

Third Semester B.E./B.Tech. Degree Examination, June/July 2025

Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the Laplace transform of $t^5 e^{4t} \cosh 3t + 5t + \frac{\sin 2t}{t}$ (06 Marks)

- b. Express $f(t) = \begin{cases} \cos t & \text{for } 0 < t \leq \pi \\ 1 & \text{for } \pi < t \leq 2\pi \\ \sin t & \text{for } \pi > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. (07 Marks)

- c. Using convolution theorem find the inverse Laplace transform of $\frac{1}{s^3(s^2+1)}$. (07 Marks)

OR

- 2 a. Find the inverse Laplace transform of $\frac{3s+2}{s^2-s-2}$ (06 Marks)

- b. If $f(t) = \begin{cases} t & \text{for } 0 \leq t \leq \pi \\ 2\pi - t & \text{for } \pi \leq t \leq 2\pi \end{cases}$ and $f(t+2\pi) = f(t)$ then show that $L\{f(t)\} = \frac{1}{s^2} \tanh\left(\frac{\pi s}{2}\right)$ (07 Marks)

- c. Use Laplace transform to solve $y'' + 6y' + 9y = 12t^2 e^{-3t}$ under the conditions $y(0) = y'(0) = 0$. (07 Marks)

Module-2

- 3 a. Expand $f(x) = \frac{\pi-x}{2}$ in $0 \leq x \leq 2\pi$ as Fourier series expansion. (06 Marks)

- b. Obtain the Fourier series for the function

$$f(x) = \begin{cases} 1 + \frac{4x}{3} & \text{in } -\frac{3}{2} < x < 0 \\ 1 - \frac{4x}{3} & \text{in } 0 < x < \frac{3}{2} \end{cases},$$

hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ (07 Marks)

- c. Expand $f(x) = \sin x$ in Fourier half range cosine series over the interval $(0, \pi)$. (07 Marks)

OR

- 4 a. Find the Fourier series expansion of x^2 in $-\pi \leq x \leq \pi$. (06 Marks)

- b. Obtain the Fourier half range sine series for the function

$$f(x) = \begin{cases} \frac{1}{4} - x & \text{in } 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{in } \frac{1}{2} < x < 1 \end{cases}$$
 (07 Marks)

- c. A function $f(x)$ of period 2π is specified by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	7.9	7.2	3.6	0.5	0.9	6.8	7.9

Obtain the Fourier series for $f(x)$ upto the first harmonic. (07 Marks)

Module-3

- 5 a. Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{in } |x| \leq 1 \\ 0 & \text{in } |x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. (06 Marks)

- b. Find the Fourier sine and cosine transform of $f(x) = e^{-ax}$ for $a > 0$ and $x > 0$. (07 Marks)

- c. Obtain the inverse z-transform of $\frac{3z^2 + z}{(5z-1)(5z+2)}$ (07 Marks)

OR

- 6 a. Find the Fourier transform of the function $f(x) = \begin{cases} 1-|x| & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$, hence evaluate $\int_0^\infty \frac{\sin^2 x}{x^2} dx$. (06 Marks)

- b. Find the z-transform of (i) $\cos n\theta$ (ii) $\sin(n\theta)$ (07 Marks)

- c. Solve $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ with $u_0 = u_1 = 0$ using z-transform. (07 Marks)

Module-4

- 7 a. Classify the second order partial differential equations :
 (i) $u_{xx} + 2u_{xy} + u_{yy} = 0$ (ii) $(x+1)u_{xx} - 2(x+2)u_{xy} + (3+x)u_{yy} = 0$
 (iii) $y^2 u_{xx} + u_{yy} + u_x^2 + u_y^2 + 7 = 0$ (iv) $(1+x^2)u_{xx} + (5+2x^2)u_{xt} + (4+x^2)u_{tt} = 0$ (10 Marks)

- b. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ [Laplace equation] for the following square mesh with boundary values as shown in Fig.Q7(b). Use Leibmann's method for 1st iteration.

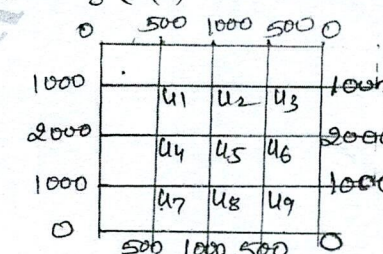


Fig.Q7(b)

(10 Marks)

OR

- 8 a. Solve the wave equation

$\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$, subject to $u(0, t) = u(4, t) = 0$, $\frac{\partial u}{\partial t}(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ by taking step length in x , $h = 1$. (10 Marks)

- b. Solve $u_t = u_{xx}$ subject to the conditions $u(0, t) = u(1, t) = 0$, $u(x, 0) = \sin(\pi x)$, $0 \leq t \leq 0.1$ by taking $h = 0.2$, by applying Bendre-Schmidt explicit formula, hence find
(i) $u(0.2, 0.04)$ (ii) $u(0.6, 0.06)$ (10 Marks)

Module-5

- 9 a. By fourth order Runge-Kutta method, solve

$\frac{d^2 y}{dx^2} = x \left(\frac{dy}{dx} \right)^2 - y^2$ for $x = 0.2$, correct to four decimal places using the initial conditions $y = 1$ and $\frac{dy}{dx} = 0$, when $x = 0$. (06 Marks)

- b. Derive Euler's equation in the standard form

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0 \quad (07 \text{ Marks})$$

- c. Show that the equation of the curve joining the points
- $(0, 1)$
- and
- $(1, 2)$
- for which

$$I = \int_0^1 \sqrt{1 + (y')^2} dx \text{ is extremum, is a straight line.} \quad (07 \text{ Marks})$$

OR

- 10 a. Apply Milne's method to find $y(0.4)$ from the $y'' + xy' + y = 0$ and initial values as $y(0) = 1$, $y(0.1) = 0.995$, $y(0.2) = 0.9801$, $y(0.3) = 0.956$, $y'(0) = 0$, $y'(0.1) = -0.0995$, $y'(0.2) = -0.196$, $y'(0.3) = -0.2867$. (07 Marks)

- b. Prove that geodesics on a plane are straight line.

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(06 Marks)

- c. Find the curve on which the functional $\int_0^{\pi/2} [(y')^2 - y^2 + 2xy] dx$ with $y(0) = y(\pi/2) = 0$ can be extremised. (07 Marks)
