMA system in which :

- The transmitted signal x (n) is a spread-spectrum signal.

- The SINR for a CDMA System will be: $\text{SINR} = \frac{P}{\frac{P_j}{N} + \frac{\sigma_w^2}{N}}$
- Thus, we observe that the ***<u>jamming power Pj</u>*** is suppressed by a factor of ***<u>N</u>*** .
- Further, as the spreading factor N increases, the jammer suppression increases, minimizing the impact of the jammer on the communication system.
    - This is termed jammer suppression in CDMA systems.
- Hence, CDMA which is inherently tolerant to jamming attacks which is highly attractive for defence applications.
- Also, the gain of N in this context of jammer suppression is also termed the jammer margin.
- Thus, the jammer margin is equal to N , i.e., the spreading length of the CDMA codes.

Advantage 2: Graceful Degradation

- Graceful degradation is another key property of CDMA-based wireless networks and also allows for much more efficient interference management, which ultimately leads to universal frequency reuse and higher spectral efficiency.

- Consider the expression for the SINR at the user 0 :

$$\begin{aligned}\mathrm{SINR} &= \frac{P_0}{\frac{P_1}{N} + \frac{P_2}{N} + \ldots + \frac{P_K}{N} + \frac{\sigma_n^2}{N}} \\ &= N \times \frac{P_0}{\sum_{k=0}^{K} P_k + \sigma_n^2}\end{aligned}$$

- At this point, assume that another user, i.e., a user with index (K + 1) joins the network.

- Let P_{K+1} denote the corresponding transmission power of this (K + 1)th user and a_{K+1}, C_{K+1} (n) denote his transmitted symbol and spreading code respectively.

- The SINR of the user 0 now changes to:

Advantage 2: Graceful Degradation

- The SINR of the user 0 now changes to:

$$\begin{aligned}\mathrm{SINR} &= \frac{P_0}{\frac{P_1}{N} + \frac{P_2}{N} + \ldots + \frac{P_K}{N} + \frac{P_{K+1}}{N} + \frac{\sigma_n^2}{N}} \\ &= N \times \frac{P_0}{\sum_{k=0}^{K+1} P_k + \sigma_n^2}\end{aligned}$$

- Thus, the addition of a new user (K + 1) with power P_{K+1} only causes an incremental interference of $\frac{P_{k+1}}{N}$ at the user 0.

- Further, in general, for any user i = (K + 1), the additional interference due to the introduction of this new user is $\frac{P_{k+1}}{N}$.

- Therefore, the addition of the new user (K + 1) does not adversely affect any single user.

- Rather, the additional interference caused by this new user is shared amongst all the existing users in the system leading to ***interference distribution***.

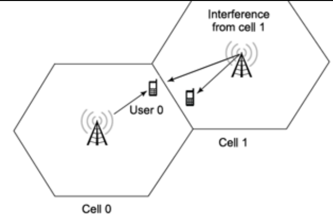
- Therefore, the addition of the new user (K + 1) does not adversely affect any single user.

- Rather, the additional interference caused by this new user is shared amongst all the existing users in the system leading to ***interference distribution***.

- This sharing of the interference by all the existing users leads to a graceful degradation of the SINR at each user.

- ***This is termed the graceful degradation property of CDMA systems.***

Advantage 3: Universal Frequency Reuse:

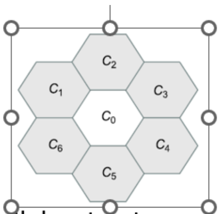


Intercell interference for the user 0 on the cell edge

- Consider a cellular network organized into cells as shown :
- . Consider two adjacent cells C0, C1
- Assume now that the same frequency f is allotted for transmission to users in both C0, C1.
- Let x0 (n) with power P0 denote the signal of the user on the frequency f in the cell 0,
- while x1 (n) with power P1 denotes the signal of the user in the cell 1.
- Since both the signals are being transmitted on the identical frequency f , they will interfere with each other.
- Hence the received signal y0 (n) at the user 0 is given as:

$$y_0(n) = \underbrace{x_0(n)}_{\text{Signal}} + \underbrace{x_1(n)}_{\text{Interferer from } C_1} + \underbrace{w(n)}_{\text{Noise}}$$

