

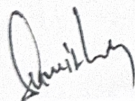
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Internal Assessment Test 1 – Sept. 2025

Subject/Code: Digital Communications /BEC503				
Date:29-09-2025	Duration: 90 mins	Max. Marks: 50	Semester: 05	Branch: ECE

Sl.	Answer any FIVE FULL Questions	Marks	CO	RBT																								
1	a) A pair of dice is rolled simultaneously. The outcome of the first dice is considered to be x and that of second as y. These events are defined as $A = \{(x, y) \text{ such that } (x+y) \text{ is exactly divisible by } 2\}$ $B = \{(x, y) \text{ such that } 3 \leq (x+y) \leq 7\}$ Which event contains more information? Justify b) The output of an information source consists of 128 symbols, 16 of which occur with probability of $1/32$ and remaining occurs with a probability of $1/224$. The source emits 1000 symbols/ sec. assuming that the symbols are chosen independently, find the rate of information of the source.	6+4	CO3	L3																								
2	i) Construct Binary codes for the source with probabilities shown using Shannon's binary encoding procedure. Compute the coding efficiency and redundancy. $P = \{0.32, 0.24, 0.21, 0.17, 0.04, 0.02\}$ Draw the code tree. ii) Determine which of the following codes shown in the table are prefix codes using Kraft's Mcmillan inequality. <table> <tr> <td>A</td> <td>B</td> <td>C</td> <td>D</td> </tr> <tr> <td>00</td> <td>11</td> <td>01</td> <td>101</td> </tr> <tr> <td>10</td> <td>101</td> <td>001</td> <td>1101</td> </tr> <tr> <td>110</td> <td>011</td> <td>000</td> <td>1010</td> </tr> <tr> <td>0110</td> <td>110</td> <td>100</td> <td>1100</td> </tr> <tr> <td>010</td> <td>1110</td> <td>110</td> <td>1111</td> </tr> </table>	A	B	C	D	00	11	01	101	10	101	001	1101	110	011	000	1010	0110	110	100	1100	010	1110	110	1111	6+4	CO3	L3
A	B	C	D																									
00	11	01	101																									
10	101	001	1101																									
110	011	000	1010																									
0110	110	100	1100																									
010	1110	110	1111																									
3	Construct Binary codes for the source with probabilities shown using Huffman's encoding procedure. Compute the coding efficiency and redundancy $P = \{0.02, 0.30, 0.02, 0.20, 0.03, 0.15, 0.06, 0.08, 0.10, 0.04\}$ Use AS LOW AS POSSIBLE	10	CO3	L3																								
4	For the given channel matrix, calculate all the entropies, given $p(x_1) = 0.5$, $p(x_2) = 0.2$ and $p(x_3) = 0.3$. <table> <tr> <td>$P(Y/X) =$</td> <td>0.4</td> <td>0.35</td> <td>0.25</td> </tr> <tr> <td></td> <td>0.1</td> <td>0.8</td> <td>0.1</td> </tr> <tr> <td></td> <td>0.2</td> <td>0.4</td> <td>0.4</td> </tr> </table> Find $H(X)$, $H(Y)$, $H(X, Y)$, $H(Y/X)$ and $I(X, Y)$ and justify your answers by verification.	$P(Y/X) =$	0.4	0.35	0.25		0.1	0.8	0.1		0.2	0.4	0.4	10	CO3	L3												
$P(Y/X) =$	0.4	0.35	0.25																									
	0.1	0.8	0.1																									
	0.2	0.4	0.4																									

