

Fourth Semester B.E. Degree Examination, June/July 2025
Engineering Statistics and Linear Algebra

18EC44

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. The pdf for the random variable X is given by

$$f_X(x) = \begin{cases} 0.5303\sqrt{x}, & 0 < x < 2 \\ 0, & \text{Otherwise} \end{cases}$$

Find:

- i) The mean
 ii) The mean of the square
 iii) The variance of the random variable X.

(06 Marks)

- b. The following is the Pdf for the random variable U

$$f_U(u) = \begin{cases} c \exp\left(\frac{-u}{2}\right), & 0 \leq u < 1 \\ 0, & \text{Otherwise} \end{cases}$$

Find the value of C and evaluate $F_U(0.5)$.

(06 Marks)

- c. Define a binomial random variable. Obtain the characteristic function of a binomial distribution and hence find mean and variance using the characteristic function. (08 Marks)

OR

- 2 a. A random variable X has a poisson distribution with a mean of 3, find $P(1 \leq x \leq 3)$.

(06 Marks)

- b. The probability distribution of a discrete random variable is as shown below:

K	-0.25	0	1	2	3.75
P(X = K)	0.2	C	0.4	0.1	2C

Find the value of C and P $\{(X > 1) / (X \geq 0)\}$.

(06 Marks)

- c. Derive mean, variance and characteristic function for exponential distributed random variable. (08 Marks)

Module-2

- 3 a. The joint Pdf $f_{XY}(x, y) = c$, a constant, when $(0 < x < 3)$ and $(0 < y < 3)$ and is 0 otherwise.

- i) What is the value of the constant c?
 ii) What are the Pdfs for X and Y?
 iii) What is $F_{XY}(x, \infty)$ when $(0 < x < 3)$ and $(0 < y < 3)$?

(06 Marks)

- b. The random variable X is uniformly distributed between ± 1 .

- i) Find the mean and variance of Y if $Y = \frac{1}{37} \sum_{i=1}^{37} X_i$
 ii) Find the mean and variance of Z if $Z = \sum_{i=1}^{37} X_i$

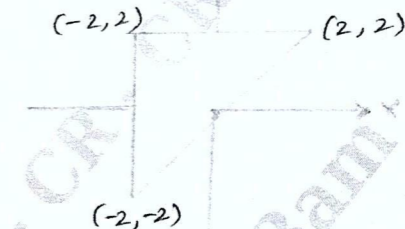
In these two sums, the x_i 's are IID.

(06 Marks)

- c. X and Y are correlated random variables with a correlation coefficient of $\rho = 0.6$, $\mu_X = 3$, $\text{Var}[X] = 49$, $\mu_Y = 144$, $\text{Var}[Y] = 144$. The random variables U and V are obtained using $U = X + CY$ and $V = X - CY$. What values can C have if U and V are uncorrelated? (08 Marks)

OR

- 4 a. Prove that sum of the two independent Gaussian random variables is also Gaussian. (06 Marks)
 b. Shown in Fig.Q.4(b) is a region in the x, y plane where bivariate Pdf $f_{xy}(x, y) = c$, elsewhere the Pdf is 0.



- i) Find the value of c
 ii) Evaluate $F_{xy}(1, 1)$

Fig.Q.4(b)

(06 Marks)

- c. Write a note on:

- i) Student's t random variable
 ii) Chi-square random variable.