- Hence, the SINR at the user 0 is given as SINR = $\frac{P_0}{P_1+\sigma_W^2}$.
- This is similar to the jamming interference case described before.
- Thus, if the same frequency f is allocated in adjacent cells, it will cause heavy interference and degradation of user SINR results from adjacent cell interference.
- Thus, in a typical 1G or 2G cellular network such as GSM, only a fraction of the total available frequencies are allocated in each cell, carefully avoiding the allocation of the same frequency in adjacent cells.



- For instance, as can be seen from the hexagonal-lattice based cellular structure, each hexagonal cell has 6 neighbours.
- Hence, to avoid adjacent cell interference, any of the frequencies allocated to C_0 cannot be allocated to its neighbours C_1, C_2, ..., C_6.
- This holds true for all the cells in the network.
- Hence, only 1/7 of the total available frequency bands can be allocated to each cell.

- This factor 1/7 is termed the frequency-reuse factor of the cellular network.

- Thus, since only a fraction of the frequencies are used in the cell, the total spectral efficiency is proportional to the frequency-reuse factor, resulting in a rate which is 1/7 compared to that of using all the available bandwidth, since the capacity is linearly related to bandwidth.

<u>In the context of a CDMA network:</u>

- Assuming that the same frequency f is allotted for transmission to users in both C0, C1.
- However, let x0 (n) with power P0 is now transmitted on code C_0 (n), while x_1 (n) with power P_1 is transmitted in the cell 1 on the random code c_1 (n).

- Hence, now similar to the jammer scenario in a CDMA system, the interference caused by the user on the identical frequency f in the adjacent cell is now reduced by a factor of N to P1/N . Therefore, the SINR is now given as,

- Hence, now similar to the jammer scenario in a CDMA system, the interference caused by the user on the identical frequency f in the adjacent cell is now reduced by a factor of N to P1/N . Therefore, the SINR is now given as:

$$\mathrm{SINR} = \frac{P_0}{\frac{P_1}{N} + \frac{\sigma_w^2}{N}}$$

- Thus, the interference of each user is limited to a fraction 1/N of the interferer power

- This is a great advantage of CDMA, which implies that the same frequency bands can be used in all cells across the network.

- Another way of stating this is that the fraction of bands used in each cell is 1, i.e., all the bands.

- Therefore, this is termed universal frequency reuse or equivalently, as a cellular network with frequency reuse factor 1.

- Thus, compared to GSM, which uses only 1/7 of the frequency bands in each cell, CDMA can use all the available frequency bands in each cell.

- This right away leads to an increase of the spectral efficiency and resulting capacity by a factor of 7.

- Thus, CDMA-based cellular networks have a much higher capacity compared to conventional 1G and 2G cellular networks.

7.	With the help of a neat block diagram, explain the implementation of the OFDM transmitter and receiver using IFFT/FFT.

- Consider the MCM transmit signal s (t).
- It is band-limited to the bandwidth B (total bandwidth).

- The associated sampling time is Ts = 1/B.

- Consider now the composite MCM signal given:

$$y(t) = s(t) = \sum_i X_i e^{j2\pi f_i t} = \sum_i X_i e^{j2\pi i \frac{B}{N} t}$$

- The u th sample at time instant $t = uT_s = \frac{u}{B}$ is given as:

$$s(uT_s) = x(u) = \sum_i X_i e^{j2\pi i \frac{B}{N}\frac{u}{B}}$$

$$x(u) = \underbrace{\sum_i X_i e^{j2\pi \frac{iu}{N}}}_{\text{DFT}}$$

General definition of DFT:

$$F[n] = \sum_{k=0}^{N-1} f[k] e^{-j\frac{2\pi}{N}nk} \quad (n = 0 : N-1)$$

Prepared By: Dr. P. Susheelkumar S., Professor, ECE, CMRIT, Bangalore ||

Courtesy: Aditya Jagannatham, Author – Principles of Wireless Communication Systems

From the above equivalence we can say that:

- The RHS of expression represents a DFT

- Hence $x(u)$ in time domain should be the inverse DFT (IDFT) coefficient of the information symbols X (0) , X (1) , . . . , X (N − 1) at the u th time point.