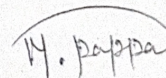
5	Consider a systematic (8,4) linear Block code for which the syndrome bits are $s_1 = r_1 + r_2 + r_4 + r_5$, $s_2 = r_1 + r_2 + r_3 + r_6$, $s_3 = r_1 + r_3 + r_4 + r_7$, $s_4 = r_2 + r_3 + r_4 + r_8$ Construct [G] and [H] matrices and find the minimum distance of this code by generating all the possible code words. Draw the syndrome calculation circuit.	10	CO4	L3
6	Design a feedback shift register for a (8,5) cyclic code with generator polynomial $g(x) = 1 + x + x^2 + x^3$. Find the code vector for the message 11011 in systematic form and Non-systematic form. List all the states of the register and verify the value using the division method.	10	CO5	L3
7	i) Consider the following source with Probabilities $S = \{A, B, C, D, E, F\}$ $P = \{0.4, 0.2, 0.2, 0.1, 0.08, 0.02\}$ Construct the Binary codewords using Shannon-Fano's Algorithm. ii) Define Entropy and list the properties of Entropy.	5+5	CO4	L3



CI



CCI



HOD

Digital Communications BEC-503

SAQ-1 Solutions (2025-2026)

Dr. Susmitha Alawar

1-a): Source S :
 $(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)$
 $(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)$
 $(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)$
 $(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)$
 $(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)$
 $(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)$ $\rightarrow 1M$

Event A $\rightarrow [(x,y) \text{ such that } (x+y) \text{ is exactly divisible by } 2]$

$$P(A) = \frac{18}{36} = \frac{1}{2} \rightarrow 1M$$

$$I(A) = \log\left(\frac{1}{P}\right) = \log\left(\frac{1}{\frac{1}{2}}\right) = \log_2 2 = 1 \text{ bit} \rightarrow 1M$$

Event B $\rightarrow [(x,y) \text{ such that } 3 \leq (x+y) \leq 7]$

$$P(B) = \frac{20}{36} \rightarrow 1M$$

$$I(B) = \log\left(\frac{1}{\frac{20}{36}}\right) = \log\left(\frac{36}{20}\right) = 0.847 \text{ bits} \rightarrow 1M$$

Event A Contains More information. $\rightarrow 1M.$

b) Total Symbols $128 = M.$

$$\text{Entropy } H(S) = \sum_{i=1}^M p_i \log \frac{1}{p_i}$$

$$= 16 \times \frac{1}{32} \log(32) + 112 \times \frac{1}{224} \log(224)$$

$$H(S) = 6.403 \text{ bits/sym} \rightarrow 3M$$

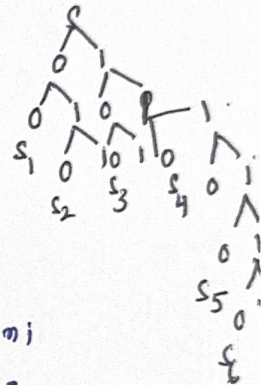
$$v_s = 1000 \text{ Sym/sec.}$$

$$R = H(S) \cdot v_s = 6.403 \times 1000 = 6403 \text{ bits/sec.} \rightarrow 1M$$

2.2)

Symbol	P_i	n_i	F_i	$(F_i)_{\text{binary}}$	codeword
s_1	0.32	2	0	0.00	00
s_2	0.24	3	0.32	0.0101	010
s_3	0.21	3	0.56	0.1000	100
s_4	0.17	3	0.77	0.1100	110
s_5	0.04	5	0.94	0.11110	11110
s_6	0.02	6	0.98	0.111110	111110

Code Tree — 1M



$$H(s) = 2.223 \text{ bits/sym} \rightarrow 1M$$

$$H_N = \bar{L} = 2.82 \text{ bits/sym} \rightarrow 1M$$

$$\eta = \frac{2.223}{2.82} = 78.8\% \rightarrow 1M$$

b)	A	n_i	B	n_i	C	n_i	D	n_i
	00	2	11	2	01	2	101	3
	10	2	101	3	001	3	1101	4
	110	3	011	3	000	3	1010	4
	0110	4	110	3	100	3	1100	4
	010	3	1110	4	110	3	1111	4

Kraft's

$$\sum_{i=1}^n 2^{-n_i} \leq 1$$

McMillan Inequality

Codeset A — $2^{-2} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-3} = 0.8125 \leq 1 \rightarrow 1M$

It is a prefix code.