(08 Marks)

Module-3

- 5 a. For the random process $X(t) = A \cos(\omega_c t + \theta)$, A and ω_c are constants θ is a random variable, uniformly distributed between $\pm \pi$. Show that this process is ergodic. (06 Marks)
 b. Define random process and discuss the terms Strict-Sense Stationary (SSS) and Wide Sense Stationary WSS associated with a random process. (06 Marks)
 c. The random process $X(t)$ has the auto correlation function

$$R_x(t) = \begin{cases} 10 \left(1 - \frac{|t|}{\tau_N} \right), & -\tau_N \leq t \leq \tau_N \\ 0, & \text{Otherwise} \end{cases}$$

CMRIT LIBRARY
 BANGALORE - 560 037

The random process Y(t) is independent of X(t) and has the auto correlation function

$$R_Y(\tau) = \frac{13 \sin(W_B \tau)}{W_B \tau}, \text{ where } \frac{2\pi}{W_B} \geq \tau_N. \text{ The random process } Z(t) = X(t) + Y(t). \text{ For } Z(t),$$

find its auto correlation function, its total power, its dc power.

(08 Marks)

OR

- 6 a. $x(t)$ and $y(t)$ are independent, jointly wide-sense stationary random process given by $x(t) = A \cos(\omega_1 t + \theta_1)$ and $y(t) = B \cos(\omega_2 t + \theta_2)$. If $W(t) = X(t) Y(t)$ then find ACF $R_w(\tau)$. (06 Marks)
- b. The random process $X(t)$ and $Y(t)$ are jointly wide-sense stationary and they are independent. Given that $W(t) = X(t) Y(t)$ and $R_X(\tau) = 10 \exp\left(\frac{-|\tau|}{3}\right)$, $-\infty < \tau < \infty$,

$$R_y(\tau) = \begin{cases} 11 \left(\frac{3-|\tau|}{3} \right), & -3 \leq \tau < 3 \\ 0, & \text{Otherwise} \end{cases}$$

For $W(t)$, find its autocorrelation function, its total power, its dc power and its ac power.

(06 Marks)

- c. Suppose that the PSD input to a linear system is $S_X(\omega) = k$. The cross-correlation of the input $X(t)$ with the output $Y(t)$ of the linear system is found to be

$$R_{XY}(\tau) = K \begin{cases} e^{-\tau} + 3e^{-2\tau}, & \tau \geq 0 \\ 0, & \tau < 0 \end{cases}$$

What is the power filter function $|H(3\omega)|^2$?

(08 Marks)

Module-4

- 7 a. Determine whether the vectors $(1, 1, 1)$, $(2, 2, 0)$ and $(3, 0, 0)$ is linearly independent or dependent. (06 Marks)
- b. Find the basis and dimension of the subspace $V_3(R)$ spanned by the vectors $(1, -2, 3)$, $(1, -3, 4)$ and $(-1, 1, -2)$. (06 Marks)
- c. If $u_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$; $u_2 = \begin{pmatrix} -1/4 \\ 1 \\ 1/4 \end{pmatrix}$; $u_3 = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$

Show that u_1, u_2, u_3 are pairwise orthogonal vectors. Hence find the orthonormal vectors of u_1, u_2, u_3 . (08 Marks)

OR

- 8 a. Find the column space and null space of

$$A = \begin{bmatrix} 1 & -5 & 7 & 0 \\ 1 & -1 & 0 & 0 \\ -2 & 10 & -14 & 0 \end{bmatrix}$$

(06 Marks)

- b. Find a least squares solution of the inconsistent system $AX = b$ for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$; $b = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}$.

(06 Marks)

- c. Let $W = \text{span} \{X_1, X_2\}$ where

$$X_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}, X_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}. \text{ Construct an orthogonal basis } [v_1, v_2] \text{ for } w.$$

(08 Marks)

Module-5

- 9 a. Reduce the matrix A to upper triangular matrix U and find the determinant by product of pivots,

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}$$

(05 Marks)

- b. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

(05 Marks)

- c. Diagonalize the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(10 Marks)

OR

- 10 a. Write the properties of determinants. (05 Marks)
- b. Test the matrix for positive definiteness of

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(05 Marks)

CMRIT LIBRARY
BANGALORE - 560 037

- c. Find a singular value decomposition of $A = \begin{pmatrix} 2 & -1 \\ 2 & 2 \end{pmatrix}$.

(10 Marks)