- Thus, the Inverse Fast Fourier Transform (IFFT) can be conveniently employed to generate the sample (composite) MCM signal
  (which was proposed by Weinsten and Ebert in 1971)

- Hence it drastically reduces the complexity of implementing an OFDM system since it eliminates the need for the bank of modulators corresponding to the different subcarrier frequencies.

- This technique, where the MCM signal is generated by employing the IFFT operation is termed ***Orthogonal Frequency Division Multiplexing, or OFDM***.

Reception and Detection:

- At the receiver, to recover the information symbols, one can correspondingly employ an FFT operation.

- Schematic figures of the OFDM transmitter and receiver with the IFFT and FFT blocks are given below:

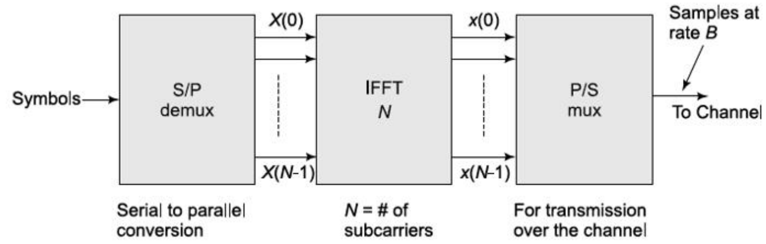


Figure 7.4 *OFDM transmitter schematic with IFFT*

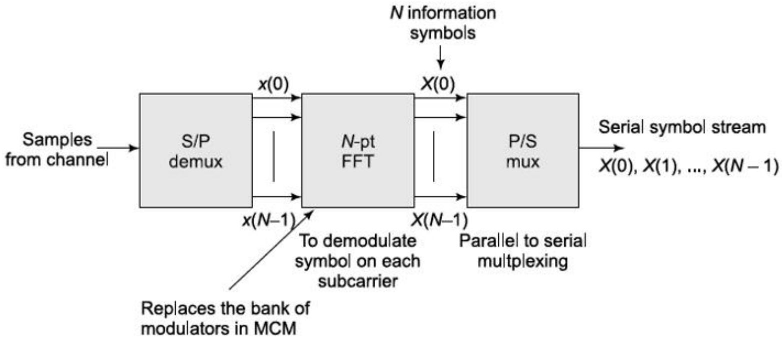


Figure 7.5 *OFDM receiver schematic with FFT*

8.	Detail the Correlation Properties of Random CDMA Spreading Sequences

Correlation Properties of Random CDMA Spreading Sequences:

- Let $C_k(i)$, $0 \leq i \leq N-1$ be the Chip sequence assigned for kth user
- Hence $P(C_k(i) = +1) = P(C_k(i) = -1) = 1/2$.
- Hence we have: $$\mathrm{E}\{c_k(i)\} = \frac{1}{2} \times (+1) + \frac{1}{2}(-1) = 0.$$
- And its advisable or important to choose sequences containing Independent Identically Distributed (IID) chips, and satisfying the property :

$$\mathrm{E}\{c_k(i)\, c_k(j)\} = \mathrm{E}\{c_k(i)\}\, \mathrm{E}\{c_k(j)\} = 0 \times 0 = 0$$