Code set - B $\rightarrow$ 1M

$$2^{-2} + 2^{-3} + 2^{-3} + 2^{-3} + 2^{-4} = 0.6875 \leq 1$$

Though Kraft's McMillan Inequality is satisfied, the code word lengths are perfect to construct prefix set but the given set B is not prefix code.

code set c — 1M

$$2^{-2} + 2^{-3} + 2^{-3} + 2^{-3} + 2^{-3} = 0.75 \leq 1.$$

It is a prefix code.

Codeset D

$$2^{-3} + 2^{-4} + 2^{-4} + 2^{-4} + 2^{-4} = 1$$

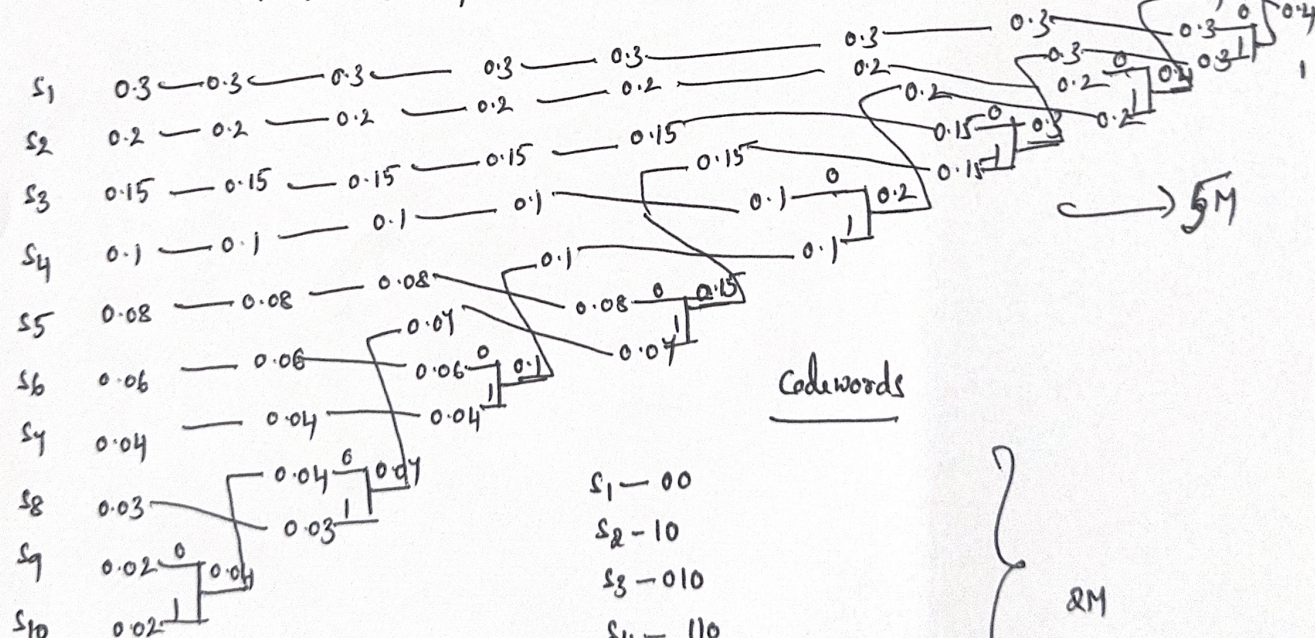
Nda puefia Cde.

③

$$P = \{0.02, 0.3, 0.02, 0.2, 0.03, 0.15, 0.06, 0.02, 0.10, 0.04\}$$

$$P = \{0.3, 0.2, 0.15, 0.1, 0.08, 0.06, 0.04, 0.03, 0.02, 0.02\}$$

$s_1 \quad s_2 \quad s_3 \quad s_4 \quad s_5 \quad s_6 \quad s_7 \quad s_8 \quad s_9 \quad s_{10}$



Code words

$S_1 - 00$
 $S_2 - 10$
 $S_3 - 010$
 $S_4 - 110$
 $S_5 - 0110$
 $S_6 - 1110$
 $S_7 - 1111$
 $S_8 - 01111$
 $S_9 - 011100$
 $S_{10} - 011101$

2M

$$H(x) = 2.826 \text{ bits/sym} \rightarrow 1M$$

$$H_N^A = 2.86 \rightarrow 1M$$

$$\eta = 98.6\%$$

$$\text{Redundancy} = 1.4\%$$

} 1M

(4)

$$P(Y/x) = \begin{matrix} & \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} 0.4 & 0.35 & 0.25 \\ 0.1 & 0.8 & 0.1 \\ 0.2 & 0.4 & 0.4 \end{bmatrix} \end{matrix}$$

$$P(x_1) = 0.5 \quad P(x_2) = 0.2 \quad P(x_3) = 0.3$$

$$H(x) = 1.485 \text{ bits/sym} \rightarrow 1M$$

$$P(xy) = \begin{bmatrix} 0.2 & 0.175 & 0.125 \\ 0.02 & 0.16 & 0.02 \\ 0.06 & 0.12 & 0.12 \end{bmatrix} \rightarrow 2M$$

$$P(y_1) = 0.28 \quad P(y_2) = 0.455 \quad P(y_3) = 0.265 \rightarrow 2M$$

$$H(y) = 1.538 \text{ bits/sym}$$

$$H(xy) = 2.905 \text{ bits/sym}$$

$$H\left(\frac{x}{y}\right) = 1.3670 \text{ bits/sym}$$

$$H\left(\frac{y}{x}\right) = 1.4203 \text{ bits/sym}$$

$$I(xy) = 0.1184$$

} 6M.

⑥

$$s_1 = x_1 + x_2 + x_4 + x_5$$

$$s_2 = x_1 + x_2 + x_3 + x_6$$

$$s_3 = x_1 + x_3 + x_4 + x_7$$

$$s_4 = x_2 + x_3 + x_4 + x_8$$

$$S = RH^T$$

$$n = 8$$

$$n - k = 4$$

$$g - k = 4$$

$$k = 4$$

(8,4) code

$$S = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8] \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \rightarrow IM$$

$$A = [S_k \ P] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \rightarrow IM$$

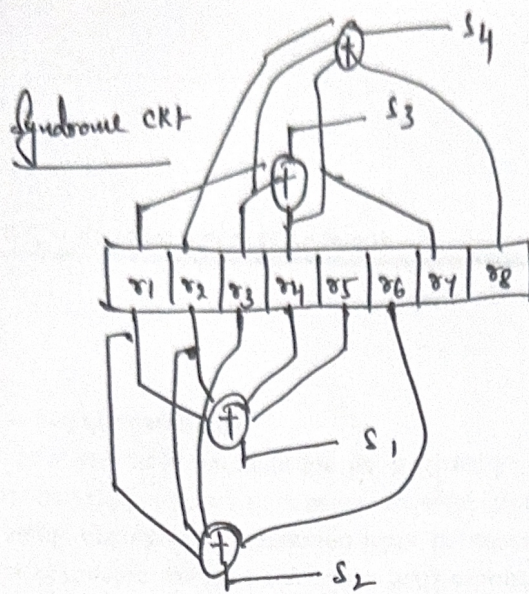
$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow IM$$

$$\begin{array}{lll} 0000 & - & 0000 \ 0000 \rightarrow 0 \\ 0001 & - & 0001 \ 1011 \rightarrow 4 \\ 0010 & - & 0010 \ 0111 \rightarrow 4 \\ 0011 & - & 0011 \ 1100 \rightarrow 4 \\ 0100 & - & 0100 \ 1101 \rightarrow 4 \\ 0101 & - & 0101 \ 0110 \rightarrow 4 \\ 0110 & - & 0110 \ 1010 \rightarrow 4 \\ 0111 & - & 0111 \ 0001 \rightarrow 4 \end{array}$$