  - The above property implies that each chip $C_k(i)$ is uncorrelated with chip $C_k(j)$.
  - Similarly we can choose independent sequences for different users, that is:

$$\mathrm{E}\{c_k(i)\, c_l(j)\} = \mathrm{E}\{c_k(i)\}\, \mathrm{E}\{c_l(j)\}$$

- Let $r_{00}(k)$ denote the autocorrelation of the chip sequence of the user 0, corresponding to a lag k = 0. This can be expressed as

$$r_{00}(k) = \frac{1}{N} \sum_{i=0}^{N-1} c_0(i)\, c_0(i-k)$$

- The average or Expected Value of $r_{00}(k)$ can be expressed as:

$$\begin{aligned} \mathrm{E}\{r_{00}(k)\} &= \mathrm{E}\left\{\frac{1}{N} \sum_{i=0}^{N-1} c_0(i)\, c_0(i-k)\right\} \\ &= \frac{1}{N} \sum_{i=0}^{N-1} \mathrm{E}\{c_0(i)\, c_0(i-k)\} \\ &= \frac{1}{N} \sum_{i=0}^{N-1} \mathrm{E}\{c_0(i)\}\, \mathrm{E}\{c_0(i-k)\} \\ &= \frac{1}{N} \sum_{i=0}^{N-1} 0 = 0 \end{aligned}$$

Thus the average value or the expected value of the correlation $\mathrm{E}\{r_{00}(k)\}$ is zero for lags $k \neq 0$.

To compute the variance of the autocorrelation r_{00} (k), consider $r_{00}{}^2(k)$ given as:

$$r_{00}^2(k) = \frac{1}{N^2}\left(\sum_{i=0}^{N-1} c_0(i)\, c_0(i-k)\right)\left(\sum_{j=0}^{N-1} c_0(j)\, c_0(j-k)\right)$$

$$= \frac{1}{N^2}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} c_0(i)\, c_0(i-k)\, c_0(j)\, c_0(j-k)$$

- In case where $i \neq j, we\ have$

$$\mathrm{E}\{c_0(i)\, c_0(i-k)\, c_0(j)\, c_0(j-k)\} = \mathrm{E}\{c_0(i)\, c_0(i-k)\}\,\mathrm{E}\{c_0(j)\, c_0(j-k)\}$$

$$= \mathrm{E}\{c_0(i)\}\,\mathrm{E}\{c_0(i-k)\}\,\mathrm{E}\{c_0(j)\}\,\mathrm{E}\{c_0(j-k)$$

$$= 0$$

- Whereas if i=j, we have

$$\mathrm{E}\{c_0(i)\, c_0(i-k)\, c_0(j)\, c_0(j-k)\} = \mathrm{E}\{c_0(i)\, c_0(i-k)\, c_0(i)\, c_0(i-k)\}$$

$$= \mathrm{E}\left\{(c_0(i))^2\right\}\mathrm{E}\left\{(c_0(i-k))^2\right\}$$

$$= 1 \times 1 = 1$$

Hence the variance of $r_{00}(k)$: $\mathrm{E}\{r_{00}^2(k)\} = \frac{1}{N^2}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\mathrm{E}\{c_0(i)\, c_0(i-k)\, c_0(j)\, c_0(j-k)\}$

$$= \frac{1}{N^2}\sum_{i=0}^{N-1}\mathrm{E}\{c_0^2(i)\, c_0^2(i-k)\} = \frac{1}{N^2}\sum_{i=1}^{N-1} 1 = \frac{1}{N^2}\times N = \frac{1}{N}$$

Also the autocorrelation corresponding to a lag of k = 0 can be readily seen to be given as:

$$\mathrm{E}\{r_{00}(0)\} = \mathrm{E}\left\{\frac{1}{N}\sum_{i=0}^{N-1} c_0(i)\, c_0(i)\right\} = \frac{1}{N}\sum_{i=0}^{N-1}\mathrm{E}\{c_0^2(i)\}$$

$$= \frac{1}{N}\sum_{i=0}^{N-1} 1 = \frac{1}{N}\times N = 1$$

Summary:

1) The autocorrelation properties of the random spreading sequence for k = 0, is r_{00} (k) = 1.

2) For k $\neq$ 0, r_{00} (k) is a random variable with E {r_{00} (k)} = 0 and variance E $\{r_{00}{}^2(k)\}$ = 1/N .