$$\begin{array}{lll} 1000 & - & 1000 \ 1110 \rightarrow 4 \\ 1001 & - & 1001 \ 0110 \rightarrow 4 \\ 1010 & - & 1010 \ 1001 \rightarrow 4 \\ 1011 & - & 1011 \ 0010 \rightarrow 4 \\ 1100 & - & 1100 \ 0011 \rightarrow 4 \\ 1101 & - & 1101 \ 1000 \rightarrow 4 \\ 1110 & - & 1110 \ 0100 \rightarrow 4 \\ 1111 & - & 1111 \ 1111 \rightarrow 8 \end{array}$$

5M

$$d_{min} = 4 \rightarrow IM$$



→ 1M.

6. $(8,5)$ cyclic code.

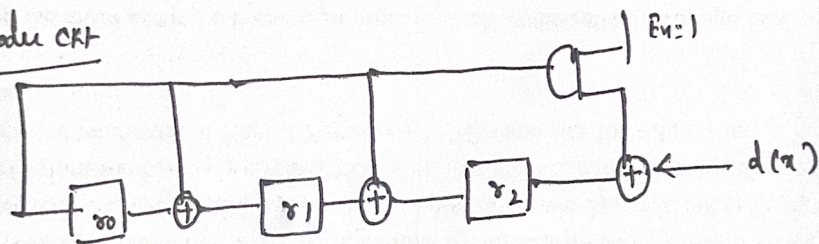
$$g(x) = 1 + x + x^2 + x^3$$

i/p → 11011

$$: 1 + g_1x + g_2x^2 + g_3x^3$$

$$g_1 = 1 \quad g_2 = 1$$

Encoder CRT



Cycle	i/p	r_0	r_1	r_2
		0	0	0
1	1	1	1	1
2	1	0	1	1
3	0	1	1	0
4	1	1	0	0
5	1	1	0	1

5M

Systematic cyclic code vector — 10111011

Non systematic cyclic code vector

$$V(x) = d(x) \cdot g(x)$$

$$= (1+x+x^3+x^4) (1+x+x^2+x^3)$$

$$= 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11}$$

$$V(x) = 1 + x^3 + x^4 + x^7$$

$$10011001 \rightarrow 3M$$

Division Method

$$R(x) = \frac{x^{n-k} \cdot d(x)}{g(x)}$$

$$\frac{x^3 (1+x+x^3+x^4)}{1+x+x^2+x^3} = \frac{x^3 + x^4 + x^6 + x^7}{x^3 + x^2 + x + 1}$$

$$\begin{array}{r} x^3 + x^2 + x + 1 \overline{) x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1} \\ \underline{x^7 + x^6 + x^5 + x^4} \\ x^3 \\ \underline{x^3 + x^2 + x} \\ x^2 + x \\ \underline{x^2 + x} \\ x \\ \underline{x} \\ 1 \end{array} \quad \left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} 2M$$

$$R(x) \rightarrow 1 + x^2$$

$$\rightarrow 101$$

Code vector — 10111001 —> Hence verified.

4. i)

$$\begin{array}{r}
 0.4 \quad 0 \quad 0 \\
 \hline
 0.2 \quad 0 \quad 1 \\
 \hline
 0.2 \quad 1 \quad 0 \\
 \hline
 0.1 \quad 1 \quad 1 \quad 0 \\
 \hline
 0.08 \quad 1 \quad 1 \quad 1 \quad 0 \\
 \hline
 0.02 \quad 1 \quad 1 \quad 1 \quad 1
 \end{array}$$

A - 00

B - 01

C - 10

D - 110

E - 1110

F - 1111

} 5M

ii) — Entropy def - 1M
 properties - 4M